Handbook on the Physics and Chemistry of Rare Earths, volume 34

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Edited by: K. A. Gschneidner, Jr., J.-C. G. Bünzli, and V. K. Pecharsky
PREFACE

Karl A. GSCHEIDNER Jr., Jean-Claude G. BÜNZLI, and Vitalij K. PECHARSKY

These elements perplex us in our researches [sic], baffle us in our speculations, and haunt us in our very dreams. They stretch like an unknown sea before us – mocking, mystifying, and murmuring strange revelations and possibilities.

Sir William Crookes (February 16, 1887)

This volume of the Handbook on the Physics and Chemistry of Rare Earths adds five new chapters to the science of rare earths, compiled by researchers renowned in their respective fields. Volume 34 opens with an overview of ternary intermetallic systems containing rare earths, transition metals and indium (Chapter 218) followed by an assessment of up-to-date understanding of the interplay between order, magnetism and superconductivity of intermetallic compounds formed by rare earth and actinide metals (Chapter 219). Switching from metals to complex compounds of rare earths, Chapter 220 is dedicated to molecular structural studies using circularly polarized luminescence spectroscopy of lanthanide systems, while Chapter 221 examines rare-earth metal-organic frameworks, also known as coordination polymers, which are expected to have many practical applications in the future. A review discussing remarkable catalytic activity of rare earths in site-selective hydrolysis of deoxyribonucleic acid (DNA) and ribonucleic acid, or RNA (Chapter 222) completes this book.

A new feature to the Handbook series has been included in this volume. An abbreviated subject index to the contents of the 222 chapters published to date. This index appears right after the Contents of Volumes 1–33. We kept its size to a minimum by including only the most general terms and plan to update the index with every new volume.

Chapter 218. Rare Earth-Transition Metal-Indides
by Yaroslav Kalychak, Vasyl’ Zaremba
Ivan Franko National University, Lviv, Ukraine
Rainer Pöttgen, Mar’yana Lukachuk, and Rolf-Dieter Hoffmann
Westfälische Wilhelms-Universität, Münster, Germany

The opening chapter of the Handbook reviews synthesis conditions, isothermal sections of phase diagrams, crystallography and basic physical properties of ternary intermetallic compounds consisting of rare-earth metals, transition metals and indium. Some of these intermetallic compounds – indides (so named because indium is usually the most electronegative
component in these ternary systems) – have been known for nearly forty years, others recently emerged as hot subjects for condensed matter physics research, especially with the discovery of heavy fermion superconductivity in a few cerium-based indium-rich intermetallics. Today, the rare earth-transition metal indides have been transformed into a playground for solid state chemists and condensed matter physicists because chemical bonding and crystallography, and therefore, physical properties of these compounds can be controlled by small changes of stoichiometry. Many systems are known to form in excess of ten individual chemical compounds. Although for the most part dominated by metallic bonding, indium-rich alloys often form well-defined clusters of indium atoms, and also a variety of low-dimensional transition metal-indium networks and chains, which just about ensures future discoveries of unusual physics, especially with respect to electronic transport and magnetic behaviors. In addition to a critical review of the up-to-date literature on the subject, Kalychak and co-authors offer the reader a challenging but very interesting subject of structure-property relationships.

Chapter 219. Unconventional Superconductivity and Magnetism in Lanthanide and Actinide Intermetallic Compounds
by Peter Thalmeier
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Gertrud Zwicknagl
Technische Universität, Braunschweig, Germany

The f-elements in general and rare earth metals in particular are celebrated for the non-conventional superconductivity, and thus this chapter of the Handbook revisits the last decade
of research uncovering some of the mysteries of the superconducting state, especially those related to heavy fermion superconductivity and co-existence of the superconducting and exotic magnetically ordered states. Thalmeier and Zwicknagl first introduce the reader to the basics by reviewing the nature of heavy quasiparticles, Kondo lattice, Fermi liquid, and spin fluctuation theory, followed by the consideration of order parameters in strongly correlated electron systems and experimental techniques most effective in detecting relevant order parameters. Justifiably, the majority of this chapter is dedicated to both the well-known unconventional superconductors, such as CeCu$_2$Si$_2$, UPt$_3$, UBe$_{13}$, and others, as well as to the newly discovered CeCoIn$_5$ (hence complementing Chapter 218 in this volume), holmium- and erbium-nickel borocarbides, and skutterides. The latter is a great addition to Chapter 212 that was written by Brian Sales and appeared last year in volume 33 of the Handbook.

Chapter 220. Circularly Polarized Luminescence Spectroscopy from Lanthanide Systems  
by James P. Riehl and Gilles Muller  
University of Minnesota, Duluth, Minnesota, USA

The phenomenon of circularly polarized luminescence, i.e., emission of partially polarized light during f–f transitions of the rare-earth ions in optically active, chiral molecules (molecules existing in two different modifications whose structures cannot be superimposed over one another), is reviewed by Riehl and Muller. They begin with the general theory, thus giving the reader a background needed to better understand experimental techniques that are used today to study the effect, followed by a critical assessment of the important structural information that has been and can be deduced while using the technique. The authors pay special attention to details and also provide their views on the future of the method as a reliable tool for probing molecular stereochemistry of lanthanide complexes both in pure forms and in mixtures.
Chapter 221. Lanthanide-Containing Coordination Polymers
by Olivier Guillou and Carole Daiguebonne
Institut National des Sciences Appliquées Rennes, France

Metal-organic frameworks, also known as coordination polymers, hold a great potential as working bodies for opto-electronic and magnetic devices, microporous materials for a variety of uses, such as size- and shape-selective separations and catalyst support, and more recently, as hydrogen storage materials. A variety of physical properties that are largely defined by the 4-f electrons of lanthanide ions combined with a similarity of their chemical properties offer an unprecedented flexibility in designing coordination polymers that are chemically similar but functionally different. The subject of this chapter is truly one of the emerging fields in rare-earth science. Guillou and Daiguebonne begin with a brief introduction to transition metal-based frameworks and then quickly move to the core by reviewing various rare earth-containing coordination polymers based on different ligands, paying special attention to those compounds that were
synthesized with a practical application in mind. Polymeric complexes based on benzene-poly-carboxylates are described in the greatest detail.

Chapter 222. Cutting DNA and RNA
by Makoto Komiyama
University of Tokyo, Tokyo, Japan

Likely the best known application of rare earths, which is related to living organisms, are the remarkably effective contrast agents in magnetic resonance imaging, yet this chapter shows that site-selective cutting (scission or hydrolysis) of deoxyribonucleic acid (DNA) and ribonucleic acid (RNA) may be the next big use for lanthanides in genetic engineering. Both DNA and RNA are biological polymers that store genetic information and pass it on via synthesis of protein molecules. According to Komiyama, future biotechnology may well rely on the use of rare-earth ions as unique catalysts that can slice DNA and RNA in order to allow their reprogramming, and thus lead to more effective bioengineered processes. The author begins with a historical overview of how the catalytic effect of Ce$^{4+}$ in the hydrolysis of DNA was discovered, and then gives the current state-of-the-art in understanding of the hydrolysis mechanism. Similarly, he describes catalytic effect of lanthanide ions in hydrolysis of RNA, followed by a review of various strategies that can be employed to improve site-selectivity in the hydrolysis of both DNA and RNA.
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RARE EARTH–TRANSITION METAL–INDIDES

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1. Introduction

Intermetallic rare earth (R)–transition metal (T)–indium compounds, i.e. indides have attracted considerable interest in recent years due to their peculiar crystal chemistry and intriguing physical properties. More than thirty years ago, the ternary indides with uranium and thorium as electropositive component, UTIn (T = Pd, Pt) and ThTIn (T = Ni, Pd, Pt), have been synthesized by Dwight et al. (1968). The first series of rare earth metal-based ternary indides, i.e. RNiIn, RRhIn, RPdIn, and RPtIn, have been synthesized in Genova by the Ferro group (Ferro et al., 1974a, 1974b) and characterized with respect to metallography and X-ray powder diffraction.

More detailed investigations of the ternary systems R–T–In including construction of the complete isothermal sections, determination of crystal structures, and chemical bonding have been performed in the last 20 years by the groups of Kalychak in Lviv and Pöttgen in Münster. The phase relations in the systems with cobalt, nickel, and copper as transition metal component have recently been reviewed by Kalychak (1997, 1998a, 1999a). Especially the nickel-based systems are extremely rich in compounds. With the early rare earth metals, more than ten different intermetallics form in these systems. Physical properties of indides have been investigated by various groups of researchers.

The crystal chemistry and chemical bonding of the R–T–In compounds strongly depend on the composition. The rare earth metal-rich compounds exhibit typical intermetallic structures with relatively large coordination numbers, as found in close packed arrangements. Compounds with a high transition metal content show a tendency for transition metal cluster formation (Kalyshak et al., 2001). In the indium rich parts of the ternary phase diagrams, compounds with interesting indium substructures exist (Hoffmann and Pöttgen, 2000; Hoffmann et al., 2000). In the case of a low rare earth metal content, the transition metal and indium atoms form two- or three-dimensional [T\text{In}_y] networks, in which the rare earth metal atoms fill cages or channels. Many of these crystal structures are very complicated, however, in some cases they can easily be described by an intergrowth of various slabs of simple binary and/or ternary structure types.
The rare earth transition metal indides have not only attracted interest in the chemists community, but also the physicists have intensively investigated such materials, mainly with respect to the largely varying magnetic and electrical properties. The primary interest concerns ferromagnetic, intermediate-valent, and heavy fermion materials. Prominent examples are the 22 K ferromagnet EuRhIn (Pöttgen et al., 1999c), mixed-valent CeRhIn (Adroja et al., 1989), valence-fluctuating CeNiIn (Fujii et al., 1989; Kurisu et al., 1990), or the intermediate heavy-fermion system Ce$_2$Ni$_6$In$_{11}$ (Tang et al., 1995). Gadolinium based indides are currently under investigation due to their potential technological applications for magnetic refrigeration (Canepa et al., 2002b). Finally, it is worthwhile to mention the very recent investigations on the new heavy fermion materials Ce$_2$Ti$_1$In$_8$ and Ce$_2$Ti$_1$In$_5$ ($T =$ Co, Rh, Ir). These indides have been thoroughly investigated in the last two years (Hegger et al., 2000; Moshopoulou et al., 2001). Precise and detailed physical property investigations have been performed on large single crystals.

With an increasing interest in hydrogen storage materials, various ternary indides have been studied with respect to their hydride formation. The hydrogen absorption has a strong influence on the physical properties. A prominent example is valence fluctuating CeNiIn that exhibits ferromagnetic ordering upon hydrogenation to CeNiInH$_{1.8}$ (Chevalier et al., 2002).

In this chapter, we review the synthesis techniques and the crystal chemistry of rare earth–transition metal–indides. Furthermore, we focus on phase relations and physical properties and discuss structure-property relationships.

2. Synthesis conditions

Rare earth–transition metal–indides can be prepared directly from the elements. In view of the highly electropositive character of the rare earth metals, the use of large rare earth metal pieces is highly recommended in order to avoid surface contaminations, mostly by oxides. The air (humidity) sensitive rare earth metals lanthanum, cerium, praseodymium, neodymium, samarium, europium, and ytterbium must be kept in Schlenk tubes under dried argon. The argon can easily be dried by standard procedures (Brauer, 1981; Brauer and Herrmann, 2000). The transition metals are used in the form of pieces or wires, while most noble metals are available as powders or foils. Indium is commercially available in the form of tear drops or granules. These are very soft and ductile and can be easily cut mechanically. The reactions of the rare earth metals with a transition metal and indium are strongly exothermic. The rare earth metals are often available in the form of sublimed ingots. Direct use of such ingots can lead to shattering during the reaction. In many cases, it is useful to arc-melt pieces of the rare earth metal first in order to reduce the shattering.

A very fast and efficient synthesis technique is arc-melting. This technique can be used for most rare earth–transition metal–indides. On the laboratory scale, arc-melting can easily be performed on water-cooled copper chills or within a water-cooled conical copper crucible (Pöttgen et al., 1999b). A non-consumable cerium dioxide doped tungsten electrode is used and the welding generator should be equipped with a high-frequency supply. This avoids
touching the crucible or the sample allowing a contact-free ignition of the arc. With a miniaturized arc-melting furnace, samples up to 2 g can be synthesized. Larger copper crucibles can be used in order to melt small bars for physical property investigations. If only polycrystalline samples are obtained by arc-melting, annealing of the arc-melted buttons in a water-cooled silica ampoule in a high-frequency furnace (Niepmann et al., 2000) can significantly increase the crystallinity. This annealing technique is especially useful for high-melting temperature products.

Many of the ternary indides directly crystallize from the melt. Often small single crystals already form during the solidification process. As an example, in fig. 1 we present a scanning electron micrograph of an arc-melted button from the solid solution TmNi$_{1+x}$In$_{1-x}$ (Lukachuk et al., 2003). In most cases, however, the arc-melted buttons are annealed in evacuated, sealed silica tubes at temperatures between 700 and 1300 K in order to achieve homogenization. The samples can be wrapped in tantalum or zirconium foil in order to avoid a reaction with the ampoule material. These annealing procedures are very important for the study of the isothermal sections of the phase diagrams. Compounds that form peritectically can only be obtained this way as single phase materials.

If one of the reaction components has a comparatively low boiling temperature, significant evaporation can occur during the arc-melting procedures. This is especially the case for the rare earth metals samarium (boils at 2050 K), europium (1870 K), thulium (2000 K), and

![Fig. 1. Scanning electron micrograph of an arc-melted button taken from the solid solution TmNi$_{1+x}$In$_{1-x}$](image-url)
ytterbium (1470 K), and the transition metals manganese (2235 K), zinc (1180 K), and silver (2485 K). In these cases, the preparation technique needs to be modified. One possibility is high-frequency melting of the elements in a water-cooled sample chamber (Kußmann et al., 1998). Alumina, zirconium dioxide, or glassy carbon are suitable crucible materials. This special sample chamber is equipped with an observation window, which allows a direct observation of the reaction process. A large advantage of this technique is the relatively fast reaction procedure, which is nearly as fast as arc-melting. Typical annealing times are between one to four hours. In most cases, however, the high-frequency techniques lead to a better crystal quality.

In the case of europium and ytterbium as rare earth metal component, the synthesis conditions need to be more sophisticated. A significant problem arises, if europium or ytterbium with relatively low boiling temperatures (1870 K for Eu and 1470 K for Yb) are reacted with the high melting noble metals (e.g. melting temperature is 2045 K for Pt and 2240 K for Rh) and indium. In these cases, during an arc-melting procedure, europium or ytterbium might have already been evaporated before the noble metal melts. This leads to a significant loss of the rare earth metal component in such a quasi-open system. Europium and ytterbium based indides are therefore synthesized in sealed inert metal tubes, mainly made of molybdenum, niobium, or tantalum (Corbett, 1983). Such tubes have small volumes (1–2 cm³).

Two different heating procedures can be used for reaction of the elements. The tantalum tubes can be sealed in evacuated silica tubes and annealed in conventional resistance furnaces. The better procedure is the annealing of the tubes in a water-cooled sample chamber of a high-frequency furnace (Pöttgen et al., 1999d). Again, the high-frequency annealing produces the better crystals. This way, the series of EuTIn and YbTIn indides (Pöttgen and Johrendt, 2000; Pöttgen et al., 2001) have been synthesized in quantitative yields for structure determination and measurements of physical properties.

Each synthesis technique discussed above mostly yields polycrystalline samples. Single crystals have only a size of several tenths to a few hundreds of μm; they can be used for single crystal structure determination. Larger single crystals for physical property investigations are not available with these techniques. They can be obtained in a different way. On a millimeter scale, single crystals can be grown in a tri-arc furnace equipped with the Czochralski technique. Suitable pieces can be cut from the crystals with a diamond wire saw. Crystal growth with the tri-arc technique is only possible for compounds that directly crystallize from the melt and where all components have comparable and high boiling temperatures.

A second technique is the growth of single crystals in an indium flux. Here, two different strategies may be applied. First it might be important to grow small single crystals from polycrystalline samples for X-ray structure determination. Suitable crucible materials for such indium fluxes are niobium, tantalum, and alumina or zirconium dioxide. Well-shaped single crystals of CeNiIn₂ (Zaremba et al., 2002b), Tb₆Pt₁₂In₂₃, and Dy₂Pt₇In₁₆ (Zaremba et al., 2002d) have recently been obtained with this technique. Larger single crystals can be grown by various metal flux techniques (Canfield and Fisk, 1992; Kanatzidis et al., 2004).
3. Ternary systems – compounds and phase relations

The ternary rare earth–transition metal–indium systems contain a vast number of individual intermetallic compounds. In this section we summarize the crystallographic data (lattice parameters) of all known indides. The compounds are grouped with respect to the \( d \) element component.

3.1. \( R–3d\)-metal–In systems

To date, the systems \( R–\{Co, Ni, Cu\}–In \) have been studied most exhaustively. The isothermal sections of the systems \( R–Cu–In \) have been constructed and published (Kalychak, 1998a). Other systems were investigated for the existence of isostructural compounds. With iron as transition metal component, only the \( Pr–Fe–In \) system has been studied. In the titanium, vanadium, and chromium systems Kalychak (2003) has studied only some alloys, which indicated that the components do not mix in the region of low contents of rare earth elements. The \( Pm–3d\)-metal–In systems have not been studied at all.

3.1.1. The \( R–Mn–In \) systems

The compounds with equiatomic composition \( RMnIn \) (Dhar et al., 2002b) crystallize with the cubic \( MgCu_2 \) type (SG \( Fd\bar{3}m \), \( a = 832.5 \) pm for \( NdMnIn \)) and the hexagonal \( MgZn_2 \) type (SG \( P6_3/mmc \), \( a = 577.8 \), \( c = 941.2 \) pm for \( GdMnIn \); \( a = 573.4 \), \( c = 930.9 \) pm for \( DyMnIn \); \( a = 568.6 \), \( c = 926.0 \) pm for \( ErMnIn \), and \( a = 574.1 \), \( c = 934.5 \) pm for \( YMnIn \)). The isothermal sections of the phase diagrams of these ternary systems have not been investigated.

3.1.2. The \( R–Fe–In \) systems

Weitzer and Rogl (1990) proposed the partial isothermal section of the \( Pr–Fe–In \) phase diagram (fig. 2) and found the indide \( Pr_6Fe_{13}In \) (structure type \( Nd_6Fe_{13}Si \); SG \( I4/mcm \), \( a = 810.32(14) \), \( c = 2352.75(89) \) pm). The other \( R–Fe–In \) systems have been investigated in order to find isotypic \( R_6Fe_{13}In \) indides. Two additional compounds, i.e. \( Nd_6Fe_{13}In \ (a = 808.8(1), c = 2343.1(9) \) pm) and \( Sm_6Fe_{13}In \ (a = 806.5(1), c = 2320.2(2) \) pm) with the same crystal structure have been found (Weitzer et al., 1993).

3.1.3. The \( R–Co–In \) systems

The ternary phase diagram \( Ce–Co–In \) has been constructed by investigating the isothermal section of the system at 870 K in the region of 0–33.3 atomic percent cerium and at 670 K in the region of 33.3–100 atomic percent cerium. According to Kalychak (1999b), only two ternary compounds, \( CeCoIn_5 \) and \( Ce_2CoIn_8 \), exist in the region with high indium content (fig. 3). In the other \( R–Co–In \) systems, the compounds with similar compositions were investigated and more than 120 ternary compounds have been found. The crystal structures for 68 of them have been determined by X-ray data. It was generally observed that in all ternary systems no solid solutions based on binary compounds exist, and ternary compounds form at constant stoichiometry showing practically no homogeneity ranges. The basic crystallographic data of all known ternary \( R_xCo_yIn_z \) compounds are listed in table 1.
3.1.4. *The R–Ni–In systems*

The isothermal sections of the phase diagram of the ternary Ce–Ni–In system at 870 K and at 670 K in the region of 0–33.3 and of 33.3–100 atomic percent cerium, respectively, have been built. Twelve ternary indides have been found in the Ce–Ni–In system (fig. 4) (Kalychak, 1998b). A small range of homogeneity was found for CeNi$_x$In$_{2-x}$, $x = 0.2–0.3$ (#9 in fig. 4), which is characteristic for the indides with the AlB$_2$ structure. Furthermore, the crystal structure for ten more compounds has been studied. Ce$_4$Ni$_7$In$_8$ exists only above 870 K. The crystal structures of $\sim$Ce$_3$NiIn$_2$ and $\sim$Ce$_5$NiIn$_3$ are unknown. The ranges of homogene-
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<th>b (pm)</th>
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<td>Lu₁₄Co₁₂In₃</td>
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<td>2300.2(9)</td>
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**Table 1**

Crystallographic data of ternary R–Co–In compounds

continued on next page
ity of the compounds with AlB₂ type structure have been established by Baranyak (1991). The compounds RNi₀.3₃In₁.₇₃ (R = La and Pr) show no homogeneity range, while the compounds with neodymium and samarium as R component have small homogeneity ranges with the compositions as stated in the formulae, RNi₀.₅₋₀.₂In₁.₇₋₁.₈.

In other ternary systems, R–Ni–In, compounds with the same compositions as in the Ce–Ni–In system have been studied. In contrast to the cobalt systems some compounds in the R–Ni–In systems have variable composition, i.e. they have extended homogeneity ranges or they form solid solutions. The equiatomic indides in the {Y, La, Ce, Pr, Nd, Sm, Gd, Tb, Dy, Ho, Er, and Tm}–Ni–In systems have been investigated by Ferro et al. (1974a). The other ternary indides R_yNi₀.₅In₂ have been prepared for the first time by the Kalychak group. Nearly 180 ternary compounds exist at the thermal conditions mentioned above and the crystal structures have been determined for 127 of these nickel-based indides (table 2).
Table 2
Crystallographic data of ternary $R$–Ni–In compounds

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<th>Compound</th>
<th>Str. type</th>
<th>S.G.</th>
<th>$a$ (pm)</th>
<th>$b$ (pm)</th>
<th>$c$ (pm)</th>
<th>Ref.</th>
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\( \chi = 0.18 \)

\( \chi = 0.52 \)

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References
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20. Baranyak et al. (1988b)
33. Mulder et al. (1998)
34. Buschow (1975)
35. Zaremba et al. (1987b)
36. Zaremba et al. (1991)
37. Lukachuk et al. (2002)
38. Sysa et al. (1988)

3.1.5. The R–Cu–In systems
Isothermal sections of the phase diagrams of each R–Cu–In system have been built for the whole area of compositions. Only the Sc–Cu–In system has not been investigated. The \{La, Ce, Pr, Nd, Sm\}–Cu–In systems have been investigated at 870 K in the region of 0–33.3 atomic percent \( R \) and at 670 K in the region of 33.3–100 atomic percent \( R \). The \{Eu, Yb\}–
Cu–In systems have been investigated at 670 K and the {Y, Gd, Tb, Dy, Ho, Er, Tm, Lu}–Cu–In systems have been investigated at 870 K. The existence of 13 compounds with equiatomic composition RCuIn is confirmed (Kalychak et al., 1998), which were first investigated by Dwight (1976) in each system. Yb$_{0.4}$Cu$_2$In$_{0.6}$ reported by Felner and Nowik (1987) is also confirmed. The formation of homogeneity ranges is typical for the R–Cu–In systems and they are much broader than in the R–Ni–In systems. There exist 102 ternary compounds in the copper based systems under the thermal conditions mentioned above. The crystal structures have been investigated for 92 of them (table 3).

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<td></td>
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<tr>
<td>(\text{Er}<em>{1.0-0.64}\text{In}</em>{1.0-1.36})</td>
<td>Zr(\text{NiAl})</td>
<td>(P\bar{6}2m)</td>
<td>740.8(1)</td>
<td>398.3(1)</td>
<td>4, 23</td>
<td></td>
</tr>
<tr>
<td>(\text{ErCu}<em>{0.44-0.35}\text{In}</em>{1.56-1.65})</td>
<td>Al(\text{B}_{2})</td>
<td>(P6/mmm)</td>
<td>469.7</td>
<td>359.8</td>
<td>23</td>
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<tr>
<td>(\text{Er}_{2}\text{CuIn})</td>
<td>Mo(\text{FeB}_{2})</td>
<td>(P4/\text{mbm})</td>
<td>741.4(3)</td>
<td>372.3(2)</td>
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</tr>
<tr>
<td>(\text{TmCu}<em>{5}\text{In}</em>{0.9})</td>
<td>Th(\text{Mn}_{1.2})</td>
<td>(I4/mmm)</td>
<td>915.0(21)</td>
<td>538.3(1)</td>
<td>15</td>
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</tr>
<tr>
<td>(\text{TmCu}_{4}\text{In})</td>
<td>Mg(\text{Cu}_{4})(\text{Sn})</td>
<td>(F\bar{4}3m)</td>
<td>715.3(1)</td>
<td>4</td>
<td></td>
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<tr>
<td>(\text{TmCu}<em>{5-4}\text{In}</em>{0.1})</td>
<td>Au(\text{Be}<em>{5})-(\text{MgCu}</em>{4})(\text{Sn})</td>
<td>(F\bar{4}3m)</td>
<td>699.1–715.3</td>
<td>24</td>
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<td></td>
</tr>
<tr>
<td>(\text{TmCu}_{2}\text{In})</td>
<td>Mn(\text{Cu}_{2})(\text{Al})</td>
<td>(Fm\bar{3}m)</td>
<td>653.9(2)</td>
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</tr>
<tr>
<td>(\text{TmCuIn})</td>
<td>Zr(\text{NiAl})</td>
<td>(P\bar{6}2m)</td>
<td>739.8(1)</td>
<td>386.8(1)</td>
<td>4</td>
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</tr>
<tr>
<td>(\text{TmCu}<em>{1.0-0.56}\text{In}</em>{1.0-1.44})</td>
<td>Zr(\text{NiAl})</td>
<td>(P\bar{6}2m)</td>
<td>739.9–739.1</td>
<td>386.8–394.3</td>
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<tr>
<td>(\text{Tm}<em>{0.4-0.3}\text{In}</em>{1.6-1.7})</td>
<td>Al(\text{B}_{2})</td>
<td>(P6/mmm)</td>
<td>469.4</td>
<td>359.5</td>
<td>24</td>
<td></td>
</tr>
<tr>
<td>(\text{Tm}_{2}\text{CuIn})</td>
<td>Mo(\text{FeB}_{2})</td>
<td>(P4/\text{mbm})</td>
<td>739.3(1)</td>
<td>369.6(1)</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>(\text{YbCu}<em>{5}\text{In}</em>{0.9})</td>
<td>Th(\text{Mn}_{1.2})</td>
<td>(I4/mmm)</td>
<td>921.9(2)</td>
<td>538.9(2)</td>
<td>15</td>
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<tr>
<td>(\text{YbCu}_{4}\text{In})</td>
<td>Mg(\text{Cu}_{4})(\text{Sn})</td>
<td>(F\bar{4}3m)</td>
<td>714.6(1)</td>
<td>4</td>
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</tr>
<tr>
<td>(\text{Yb}_{2}\text{CuIn})</td>
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<td>(P4/\text{mbm})</td>
<td>715.75(6)</td>
<td>25</td>
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<td>(\text{YbCu}<em>{4-4.2}\text{In}</em>{0.2-1.0})</td>
<td>Mg(\text{Cu}_{4})(\text{Sn})</td>
<td>(F\bar{4}3m)</td>
<td>703.6–714.5(3)</td>
<td>26</td>
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</table>

continued on next page
Six ternary compounds exist in the Y–Cu–In system (fig. 5); (Kalychak and Bakar, 1989). The crystal structures are all known. YCu_{4.64–4.00}In_{0.36–1.00} and YCu_{0.53–0.32}In_{1.47–1.68} have extended homogeneity ranges.
Eight ternary compounds exist in the La–Cu–In system (fig. 6) (Dmytrakh and Kalychak, 1990). The crystal structure of ~LaCu$_2$In$_3$ is unknown. LaCu$_{5.2-4.3}$In$_{0.8-1.7}$ and LaCu$_{0.5-0.2}$In$_{1.5-1.8}$ have homogeneity ranges as stated in the formulae. A small homogeneity region is also characteristic for LaCu$_{0}$In$_2$.

Nine ternary indides have been discovered in the Ce–Cu–In system (fig. 7) (Baranyak and Kalychak, 1991). So far, only the crystal structure of ~CeCu$_{4.2}$In$_{4.8}$ has not been de-
Homogeneity regions are present in \( \text{CeCu}_{9.0-8.0}\text{In}_{2.0-3.0} \), \( \text{CeCu}_{5.1-4.2}\text{In}_{0.9-1.8} \), and \( \text{CeCu}_{0.8-0.4}\text{In}_{1.2-1.6} \). \( \text{CeCu}_{2-x}\text{In}_{2-y} \) is stable only above 870 K (Baranyak et al., 1990). A homogeneity range \( \text{CeCu}_{6-x}\text{In}_x \) (\( x = 0-1.75 \)) has been reported by Kasaya et al. (1996).

In the Pr–Cu–In system (fig. 8), seven ternary compounds and their crystal structures are known (Kalychak, 1998a). The indides \( \text{PrCu}_{9.0-8.1}\text{In}_{2.0-2.9} \), \( \text{PrCu}_{5.1-4.2}\text{In}_{0.9-1.8} \) and \( \text{PrCu}_{0.5-0.2}\text{In}_{1.5-1.8} \) have homogeneity ranges as indicated in the formulæ.
In the Nd–Cu–In system (fig. 9), eight ternary compounds with known crystal structures exist (Baranyak and Kalychak, 1993). NdCu_{9.0-8.4}In_{2.0-2.6}, NdCu_{4.9-4.0}In_{1.1-2.0} and NdCu_{0.7-0.3}In_{1.3-1.7} have extended homogeneity ranges with respect to Cu/In mixing.
Eight ternary indides have been found in the Sm–Cu–In system (fig. 10) (Baranyak and Kalychak, 1993). The crystal structures of these indides have been determined. Homogeneity regions are present in SmCu9.0–8.0In2.0–3.0, SmCu4.9–4.0In1.1–2.0, and SmCu0.6–0.2In1.4–1.8.
Nine compounds of practically constant stoichiometry in the Eu–Cu–In system (fig. 11) are known (Kalychak, 1998a). The crystal structures have been investigated for EuCu$_{6.5}$In$_{6.5}$, EuCu$_{8.5}$In$_{2.5}$, EuCuIn$_4$, and EuCu$_{0.5}$In$_{1.5}$.

In the Gd–Cu–In system (fig. 12), seven ternary compounds exist (Bakar and Kalychak, 1990) and their crystal structures are known. GdCu$_{9.0-8.4}$In$_{2.0-2.6}$, GdCu$_{4.7-4.0}$In$_{0.3-1.0}$, and...
GdCu$_{0.6-0.4}$In$_{1.4-1.6}$ have homogeneity ranges. Six ternary indides have been found in the Tb–Cu–In system (fig. 13) (Kalychak et al., 1989b) and their crystal structures have been determined. Binary TbCu$_5$ dissolves up to 16.7 atomic percent indium and a homogeneity range is present in TbCu$_{0.56-0.32}$In$_{1.44-1.68}$.
The Dy–Cu–In system (fig. 14) is very similar to the system with terbium (fig. 13) (Kalychak et al., 1989b), except for the difference in the limits of the homogeneity range of DyCu$_{0.65-0.32}$In$_{1.35-1.68}$. 

The Dy–Cu–In system (fig. 14) is very similar to the system with terbium (fig. 13) (Kalychak et al., 1989b), except for the difference in the limits of the homogeneity range of DyCu$_{0.65-0.32}$In$_{1.35-1.68}$.
In the Ho–Cu–In system (fig. 15) five ternary indides have been found (Kalychak and Bakar, 1994) and their crystal structures have been established. HoCu$_5$ dissolves up to 16.7 atomic percent indium, while the homogeneity range for HoCu$_{0.41-0.29}$In$_{1.59-1.71}$ is smaller.

Six ternary indides have been found in the Er–Cu–In system, whose phase diagram is shown in fig. 16 (Kalychak and Bakar, 1994). The crystal structures have been established for all compounds. Binary ErCu$_5$ dissolves up to 16.7 atomic percent indium and homogeneity regions are present in ErCu$_{1.0-0.64}$In$_{1.0-1.36}$ and ErCu$_{0.44-0.33}$In$_{1.56-1.65}$.

Seven ternary compounds form in the Tm–Cu–In system (fig. 17; Kalychak and Bakar, 1996). A solid solution based on TmCu$_5$ extends up to 16.7 atomic percent indium into the ternary system. The crystal structures of all Tm–Cu–In indides have been determined. TmCu$_{1.0-0.56}$In$_{1.0-1.44}$ with ZrNiAl type structure has an extended homogeneity range. TmCu$_{0.4-0.3}$In$_{1.6-1.7}$ with AlB$_2$ structure, however, has a limited homogeneity range. The crystal structure of ∼Tm$_6$CuIn is unknown.

In the Yb–Cu–In system (fig. 18; Kalychak, 1998a), the crystal structures of YbCu$_{4.8-4.0}$In$_{0.2-1.0}$, YbCu$_{5.1}$In$_{6.9}$, and Yb$_2$Cu$_2$In (Giovannini et al., 2001; Tsujii et al., 2001) have been investigated. The crystal structures of two other compounds are still unknown.

In the Lu–Cu–In system (fig. 19), five ternary compounds (Kalychak and Bakar, 1996) exist with known crystal structures. LuCu$_5$ dissolves up to 16.7 at. percent indium and LuCu$_{1.0-0.41}$In$_{1.0-1.59}$ has an extended homogeneity range. The crystal structure of ∼Lu$_6$CuIn is not known.

3.1.6. The R–Zn–In systems
In the R–Zn–In systems only the compounds of equiatomic composition have been investigated and they crystallize in the CaIn$_2$ or CeCu$_2$ (KHg$_2$) type. Their crystallographic data are listed in table 4.

<table>
<thead>
<tr>
<th>Compound</th>
<th>Str. type</th>
<th>S.G.</th>
<th>a (pm)</th>
<th>b (pm)</th>
<th>c (pm)</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>LaZnIn</td>
<td>CaIn$_3$</td>
<td>$P6_3/mmc$</td>
<td>479.6(3)</td>
<td>776.8(5)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>CeZnIn</td>
<td>CaIn$_3$</td>
<td>$P6_3/mmc$</td>
<td>478.7(3)</td>
<td>763.1(5)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>PrZnIn</td>
<td>CaIn$_3$</td>
<td>$P6_3/mmc$</td>
<td>478.6(3)</td>
<td>753.7(5)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>NdZnIn</td>
<td>CaIn$_3$</td>
<td>$P6_3/mmc$</td>
<td>471.9(3)</td>
<td>753.1(5)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>SmZnIn</td>
<td>CaIn$_3$</td>
<td>$P6_3/mmc$</td>
<td>469.7(3)</td>
<td>741.4(5)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>EuZnIn</td>
<td>KHg$_2$</td>
<td>$Imma$</td>
<td>482.3(1)</td>
<td>780.7(2)</td>
<td>2</td>
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<tr>
<td>GdZnIn</td>
<td>CaIn$_3$</td>
<td>$P6_3/mmc$</td>
<td>467.6(3)</td>
<td>733.5(5)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>DyZnIn</td>
<td>CaIn$_3$</td>
<td>$P6_3/mmc$</td>
<td>464.9(3)</td>
<td>723.9(5)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>HoZnIn</td>
<td>CaIn$_3$</td>
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<td>463.6(3)</td>
<td>721.4(5)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>ErZnIn</td>
<td>CaIn$_3$</td>
<td>$P6_3/mmc$</td>
<td>461.6(3)</td>
<td>718.4(5)</td>
<td>1</td>
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<tr>
<td>YbZnIn</td>
<td>CaIn$_3$</td>
<td>$P6_3/mmc$</td>
<td>472.5(3)</td>
<td>738.2(5)</td>
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<tr>
<td>YbZnIn</td>
<td>KHg$_2$</td>
<td>$Imma$</td>
<td>473.4</td>
<td>737.0</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

References
3.2. R–4d-metal–In systems

The isothermal sections of the phase diagrams of the R–4d-metal–In systems have not been studied prior to our own experimental investigations. Only the ternary compounds with equiatomic compositions in the R–{Rh, Pd, Ag, Cd}–In systems were known in the literature. The systems with zirconium, niobium, molybdenum, and technetium as 4d-element component have not been studied to date. The Pm–4d-metal–In systems have not been investigated at all.

3.2.1. The R–Ru–In systems

The phase diagrams of the R–Ru–In systems have not been studied at the present time. According to Kurenbaeva et al. (2002), only CeRuIn₂ with MgCuAl₂ type structure, SG Cmcm, a = 453.58(17), b = 1000.27(23), c = 767.73(12) pm is known.

3.2.2. The R–Rh–In systems

Several compounds with equiatomic and other simple stoichiometries have been studied. The isothermal sections of the phase diagrams for these ternary systems have not been investigated. The crystallographic data for the known compounds are listed in table 5. The structurally most peculiar compound in the La–Rh–In system is La₁₁₈Rh₃In₂ (Esmaeilzadeh et al., 2002), which crystallizes with an incommensurate structure.

Table 5
Crystallographic data of ternary R–Rh–In compounds

<table>
<thead>
<tr>
<th>Compound</th>
<th>Str. type</th>
<th>S.G.</th>
<th>a (pm)</th>
<th>b (pm)</th>
<th>c (pm)</th>
<th>Ref.</th>
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<td>YRhIn₅</td>
<td>HoCoGa₅</td>
<td>P4/mmm</td>
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<td>743.5(4)</td>
<td>1</td>
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<tr>
<td>YRhIn</td>
<td>Fe₂P</td>
<td>P6₂m</td>
<td>748</td>
<td>388</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>LaRhIn₅</td>
<td>HoCoGa₅</td>
<td>P4/mmm</td>
<td>467.5(1)</td>
<td>759.6(4)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>La₂RhIn₈</td>
<td>Ho₂CoGa₈</td>
<td>P4/mmm</td>
<td>467.68(3)</td>
<td>759.88(12)</td>
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<td>La₃RhIn₂</td>
<td>MgCuAl₂</td>
<td>Cmcm</td>
<td>469.9(4)</td>
<td>1233.6(7)</td>
<td>5</td>
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<tr>
<td>LaRhIn</td>
<td>Fe₂P</td>
<td>P6₂m</td>
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<td>412.3</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>La₃Rh₂In</td>
<td>Mo₂FeB₂</td>
<td>P₄/mmb</td>
<td>761.0</td>
<td>412.9</td>
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<tr>
<td>CeRhIn₅</td>
<td>HoCoGa₅</td>
<td>P4/mmm</td>
<td>464.9(1)</td>
<td>754.0(3)</td>
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</tr>
<tr>
<td>Ce₂RhIn₈</td>
<td>Ho₂CoGa₈</td>
<td>P4/mmm</td>
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<td>754.2</td>
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<td>Fe₂P</td>
<td>P6₂m</td>
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<td>1224.4(5)</td>
<td>13</td>
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<td>Ce₂Rh₂In</td>
<td>Mo₂FeB₂</td>
<td>P₄/mmb</td>
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<tr>
<td>PrRhIn₅</td>
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<td>446.0(1)</td>
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<td>446.0(1)</td>
<td>1017.3(2)</td>
<td>5</td>
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continued on next page
1023 K (fig. 20). Besides the known ternary indides CePdIn (Fe2P type), CePdIn2 (MgCuAl2

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Table 5, continued

<table>
<thead>
<tr>
<th>Compound</th>
<th>Str. type</th>
<th>S.G.</th>
<th>a (pm)</th>
<th>b (pm)</th>
<th>c (pm)</th>
<th>Ref.</th>
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<td>ZrNiAl</td>
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<td>Pr2RhIn</td>
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<td>393.83</td>
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<tr>
<td>NdRhIn5</td>
<td>HoCoGa5</td>
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<td>463.6(2)</td>
<td>751.0(4)</td>
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<td>Nd2RhIn5</td>
<td>HoCoGa5</td>
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<td>463.0(3)</td>
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<tr>
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<td>Pnma</td>
<td>744.4(1)</td>
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<td>GdRhIn5</td>
<td>HoCoGa5</td>
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<td>460.1(1)</td>
<td>744.5(3)</td>
<td>1</td>
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<tr>
<td>Gd2RhIn5</td>
<td>Ho2CoGa8</td>
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References


3.2.3. The R–Pd–In systems

Giovannini et al. (2003) have built the isothermal section of the Ce–Pd–In phase diagram at
1023 K (fig. 20). Besides the known ternary indides CePdIn (Fe2P type), CePdIn2 (MgCuAl2
Fig. 20. Isothermal section of the phase diagram in the Ce–Pd–In system at 1023 K. \( \tau_1 \) – left branch Ce\(_{2+x}\)Pd\(_{1.85}\)In\(_{1-x}\); \( \tau_1' \) – right branch Ce\(_{1.95}\)Pd\(_{2+2x}\)In\(_{1-x}\); \( \tau_2 \) – Ce\(_8\)Pd\(_{24}\)In; \( \tau_3 \) – Ce\(_2\)Pd\(_7\)In\(_3\); \( \tau_4 \) – Ce\(_{2+4}\)Pd\(_{1.85}\)In\(_{1-x}\); \( \tau_5 \) – Ce\(_{2+4}\)Pd\(_{1.85}\)In\(_{1-x}\); \( \tau_6 \) – CePdIn; \( \tau_7 \) – \( \sim \)Ce\(_{47}\)Pd\(_{17}\)In\(_{36}\); \( \tau_8 \) – Ce\(_{2}\)Pd\(_{2}\)In\(_2\); \( \tau_9 \) – CePdIn \(_2\); \( \tau_{10} \) – \( \sim \)Ce\(_{22}\)Pd\(_{2}\)In\(_{3}\); \( \tau_{11} \) – \( \sim \)Ce\(_{22}\)Pd\(_{2}\)In\(_{3}\); \( \tau_{12} \) – \( \sim \)Ce\(_{15}\)Pd\(_{2}\)In\(_{36}\); \( \tau_{13} \) – \( \sim \)Ce\(_{15}\)Pd\(_{2}\)In\(_{36}\). Taken from Giovannini et al. (2003).

type), CePd\(_2\)In (GdPt\(_2\)Sn type), Ce\(_8\)Pd\(_{24}\)In (Ce\(_8\)Pd\(_{24}\)Sb type) and double-branched solid solutions Ce\(_{1.95}\)Pd\(_{2+2x}\)In\(_{1-x}\) and Ce\(_{2+4}\)Pd\(_{1.85}\)In\(_{1-x}\), eight novel ternary compounds were revealed in the system. The isothermal sections of the phase diagrams of the other ternary systems have not been investigated, however, many ternary indides have been discovered. The lattice parameters of these compounds are listed in table 6.

3.2.4. The \( R \)–Ag–In systems

Only the quasi-binary sections between the isostructural compounds \( R \)Ag and \( R \)In with CsCl type structure have been investigated. According to Ihrig et al. (1973), a continuous solid solution between the isostructural compounds LaAg and LaIn (CsCl type) occurs in the system La–Ag–In. Within the solid solution, a cubic-to-tetragonal distortion at low temperatures is observed. The transition temperature is a function of the composition and is the highest at 180 K for LaAg\(_{0.5}\)In\(_{0.5}\). The structural deformation has been confirmed by Maetz et al. (1980). According to Ihrig and Methfessel (1976) the solid solution between CeAg and CeIn also shows a tetragonal distortion at low temperatures \( (T \leq 40 \text{ K}) \). The structures and physical properties of the alloys within the solid solutions between \( R \)Ag and \( R \)In were studied by various authors (Sekizawa and Yasukōchi, 1964; Buschow et al., 1972). Data for \( R \) = Gd (Yagasaki et al., 1978b, 1980), \( R \) = Tb and Dy (Lal and Methfessel, 1981), and for \( R \) = Ho, Er, and Tm, show that above 8 K they have the cubic CsCl structure. According to Lal (1982), a tetragonal deformation occurs below 8 K for GdAg\(_{0.5}\)In\(_{0.5}\). The crystallographic data of the other known compounds are given in table 7.
Table 6
Crystallographic data of the ternary R–Pd–In compounds

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### References

1. Dwight and Kimball (1987)
3. Savitskiy et al. (1980)
4. Jorda et al. (1983)
5. Ferro et al. (1974a)
7. Zaremba et al. (2002a)
10. Xue et al. (1992)
11. Kaczorowski et al. (1996b)
12. Ijiri et al. (1996)
13. Xue et al. (1993)
15. Hoffmann et al. (2000)
16. Galadzhun et al. (1999a)
17. Cirafici et al. (1985)
18. Pottgen (1996b)
20. Daniš et al. (2002)
21. Zaremba et al. (2002c)
22. Giovannini et al. (2001)

### Table 6, continued

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### Crystallographic data of ternary R–Ag–In compounds

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<th>(c) (pm)</th>
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### Table 7 (continued)

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\(^a\) \(x = 0–0.6\)
\(^b\) \(x = 0–0.5\)
\(^c\) \(x = 0–0.6\)
### Table 7, continued

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<th>(a) (pm)</th>
<th>(c) (pm)</th>
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<td>YbAg(_2)In(_4)</td>
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<tr>
<td>Ho(_2)AgIn</td>
<td>MnCu(_2)Al</td>
<td>(Fm\bar{3}m)</td>
<td>689.3(15)</td>
<td>3</td>
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<tr>
<td>Ho(_2)AgIn</td>
<td>CaIn(_2)</td>
<td>(P6_3/mmc)</td>
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<tr>
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<td>YbAg(_2)In(_4)</td>
<td>(Im\bar{3})</td>
<td>1536.2(3)</td>
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<td>720.7(5)</td>
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\(^a\) Lattice parameters at 20 K.
\(^b\) Lattice parameters at 100 K.
\(^c\) RT modification, CeAg\(_x\)In\(_{1-x}\), \(x = 0.3–1\), \(a = 386.0–378.0\) pm at 300 K.
\(^d\) LT modification, CeAg\(_x\)In\(_{1-x}\), \(x = 0.3–1\), \(a = 374.1–377.1\), \(c = 400.0–377.1\) pm at 40 K.

### References


#### 3.2.5. The \(R–Cd–In\) systems

In the \(R–Cd–In\) systems, 10 ternary compounds with equiatomic composition have been found (Rossi et al., 1981). They crystallize with the hexagonal CaIn\(_2\) type structure. The basic crystallographic data are listed in table 8.

#### 3.3. \(R–5d\)-metal–In systems

No information has been found in the literature for phase equilibria and isothermal sections of ternary \(R–5d\)-metal–In systems. Only a few compounds with ideal compositions have been investigated. The systems with hafnium, tantalum, tungsten, rhenium, and osmium as \(5d\)-element have not been studied. The \(Pm–5d\)-metal–In systems have not been investigated at all.
3.3.1. The \( R \text{–Ir–In} \) systems

So far, only some ternary indides of compositions \( R\text{IrIn}_5 \) and \( R_2\text{IrIn}_8 \) are known. Their crystallographic data are listed in table 9.

3.3.2. The \( R \text{–Pt–In} \) systems

The ternary equiatomic \( R\text{PtIn} \) indides are known for a long time. They have been originally investigated by Ferro et al. (1974b). Crystallographic data of all known ternary compounds are presented in table 10.
<table>
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<tr>
<th>Compound</th>
<th>Str. type</th>
<th>S.G.</th>
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<th>b (pm)</th>
<th>c (pm)</th>
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According to Marazza et al. (1975), the indides RAu₂In crystallize with tungsten (or CsCl) type structure. However, the latest investigations by Besnus et al. (1985, 1986) showed that these indides belong to the MnCu₂Al type. YbAuIn has a pronounced homogeneity range (Pöttgen and Grin, 1997). The crystallographic data are given in Table 11.

3.3.3. The R–Au–In systems

According to Marazza et al. (1975), the indides RAl₂In crystallize with tungsten (or CsCl) type structure. However, the latest investigations by Besnus et al. (1985, 1986) showed that these indides belong to the MnCu₂Al type. YbAuIn has a pronounced homogeneity range (Pöttgen and Grin, 1997). The crystallographic data are given in Table 11.
<table>
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References
1. Dwight and Kimball (1987)
3. Marazza et al. (1975)
4. Besnus et al. (1985)
5. Besnus et al. (1986)
6. Rossi et al. (1977)
7. Pöttgen (1994)
8. Galadzhun et al. (2000a)
9. Kaczorowski et al. (1996a)
10. Galadzhun et al. (2000c)
11. Galadzhun et al. (1999b)
12. Hoffmann et al. (1999)
13. Hoffmann et al. (2000)
14. Pöttgen (1996b)
15. Pöttgen et al. (1998)
17. Galadzhun et al. (1999a)
19. Giovannini et al. (2001)
4. Structure types of ternary indides of rare-earth and transition metals

The various ternary $R$–$d$-metal–In indides, the corresponding structure types, and the lattice parameters have been listed in the previous section (tables 1–11). These ternary indides crystallize with 58 different structure types. 24 of them represent new structure types of intermetallic compounds. They have been determined first for these indium compounds. In the following paragraphs, we list the atomic coordinates and details on the coordination polyhedra.

4.1. Structure type NaZn$_{13}$ (Shoemaker et al., 1952) (fig. 21). SG Fm$\bar{3}$c; $Z = 8$; $a = 1249.2(3)$ pm for LaCu$_{6.5}$In$_{6.5}$. 8La: 8$a$ 1/4 1/4 1/4; 8$M$1: 8$b$ 0 0 0; 96$M$2: 96$i$ 0 $y$ $z$ ($y = 0.1810$, $z = 0.1217$, $M$1 = $M$2 = Cu$_{0.5}$In$_{0.5}$) (Kalychak et al., 1984a). The CP of the lanthanum atoms have 24 vertices (CN = 24), for the $M$1 and $M$2 atoms of the mixed Cu–In sites the CP are icosahedra (CN = 12). The shortest distances are La–M 361.3 pm and M–M 272.7 pm. The isotypic compound with cerium shows defects on the cerium and copper sites: 6.14Ce: 8$a$; 96(Cu 0.68 In 0.32 ): 96$i$ ($y = 0.1810$, $z = 0.1223$); 5.18Cu: 8$b$. The composition of this compound is described by the formula Ce$_{1−x}$(Cu$_{0.68}$In$_{0.32}$)$_{12}$Cu$_{1−y}$ with $x = 0.22$ and $y = 0.36$ (Baranyak et al., 1990).

4.2. Structure type LaNi$_7$In$_6$ (fig. 22). SG Ibam; $Z = 4$; $a = 806.6(2)$, $b = 924.8(2)$, $c = 1246.5(2)$ pm. 4La: 4$a$ 0 0 1/2 0; 4Ni1: 4$d$ 0 1/2 0; 8Ni2: 8$j$ $x$ $y$ 0 ($x = 0.0758$, $y = 0.2388$); 16Ni3: 16$k$ $x$ $y$ $z$ ($x = 0.6245$, $y = 0.08896$, $z = 0.17140$); 16In1: 16$k$ ($x = 0.17351$, $y = 0.69254$, $z = 0.13737$); 8$M$: 8$j$ ($x = 0.17391$, $y = 0.95635$, $M = 0.180$Ni + 0.820In). The CP of the lanthanum atoms have 20 vertices (CN = 20). The CP of the Ni1 atoms are icosahedra (CN = 12) and the Ni2 and Ni3 atoms are situated in defective icosahedra.

Fig. 21. Projection of the LaCu$_{6.5}$In$_{6.5}$ structure on the $xy$ plane and coordination polyhedra of the atoms: La (a), $M$ (b, c) ($M$ = Cu$_{0.5}$In$_{0.5}$).
Fig. 22. Projection of the LaNi\textsubscript{7}In\textsubscript{6} structure on the \(yz\) plane and coordination polyhedra of the atoms: La (a), Ni (b–e) and \(M\) (f) (\(M = \text{Ni}_{0.18}\text{In}_{0.82}\)).

(CN = 10 and 11, respectively). The CP of the indium atoms and the mixed site are distorted icosahedra with one additional atom (CN = 13). The shortest distances are La–In (346.6 pm), La–M (344.1 pm), Ni–Ni (249.2 pm), Ni–In (259.9 pm), Ni–M (266.1 pm), M–M (291.9 pm), M–In (298.1 pm), and In–In (324.6 pm) (Kalychak et al., 2001).

4.3. Structure type ThMn\textsubscript{12} (Florio et al., 1952) (fig. 23). SG \(I4/mmm\); \(Z = 2; a = 915.2(2), c = 539.0(2)\) pm for ErCu\textsubscript{5.1}In\textsubscript{6.9}. 2Er: 2a 0 0 0; 8\(M1\): 8f 1/4 1/4 1/4; 8In: 8i x 0 0 (\(x = 0.3397\)); 8\(M2\): 8j x 1/2 0 (\(x = 0.2968\)) (\(M1 = \text{Cu}_{0.850}\text{In}_{0.150}; M2 = \text{Cu}_{0.422}\text{In}_{0.578}\)) (Sysa et al., 1989). The CP of the erbium atoms have 20 vertices (CN = 20), the CP of the \(M1\) atoms are icosahedra (CN = 12), the CP of \(M2\) atoms are polyhedra with 13 vertices, which derive from icosahedra (CN = 12 + 1). The indium atoms have polyhedra with 14 vertices, which can be described as hexagonal antiprisms with capped bases (CN = 12 + 2). The shortest distances are Er–In (310.9 pm), In–In (293.4 pm), and \(M2–M2\) (263.0 pm). Isostructural YbAg\textsubscript{5.4}In\textsubscript{6.6} has the following distribution of the atoms: 2Yb: 2a; 8Ag: 8i (\(x = 0.339\)); 8In: 8j (\(x = 0.289\)); 8(\(\text{Ag}_{0.35}\text{In}_{0.65}\)): 8f (Sysa, 1991).

4.4. Structure type EuAg\textsubscript{4}In\textsubscript{8} (fig. 24). SG \(P6/mmm\); \(Z = 3; a = 992.5(6), c = 972.7(5)\) pm.

1Eu1: 1a 0 0 0; 2Eu2: 2d 2/3 1/3 1/2; 12Ag: 12a x 2x z (\(z = 0.1670, z = 0.2416\)); 2In1: 2e 0 0 z (\(z = 0.3434\)); 4In2: 4h 2/3 1/3 z (\(z = 0.1487\)); 6In3: 6i 1/2 0 z (\(z = 0.2764\)); 6In4: 6k x 0 1/2 (\(x = 0.3008\)); 6In5: 6j x 0 0 (\(x = 0.3469\)) (Sysa et al., 1994a). The europium CP are face capped hexagonal prisms (CN = 20), the silver CP are icosahedra (CN = 12), the CP
of the In1, In2 and In5 atoms are base capped hexagonal antiprisms (CN = 14), and the CP of the In5 atoms are similar to the CP of the indium atoms in the ThMn12 type. The CP for the In3 atoms are hybrids of the CP for the Cu2 atoms in the CaCu5 structure and icosahedra (CN = 12). The In4 CP are distorted icosahedra (CN = 12). The shortest distances are Eu–In (334.0 pm), Ag–Ag (287.1 pm), Ag–In (288.2 pm), and In–In (289.3 pm).

4.5. Structure type Dy2Pt7In16 (fig. 25). SG Cmmm; Z = 2; a = 1211.1(2), b = 1997.8(3), c = 439.50(6) pm. 4Dy1: 4h x 0 1/2 (x = 0.82979); 2Pt1: 2c 1/2 0 1/2; 4Pt2: 4j 0 y 1/2
Fig. 25. The projection of the Dy$_2$Pt$_7$In$_{16}$ structure on the $xy$ plane and coordination polyhedra of the atoms: Dy (a), Pt (b–d) and In (e–i).

(y = 0.27558); 8Pt3: $8p$ $x$ $y$ 0 ($x = 0.27619$, $y = 0.39055$); 4In1: $4g$ $x$ 0 0 ($x = 0.63185$); 8In2: $8q$ $x$ $y$ 1/2 ($x = 0.13008$, $y = 0.38679$); 8In3: $8p$ ($x = 0.13473$, $y = 0.27644$); 8In4: $8q$ ($x = 0.12466$, $y = 0.16317$); 4In5: $4i$ 0 $y$ 0 ($y = 0.92644$) (Zaremba, 2003d). The CP for Pt1 and Pt2 are cubes (CN = 8) and trigonal prisms for Pt3 with four additional equatorial atoms (CN = 10). For the indium atoms, In1 and In5, the CP are tetragonal prisms with three additional equatorial atoms (CN = 11). The 11-vertices polyhedron of In2 is similar to the In2 coordination in the structure of HoCoIn$_5$. In1 and In2 have 11-vertices polyhedra and a trigonal prism with four additional equatorial atoms (CN = 10) occurs for the In4 atoms. The shortest distances are Pt–In (270.6 pm), In–In (298.5 pm), and Dy–Pt (316.7 pm).

4.6. *Structure type YNi$_9$In$_2$* (fig. 26). SG $P4/mbm$; Z = 2; $a = 822.2(2)$, $c = 482.7(2)$ pm. 2Y: 2$a$ 0 0 0; 8Ni1: 8$j$ $x$ $y$ 1/2 ($x = 0.195$, $y = 0.061$); 2Ni2: 2$c$ 0 1/2 1/2; 8Ni3: 8$k$ $x$, 1/2 + $x$, $z$ ($x = 0.326$, $z = 0.252$); 4In: 4$g$ $x$, 1/2 + $x$, 0 ($x = 0.111$) (Kalychak et al., 1984b). The CP of the yttrium atoms have 20 vertices and they are similar to the thorium polyhedra in the ThMn$_{12}$ type (CN = 20). The CP of the Ni1, Ni2, and Ni3 atoms are icosahedra (CN = 12). The indium atoms have 15 neighbors. The CP can be described as hexagonal antiprisms, in which one of the bases is capped by one atom and the opposite one by two atoms (CN = 15). The shortest distances are Y–Ni (294.0 pm), Ni–Ni (235.1 pm), Ni–In (269.0 pm), and In–In (258.2 pm).
4.7. Structure type CaCo$_2$Al$_8$ (Czech et al., 1983) (fig. 27). SG Pbam; $Z = 4$; $a = 1611.8(2)$, $b = 1381.7(2)$, $c = 436.44(6)$ pm for EuRh$_2$In$_8$. 4Eu: $4h$ $x$ $y$ 1/2 ($x = 0.68057$, $y = 0.15683$); 4Rh1: $4h$ ($x = 0.90356$, $y = 0.34646$); 4Rh2: $4h$ ($x = 0.59524$, $y = 0.46552$); 4In1: $4g$ $x$ 0 ($x = 0.12249$, $y = 0.16704$); 4In2: $4g$ ($x = 0.31621$, $y = 0.44956$); 2In3: $2a$

Fig. 27. Projection of the EuRh$_2$In$_8$ structure on the $xy$ plane and coordination polyhedra of the atoms: Eu (a), Rh (b, c), and In (d–l).
0 0 0; 4In4: 4g \( (x = 0.32275, y = 0.24019) \); 4In5: 4h \( (x = 0.95555, y = 0.16029) \); 4In6: 4g \( (x = 0.01161, y = 0.33048) \); 4In7: 4h \( (x = 0.74642, y = 0.39938) \); 2In8: 2d 0 1/2 1/2; 4In9: 4g \( (x = 0.36626, y = 0.02000) \) (Pöttgen and Kußmann, 2001). The europium CP are pentagonal prisms, which are capped on the bases and on three rectangular faces (CN = 15). The Rh1 and Rh2 CP are trigonal prisms with capped rectangular faces (CN = 9). The In5 CP has a pentagonal prism with three additional atoms on the rectangular faces (CN = 13). The CP of the other indium atoms are distorted cuboctahedra (CN = 12). The shortest distances are Eu–In (336.2 pm), Eu–In (325.1 pm), Rh–In (260.2 pm), and In–In (288.0 pm).

4.8. Structure type Yb\(_2\)Pd\(_6\)In\(_{13}\) (fig. 28). SG \( C2/m \); \( Z = 2 \); \( a = 1677.0(1), b = 438.17(4), c = 1339.7(1) \) pm, \( \beta = 118.53(1)^\circ \). 4Yb: 4i \( x 0 z (x = 0.17731, z = 0.27207) \); 2In1: 2a 0 0 0; 4In2: 4i \( (x = 0.51062, z = 0.17241) \); 4In3: 4i \( (x = 0.79906, z = 0.46081) \); 4In4: 4i \( (x = 0.40763, z = 0.40093) \); 4In5: 4i \( (x = 0.29942, z = 0.12454) \); 4In6: 4i \( (x = 0.65290, z = 0.07570) \); 4In7: 4i \( (x = 0.99902, z = 0.32852) \); 4Pd1: 4i \( (x = 0.81896, z = 0.26369) \); 4Pd2: 4i \( (x = 0.88380, z = 0.08234) \); 4Pd3: 4i \( (x = 0.61206, z = 0.40484) \) (Zaremba et al., 2002c). The CP of the ytterbium atoms are pentagonal prisms with five additional atoms on the equatorial rectangular faces (CN = 15). The CP for the palladium atoms are trigonal prisms with four additional atoms (CN = 10). For the indium atoms the CP are distorted cuboctahedra (CN = 12) for In1–In3 and In6. Tetragonal prisms with five additional equatorial atoms (CN = 13) are found for In4, while pentagonal prisms with five and four additional atoms occur for In5 and In7, (CN = 15 and 14), respectively. The shortest distances are Yb–In (323.0 pm), In–In (296.5 pm), and Pd–In (265.9 pm).

Fig. 28. Projection of the Yb\(_2\)Pd\(_6\)In\(_{13}\) structure on the \( xz \) plane and coordination polyhedra of the atoms: Yb (a), Pd (b–d) and In (e–k).
4.9. **Structure type LaNi$_3$In$_6$** (fig. 29). SG Pmmm; $Z = 2$; $a = 438.8(2)$, $b = 757.4(4)$, $c = 1211.0(7)$ pm. 2La: $2a$ 1/4 1/4 z ($z = 0.32187$); 2Ni1: $2b$ 1/4 3/4 z ($z = 0.46597$); 4Ni2: 4e 1/4 y z ($y = 0.37642$, $z = 0.06560$); 2In1: 2a ($z = 0.25114$); 2In2: 2a ($z = 0.01173$); 4In3: 4e ($y = 0.44791$, $z = 0.58362$); 4In4: 4e ($y = 0.54299$, $z = 0.83908$) (Kalychak et al., 1985). The CP of the lanthanum atoms are face capped pentagonal prisms (CN = 17). Ni1 and Ni2 have trigonal prisms with three and four additional atoms (CN = 9 and 10). In1 and In2 have tetragonal prisms with five additional equatorial atoms (CN = 13) and In3 and In4 are situated in strongly distorted cuboctahedra (CN = 12). The shortest distances are La–Ni (338.0 pm), La–In (332.7 pm), Ni–In (256.0 pm), Ni–Ni (263.0 pm), and In–In (299.8 pm).

4.10. **Structure type Th$_2$Ni$_{17}$** (Florio et al., 1956) (fig. 30). SG P6$_3$/mmc; $Z = 2$; $a = 835.3$, $c = 814.1$ pm for Pr$_2$Ni$_{16.0}$In$_{1.0}$. 2Pr1: 2d 1/3 2/3 3/4; 2Pr2: 2b 0 0 1/4; 12M1: 12k x 2x 0 ($x = 0.1666$); 12M2: 12j x 0 1/4 ($x = 0.3300$); 6M3: 6g 1/2 0 0; 4M4: 4f 1/3 2/3 z ($z = 0.1100$) ($M = Ni_{0.94}In_{0.06}$) (Kalychak, 1997). The CP of the praseodymium atoms are face capped hexagonal prisms (CP = 20). The CP of the M1 and M3 atoms are distorted icosahedra (CN = 12). The CP of the M2 atoms have 13 vertices (CN = 13) and the M4 CP are base capped hexagonal antiprisms (CN = 14). The shortest distances are Pr–Ni (289.5 pm) and $M–M$ (244.1 pm).

4.11. **Structure type Ho$_4$Ni$_{10}$Ga$_{21}$** (Grin et al., 1979b) (fig. 31). SG C2/m; $Z = 2$; $a = 2314.3(2)$, $b = 454.70(7)$, $c = 1940.8(2)$ pm, $\beta = 133.43(1)^\circ$ for Ce$_4$Pd$_{10}$In$_{21}$. 4Ce1: 4i x 0 z ($x = 0.90482$, $z = 0.67701$), 4Ce2: 4i ($x = 0.71229$, $z = 0.83395$), 4Pd1: 4i ($x = 0.13471$, $z = 0.88679$), 4Pd2: 4i ($x = 0.9115$, $z = 0.8865$), 4Pd3: 4i ($x = 0.26770$, $z = 0.61427$), 4Pd4: 4i ($x = 0.47021$, $z = 0.60646$), 4Pd5: 4i ($x = 0.18294$, $z = 0.68821$), 2In1: 2b 0 1/2 0, 4In2: 4i ($x = 0.06148$, $z = 0.95053$), 4In3: 4i ($x = 0.33085$, $z = 0.87224$), 4In4: 4i ($x = 0.76707$, $z = 0.70270$), 4In5: 4i ($x = 0.06836$, $z = 0.6986$), 4In6: 4i ($x = 0.5841$,

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Fig. 29. Projection of the LaNi$_3$In$_6$ structure on the yz plane and coordination polyhedra of the atoms: La (a), Ni (b, c), and In (d–g).
Fig. 30. Projection of the Pr$_2$Ni$_{16.0}$In$_{1.0}$ structure on the $xy$ plane and coordination polyhedra of the atoms: Pr (a, b) and $M$ (c–f) ($M = \text{Ni}_{0.94}\text{In}_{0.06}$).

Fig. 31. Projection of the Ce$_4$Pd$_{10}$In$_{21}$ structure on the $xz$ plane and coordination polyhedra of the atoms: La (a, b), Pd (c–g) and In (h–q).
$z = 0.5936$), $4\text{In}7$: $4i$ ($x = 0.54379, z = 0.79579$), $4\text{In}8$: $4i$ ($x = 0.41472, z = 0.8021$), $4\text{In}9$: $4i$ ($x = 0.10453, z = 0.50274$), $4\text{In}10$: $4i$ ($x = 0.69682, z = 0.49612$), $4\text{In}11$: $4i$ ($x = 0.70030, z = 0.99949$) (Zaremba et al., 2003a). The CP of both cerium atoms are pentagonal prisms with five additional indium atoms located on opposite rectangular faces and three additional palladium atoms located on opposite edges (CN 18). Trigonal prisms with three or four additional atoms are characteristic for the palladium atoms (CN 9 and CN 10). For the indium atoms, distorted and defect icosahedra (CN 12 and CN 11) and 13-vertices polyhedra are typical. The shortest distances are Ce–Pd (331.3 pm), Ce–In (321.3 pm), Pd–In (269.4 pm), and Pd–Pd (312.4 pm).

4.12. Structure type CeNi$_5$Sn (Skolozdra et al., 1981) (fig. 32). SG $P6_3/mmc$; $Z = 4$; $a = 489.4(2)$, $c = 1987.8(5)$ pm for CeNi$_5$In. 2Ce1: 2e $1/3 2/3 1/4$; 2Ce2: 2a 0 0 0; 12Ni1: $12k$ $x 2x z (x = 0.8347, z = 0.1473)$; 4Ni2: $4f$ $1/3 1/3 2/3 z (z = 0.5460)$; 2Ni3: $2d 1/3 2/3 3/4$; 2Ni4: $2b 0 0 1/4$; 4In: $4f (z = 0.0856)$ (Baranyak et al., 1992). The CP of the Ce1 atoms are face capped hexagonal prisms (CN = 20), the Ce2 polyhedra have 18 vertices (CN = 18) and the Ni1 CP are icosahedra (CN = 12). The Ni2 atoms have a 10-vertices polyhedron, which may be considered as a combination of an icosahedron and a trigonal prism (CN = 10). The Ni3 and Ni4 atoms have coordination number 12, while the indium atoms have a 14-vertices polyhedron.

Fig. 32. Projection of the CeNi$_5$In structure in the orthohexagonal setting and coordination polyhedra of the atoms: Ce (a, b), Ni (c–f) and In (g).
polyhedron (CN = 14). The shortest distances are Ce–Ni (282.5 pm), Ce–In (326.8 pm), Ni–In (261.6 pm), and Ni–Ni (242.7 pm).

4.13. Structure type Sm$_2$Co$_9$In$_3$ (fig. 33). SG Cmmm; Z = 2; a = 2283.4(4), b = 502.0(1), c = 408.42(8) pm. 4Sm: 4g x 0 0 (x = 0.3166); 2Co1: 2c 1/2 0 1/2; 4Co2: 4g (x = 0.1212); 8Co3: 8q x y 1/2 (x = 0.0931, y = 0.2490); 4Co4: 4g (x = 0.4390); 2In1: 2a 0 0 0; 4In2: 4h x 0 1/2 (x = 0.2022) (Baranyak et al., 1993). The CP of the samarium atoms are face capped pentagonal prisms (CN = 17). The Co1 (Co3) and Co4 atoms have a 12-vertices polyhedron similar to the Cu2 and Cu1 atoms in the CaCu$_5$ structure, (CN = 12), respectively. Co2 has a lower coordination number with an 11-vertices polyhedron. The In1 atoms have a face capped hexagonal prism (CN = 18) and the In2 atoms have a distorted cuboctahedron (CN = 12). The shortest distances are Sm–Co (279.5 pm), Sm–In (326.4 pm), and Co–Co (247.1 pm).

4.14. Structure type CeCu$_4$.32In$_{1.68}$ (fig. 34). SG Pnmm; Z = 8; a = 1716.9(6), b = 1090.8(4), c = 520.2(2) pm. 2Ce1: 2a 0 0 0; 2Ce2: 2d 1/2 0 0; 4Ce3: 4g x y 0 (x = 0.7585, y = 0.3844); 4Cu1: 4g (x = 0.4653, y = 0.3057); 4Cu2: 4g (x = 0.3485, y = 0.1581); 4Cu3: 4g (x = 0.1682, y = 0.0697); 4Cu4: 4g (x = 0.0942, y = 0.4530); 8Cu5: 8h x y z (x = 0.5913, y = 0.2534, z = 0.2503); 8Cu6: 8h (x = 0.8424, y = 0.1344, z = 0.2540); 4M1: 4g (x = 0.2193, y = 0.3003); 4M2: 4g (x = 0.5871, y = 0.4616); 4In1: 4g (x = 0.9474, y = 0.3085); 4In2: 4g (x = 0.6927, y = 0.0795) (M1 = Cu$_{0.50}$In$_{0.50}$; M2 = Cu$_{0.75}$In$_{0.25}$) (Kalychak et al., 1988a). The CP of the Ce1, Ce2, and Ce3 atoms are 20-, 18- and 19-vertices polyhedra, respectively. The first two are similar to the CP of the cerium atoms in the hexagonal CeNi$_5$In structure and the third one is a hybrid of both (CN = 20; 18; 19). The CP of Cu1, Cu5, Cu6, and M1 are distorted icosahedra (CN = 12). M2 is similar to the Ni3 and Ni4 CP in the CeNi$_5$In structure (CN = 12). Cu3 has only an 11-vertices polyhedron (CN = 11) and the Cu2 and Cu4 atoms each have a 10-vertices polyhedron, which is similar to those of the Ni2 atoms in CeNi$_5$In (CN = 10). The In1 and In2 CP are also identical to the indium CP in CeNi$_5$In (CN = 14). The shortest distances are Ce–Cu (298.6 pm), Ce–In (334.7 pm), Cu–Cu (254.9 pm), Cu–In (268.7 pm), and Cu–M1 (259.5 pm).
Fig. 34. Projection of the CeCu_{4.32}In_{1.68} structure on the $xy$ plane and coordination polyhedra of the atoms: Ce (a–c), Cu (d–i), M1 (j), M2 (k), and In (l, m) ($M_1 = Cu_{0.50}In_{0.50}; M_2 = Cu_{0.75}In_{0.25}$).

4.15. **Structure type YbAg_{2}In_{4}** (fig. 35). SG $Im\bar{3}$; $Z = 24$; $a = 1536.2(3)$ pm. 24Yb: 24g $0y0z$ ($y = 0.1984, z = 0.3149$); 11.8Ag1: 16 $f$ $x$ $x$ $x$ ($x = 0.1617$); 5.6Ag2: 16 $f$ ($x = 0.1448$); 18.2Ag3: 24g ($y = 0.2478, z = 0.0940$); 7.7Ag4: 24g ($y = 0.2164, z = 0.084$); 7.7Ag5: 48 $h$ $x$ $y$ $z$ ($x = 0.023, y = 0.0664, z = 0.0850$); 12In1: 12 $d$ $x$ 0 0 ($x = 0.4089$); 12In2: 12 $e$ $x$ 0 1/2 ($x = 0.1966$); 24In3: 24g ($y = 0.4038, z = 0.3497$); 48In4: 48 $h$ ($x = 0.2012, y = 0.1153, z = 0.3392$) (Sysa et al., 1998). The ytterbium CP are 16-vertices polyhedra, Ag1 and Ag2 have trigonal prisms with capped rectangular faces (CN = 9), Ag3 and Ag4 have distorted tetragonal antiprisms with a capped base (CN = 9), Ag5 has a 7-vertices polyhedron (CN = 7). In1 and In3 have icosahedra (CN = 12) and In2 and In4 have a 11-vertices polyhedron each (CN = 11). The shortest distances are Yb–Ag (329.5 pm), Yb–In (328.4 pm), Ag–Ag (240.7 pm), and In–In (277.7 pm).

4.16. **Structure type HoCoGa_{5}** (Grin et al., 1979a) (fig. 36). SG $P4/mmm$; $Z = 1$; $a = 454.7(1)$, $c = 741.1(2)$ pm for HoCoIn_{5}. 1Ho: 1a 0 0 0; 1Co: 1b 0 0 1/2; 1In1: 1c 1/2 1/2 0; 4In2: 4h 0 1/2 $z$ ($z = 0.3051$) (Kalychak et al., 1989a). The holmium CP are cuboctahedra (CN = 12), the cobalt atoms have a cube-like coordination (CN = 8), the In1 atoms have cuboctahedra (CN = 12), and the In2 atoms have a defective cuboctahedron with coordination number 11. The shortest distances are Ho–In (320.6 pm), In–In (289.0 pm), and Co–In (269.4 pm).

4.17. **Structure type Tb_{6}Pt_{12}In_{23}** (fig. 37). SG $C2/m$; $Z = 2$; $a = 2834.6(4)$, $b = 440.05(7)$, $c = 1477.1(3)$ pm, $\beta = 112.37(1)^{\circ}$. 4Tb1: 4$i$ $x$ 0 0 ($x = 0.45413, z = 0.30114$); 4Tb2: 4$i$
Fig. 35. Projection of the YbAg₂In₄ structure on the xy plane and coordination polyhedra of the atoms: Yb (a), Ag (b–d), and In (e–h). (To simplify the figure, the Ag5 atoms are not up to scale).

Fig. 36. Projection of the HoCoIn₅ structure on the xz plane and coordination polyhedra of the atoms: Ho (a), Co (b), and In (c, d).

(x = 0.82171, z = 0.92201); 4Tb3: 4i (x = 0.83410, z = 0.46938); 4Pt1: 4i (x = 0.25299, z = 0.36389); 4Pt2: 4i (x = 0.38127, z = 0.85045); 4Pt3: 4i (x = 0.32069, z = 0.08940); 4Pt4: 4i (x = 0.90070, z = 0.36563); 4Pt5: 4i (x = 0.96746, z = 0.14225); 4Pt6: 4i (x = 0.39081, z = 0.64133); 4In1: 4i (x = 0.40793, z = 0.04986); 4In2: 4i (x = 0.71338, z =
Fig. 37. Projection of the Tb₆Pt₁₂In₂₃ structure on the xz plane and coordination polyhedra of the atoms: Tb (a–c), Pt (d–i), and In (j–u).

0.43450); 2In₃: 2c 0 0 1/2; 4In₄: 4i (x = 0.76219, z = 0.08351); 4In₅: 4i (x = 0.14577, z = 0.27948); 4In₆: 4i (x = 0.04543, z = 0.33334); 4In₇: 4i (x = 0.94389, z = 0.93853); 4In₈: 4i (x = 0.32641, z = 0.28166); 4In₉: 4i (x = 0.87649, z = 0.17059); 4In₁₀: 4i (x = 0.47123, z = 0.82555); 4In₁₁: 4i (x = 0.42464, z = 0.49405); 4In₁₂: 4i (x = 0.72732, z = 0.24450) (Zaremba et al., 2002d). The CP of the terbium atoms are pentagonal prisms with five (for Tb₁ and Tb₃, CN = 15) or four (for Tb₂, CN = 14) additional atoms on the equatorial rectangles. The CP of the platinum atoms are trigonal prisms with three (for Pt₁–Pt₄, CN = 9) and four (for Pt₅–Pt₆, CN = 10) additional atoms on the equatorial rectangles. The indium atoms have distorted cuboctahedra (for In₁–In₄, In₆, In₉–In₁₂), tetragonal prisms with five additional atoms on the equatorial rectangles (CN = 13 for In₅). The In₇ atoms are coordinated by an 11-vertices polyhedron similar to the In₂ atoms in HoCoIn₅. Pentagonal prisms with four additional atoms on the equatorial rectangles (CN = 14) occur for the In₈ atoms. The shortest distances are In–In (290.2 pm), Tb–Pt (284.9 pm), and In–Pt (269.1 pm).

4.18. Structure type Gd₃Pt₄In₁₂ (fig. 38). SG P₃m1; Z = 3, a = 990.5(1), c = 1529.5(3) pm. 1Gd₁: 1a 0 0 0; 2Gd₂: 2d 1/3 2/3 z (z = 0.51246); 6Gd₃: 6i x -x z (x = 0.49152, z = 0.19482); 6Pt₁: 6i (x = 0.82826, z = 0.13791); 6Pt₂: 6i (x = 0.50511, z = 0.38982); 2In₁: 2c 0 0 z (z = 0.3741); 2In₂: 2d (z = 0.8795); 6In₃: 6g x 0 0 (x = 0.65033); 6In₄: 6i (x = 0.22110, z = 0.33341); 6In₅: 6i (x = 0.11826, z = 0.17814); 6In₆: 6i (x = 0.78757, z = 0.31395); 2In₇: 2d (z = 0.0819); 6In₈: 6h x 0 1/2 (x = 0.71296) (Rodewald et al., 2002). The Gd₁ and Gd₂ atoms have CN 18. The Gd₁ CP can be considered as a cuboctahedron with six additional atoms capping the rectangular faces. The CP of the Gd₂ atoms are hexagonal prisms with six additional atoms. Gd₃ with CN 15 has a low symmetry coordination. Both platinum sites are surrounded by tricapped trigonal prisms (CN 9). The coordination num-
Fig. 38. Projection of the Gd$_3$Pt$_4$In$_{12}$ structure on the yz plane and coordination polyhedra of the atoms: Gd (a–c), Pt (d–e), and In (f–m).

bers for the eight crystallographically different indium sites vary from CN 12 to CN 16. In$_1$ has exclusively indium neighbors, while all other indium atoms have gadolinium, platinum, and indium atoms in their coordination shell. All polyhedra contain fragments of icosahedra. The shortest distances are Gd–In (321.2 pm), Gd–Pt (299.2 pm), In–In (284.3 pm), and In–Pt (268.3 pm).

4.19. Structure type Lu$_6$Co$_{17.92}$In$_{14}$ (fig. 39). SG $Pm\bar{3}$; $Z = 1; a = 865.2(3)$ pm. 6Lu: 6g $1/2 \ 0 \ z \ (z = 0.1824)$; 1Co1: 1a $0 \ 0 \ 0$; 12Co2: 12j $x \ 0 \ z \ (x = 0.1452, z = 0.2402)$; 0.33Co3: 1b $1/2 \ 1/2 \ 1/2$; 2.70Co4: 6h $1/2 \ y \ 1/2 \ (y = 0.287)$; 1.56Co5: 6h $(y = 0.237)$; 6In1: 6f $x \ 0 \ 1/2 \ (x = 0.3210)$; 8In2: 8i $x \ x \ x \ (x = 0.2824)$ (Zaremba et al., 1990). The lutetium CP are hexagonal antiprisms with capped faces (CN = 17). Co1 and Co2 have icosahedra (CN = 12), Co4 and Co5 have polyhedra with 12 vertices (CN = 12), which can be considered as strongly distorted icosahedra or cuboctahedra. Co3 has a tetrahexahedron (CN = 14). The In$_1$ atoms have a 13-vertices polyhedron derived from an icosahedron and In2 has a distorted cuboctahedral coordination (CN = 12). The shortest distances are Lu–Lu (318.7 pm), Lu–In (314.1 pm), Co–Co (242.8 pm), Co–In (256.9 pm), and In–In (309.7 pm).

4.20. Structure type MgCu$_4$Sn (Gladyshevskii et al., 1952) (fig. 40). SG $F\bar{4}3m$; $Z = 4; a = 705.6(1)$ pm for CeNi$_4$In. 4Ce: 4a $0 \ 0 \ 0$; 16Ni: 16e $x \ x \ x \ (x = 5/8)$; 4In: 4c 1/4 1/4 1/4 (Zaremba et al., 1984). The CP of the cerium and indium atoms have 16 vertices (CN = 16).
The nickel CP are icosahedra (CN = 12). The shortest distances are Ce–Ni (292.7 pm), Ce–In (305.7 pm), and Ni–Ni (249.7 pm).

4.21. Structure type HoNi$_{2.6}$Ga$_{2.4}$ (Grin et al., 1983) (fig. 41). SG $P6/mmm$; $Z = 3$; $a = 933.4(4)$, $c = 435.6(2)$ pm for LaNi$_3$In$_2$. 1La1: 1a 0 0 0; 2La2: 2d 1/3 2/3 1/2; 6Ni: 6l x 2x 0 ($x = 0.1783$); 6M: 6k x 0 1/2 ($x = 0.2839$; $M = \text{Ni}_{0.48}\text{In}_{0.52}$); 3In: 3f 1/2 0 0 (Zaremba et al., 1993). The La1 and La2 CP are face capped hexagonal prisms (CN = 20). The nickel
polyhedra have only 11 vertices (one atom is missing when compared to the Cu2 atoms in CaCu5) (CN = 11), the M and indium atoms have a 12-vertices polyhedron each as the Cu2 atoms in CaCu5 (CN = 12). The shortest distances are La–Ni (288.2 pm), Ni–M (261.6 pm), M–M (265.0 pm), and M–In (296.9 pm).

4.22. Structure type YNiAl4 (Rykhal et al., 1972) (fig. 42). SG Cmcm; Z = 4; a = 439.8(1), b = 1660.6(5), c = 727.8(2) pm for YbNiIn4. 4Yb: 4c 0 y 1/4 (y = 0.12458); 4Ni: 4c (y = 0.77398); 8In1: 8f 0 y z (y = 0.31307, z = 0.04844); 4In2: 4c (y = 0.92806); 4In3: 4b 0 1/2 0 (Kalyuchak et al., 1988b). The ytterbium CP are face capped pentagonal prisms (CN = 17), the nickel atoms have trigonal prisms with capped rectangular faces (CN = 9). The In1, In2, and In3 atoms have distorted cuboctahedra (CN = 12). The shortest distances are Yb–In (326.0 pm), Ni–In (255.9 pm), and In–In (309.4 pm).

4.23. Structure type LaCoAl4 (Rykhal et al., 1977) (fig. 43). SG Pmma; Z = 2; a = 863.7(2), b = 422.5(1), c = 743.1(1) pm for YbRhIn4. 2Yb: 2e 1/4 0 z (z = 0.3980); 2Rh: 2e (z = 0.8063); 2In1: 2a 0 0 0; 2In2: 2f 1/4 1/2 z (z = 0.0656); 4In3: 4j x 1/2 z (x = 0.0634, z = 0.6860) (Hoffmann et al., 2000). The ytterbium CP are pentagonal prisms with capped bases and three capped rectangular faces (CN = 15). The rhodium atoms fill trigonal prisms with capped rectangular faces (CN = 9): In1–In3 have a distorted cuboctahedral coordination (CN = 12). The shortest distances are Yb–Rh (303.4 pm), Yb–In (325.0 pm), Rh–In (259.5 pm), and In–In (297.3 pm).
4.24. Structure type $Ho_2CoGa_8$ (Grin et al., 1979a) (fig. 44). SG $P4/mmm$; $Z = 1$; $a = 464.0(1)$, $c = 1225.1(3)$ pm for $Ce_2CoIn_8$. 2Ce: 2a 0 0 z ($z = 0.3105$); 1Co: 1a 0 0 0; 2In1: 2e 0 1/2 1/2; 2In2: 2h 1/2 1/2 z ($z = 0.2962$); 4In3: 4i 0 1/2 z ($z = 0.1199$) (Kalychak et al., 1989a). The CP of the cerium, In1, and In2 atoms are cuboctahedra (CN = 12). The
cobalt atoms have a compressed cube-like coordination (CN = 8) and In3 has an 11-vertices polyhedron similar to the In2 CP in HoCoGa5. The shortest distances are Ce–In (328.2 pm), Co–In (274.6 pm), and In–In (293.8 pm).

4.25. Structure type CePt2In2 (fig. 45). SG P21/m; Z = 4; \(a = 1018.9(6), b = 447.7(4), c = 1022.6(6)\) pm, \(\beta = 117.00(5)^\circ\); 2Ce1: 2e x 1/4 z (x = 0.0483, z = 0.7899); 2Ce2: 2e (x = 0.4020, z = 0.7106); 2Pt1: 2e (x = 0.0712, z = 0.1003); 2Pt2: 2e (x = 0.1574, z = 0.4189);
Au_12 atoms are coordinated by trigonal prisms with three capping atoms (CN = 3) and pentagonal prisms with six equatorial atoms (CN = 5). In_2, In_3, and In_17: In_7, In_10, and In_17: In_10: The cerium CP are pentagonal prisms with six equatorial atoms. Most indium atoms have a distorted cuboctahedral environment (CN = 12 for In_1 and In_4) or a cuboctahedron with an additional atom (CN = 13 for In_2 and In_3). The shortest distances are Ce–Pt (289.1 pm), Ce–In (328.8 pm), Pt–In (271.4 pm), and In–In (312.6 pm).

4.26. Structure type Ce_2Au_3In_5 (fig. 46). SG Pmn2_1; Z = 8; a = 465.27(8), b = 5348.3(9), c = 740.5(1) pm. All atoms are in the Wyckoff position 2 0 y z: 2Ce1: y = 0.11547, z = 0.9021; 2Ce2: y = 0.13424, z = 0.4008; 2Ce3: y = 0.28853, z = 0.4342; 2Ce4: y = 0.46181, z = 0.9379; 2Ce5: y = 0.61566, z = 0.9706; 2Ce6: y = 0.63459, z = 0.4696; 2Ce7: y = 0.78788, z = 0.4335; 2Ce8: y = 0.96172, z = 0.9330; 2Au1: y = 0.07123, z = 0.6410; 2Au2: y = 0.17904, z = 0.1412; 2Au3: y = 0.26207, z = 0.9340; 2Au4: y = 0.35112, z = 0.6848; 2Au5: y = 0.39911, z = 0.1849; 2Au6: y = 0.48762, z = 0.4344; 2Au7: y = 0.57116, z = 0.2328; 2Au8: y = 0.67868, z = 0.7340; 2Au9: y = 0.76202, z = 0.9382; 2Au10: y = 0.85087, z = 0.1898; 2Au11: y = 0.89903, z = 0.6893; 2Au12: y = 0.98763, z = 0.4380; 2In1: y = 0.02103, z = 0.1413; 2In2: y = 0.02176, z = 0.7327; 2In3: y = 0.07458, z = 0.2517; 2In4: y = 0.17539, z = 0.7530; 2In5: y = 0.22834, z = 0.2295; 2In6: y = 0.22884, z = 0.6401; 2In7: y = 0.31428, z = 0.9617; 2In8: y = 0.34894, z = 0.2992; 2In9: y = 0.40070, z = 0.7982; 2In10: y = 0.43593, z = 0.4613; 2In11: y = 0.52115, z = 0.7285; 2In12: y = 0.52145, z = 0.1421; 2In13: y = 0.57443, z = 0.6202; 2In14: y = 0.67524, z = 0.1202; 2In15: y = 0.72821, z = 0.2316; 2In16: y = 0.72873, z = 0.6393; 2In17: y = 0.81430, z = 0.9107; 2In18: y = 0.84922, z = 0.5793; 2In19: y = 0.90106, z = 0.0773; 2In20: y = 0.93558, z = 0.4087 (Galadzhun et al., 1999b). The CP of the cerium atoms are pentagonal prisms with six equatorial atoms (CN = 16). The Au_3, Au_6, Au_9, and Au_12 atoms are coordinated by trigonal prisms with three capping atoms (CN = 9), while the other gold atoms have trigonal prisms with four equatorial atoms (CN = 10). In_7, In_10, In_17,

Fig. 46. Projection of the Ce_2Au_3In_5 structure on the yz plane and coordination polyhedra of the atoms: Ce (a, b), Au (c–e), and In (f–j). Only representative polyhedra are shown.
and In20 have pentagonal prisms with five equatorial atoms (CN = 15) and the coordination polyhedra of the other indium atoms are distorted cuboctahedra (CN = 12). The shortest distances are Ce–Au (305.6 pm), Ce–In (325.3 pm), Au–In (271.6 pm), and In–In (295.1 pm).

4.27. Structure type LaRuSn3 (Eisenmann and Schäfer, 1986) (fig. 47). SG Pm\(\overline{3}n\); Z = 8; \(a = 980.4\) pm; \(1.81\)La\(1:2\) \(a\) \(000\); \(6\)La\(2:6\) \(c\) \(1/2 1/4 0\); \(8\)Pt: \(8e\) \(1/4 1/4 1/4\); \(24\)In: \(24k\) \(x y 0\) \((x = 0.15239, y = 0.30906)\) for LaPtIn3 (Galadzhun and Pöttgen, 1999). The La1 CP are icosahedra (CN = 12). La2 has a 16-vertices polyhedron, which can be described as a hexagonal prism built from platinum and indium atoms. The platinum atoms have a tricapped trigonal prismatic coordination (CN = 9) and the indium atoms occupy distorted cuboctahedra (CN = 12). The shortest distances are La–In (337.8 pm), Pt–In (269.4 pm), and In–In (298.8 pm).

4.28. Structure type Ce4Ni7In8 (fig. 48). SG Cmcm; Z = 4; \(a = 1475.2(2), b = 2418.4(3), c = 439.51(8)\) pm; \(4\)Ce1: \(4g\) \(x 0 0\) \((x = 0.18413)\); \(8\)Ce2: \(8p\) \(x y 0\) \((x = 0.17175, y = 0.17682)\); \(4\)Ce3: \(4j\) \(y 1/2\) \((y = 0.39366)\); \(8\)Ni1: \(8q\) \(x y 1/2\) \((x = 0.09462, y = 0.09494)\); \(8\)Ni2: \(8p\) \((x = 0.64297, y = 0.05444)\); \(4\)Ni3: \(4h\) \(x 0 1/2\) \((x = 0.62174)\); \(8\)Ni4: \(8p\) \((x = 0.13832; y = 0.34612)\); \(4\)In1: \(4j\) \((y = 0.19046)\); \(4\)In2: \(4i\) \(y 0\) \((y = 0.08628)\); \(8\)In3: \(8q\) \((x = 0.65787, y = 0.21613)\); \(2\)In4: \(2d\) \(0 0 1/2\); \(2\)In5: \(2b\) \(1/2 0 0\); \(8\)In6: \(8q\) \((x = 0.22498, y = 0.40620)\); \(4\)In7: \(4i\) \((y = 0.27729)\) (Baranyak et al., 1988a). The CP of Ce1 and Ce3 are hexagonal prisms, which are capped on all faces (CN = 20) and Ce2 has a pentagonal prismatic coordination with additional atoms capping each face (CN = 17). Ni1 and Ni2 have trigonal prisms with four additional atoms (CN = 10), Ni3 has a strongly distorted cuboctahedron (CN = 12), and Ni4 has trigonal prisms with three additional atoms capping the rectangular faces (CN = 9). The In1–In7 atoms have distorted cuboctahedra (CN = 12). The shortest dis-
Fig. 48. Projection of the Ce₄Ni₇In₈ structure on the \( xy \) plane and coordination polyhedra of the atoms: Ce (a–c), Ni (d–g), and In (h–n).

Distances are Ce–Ni (285.7 pm), Ce–In (333.3 pm), Ni–Ni (240.7 pm), Ni–In (248.6 pm), and In–In (303.9 pm).

4.29. Structure type \( \text{La}_3\text{Au}_4\text{In}_7 \) (fig. 49). SG \( \text{I}2/\text{m} \); \( Z = 2; a = 460.42(5), b = 1389.5(1), c = 1039.6(2) \text{ pm}, \alpha = 90.77(1)^\circ. \) 2La1: 2a 0 0 0; 4La2: 4i 0 y z (\( y = 0.31110, z = 0.00504 \)); 4Au1: 4i (\( y = 0.65405, z = 0.38435 \)); 4Au2: 4i (\( y = 0.12836, z = 0.27750 \)); 2In1: 2b 0 0 1/2; 4In2: 4i (\( y = 0.32455, z = 0.35676 \)); 4In3: 4i (\( y = 0.83943, z = 0.26837 \)); 4In4: 4i (\( y = 0.50319, z = 0.21248 \)) (Galadzhun et al., 2000a). The La1 CP are hexagonal prisms with six equatorial atoms (CN = 18), La2 has pentagonal prisms with seven equatorial atoms (CN = 17), Au1 and Au2 have trigonal prisms with three and four equatorial atoms (CN = 9 and CN = 10), and In1–In4 have distorted cuboctahedra (CN = 12). The shortest distances are La–Au (332.7 pm), La–In (331.5 pm), Au–In (270.4 pm), and In–In (291.7 pm).

4.30. Structure type \( \text{anti-Hf}_2\text{Co}_4\text{P}_3 \) (Ganglberger, 1968) (fig. 50). SG \( \text{P6}2\text{m} \); \( Z = 4; a = 1489.7(3), c = 454.90(7) \text{ pm for Eu}_2\text{Au}_3\text{In}_4. \) 3Eu1: 3g x 0 1/2 (\( x = 0.8195 \)); 2Eu2: 2c 1/3 2/3 0; 3Eu3: 3f x 0 0 (\( x = 0.4522 \)); 3Au1: 3f (\( x = 0.1789 \)); 6Au2: 6k x y 1/2 (\( x = 0.6788, y = 0.5081 \)); 3Au3: 3f (\( x = 0.6569 \)); 3In1: 3g (\( x = 0.2807 \)); 6In2: 6j x y 0 (\( x = 0.8012, y = 0.6187 \)); 6In3: 6k (\( x = 0.8762, y = 0.5265 \)); 1In4: 1a 0 0 0 (Hoffmann et al., 1999). The Eu1 and Eu2 CP are face capped hexagonal prisms (CN = 20) and face capped pentagonal prisms for Eu3, CN = 18. The gold atoms have tricapped trigonal prisms (CN = 9) and the indium atoms have either distorted cuboctahedra (CN = 12 for In1–In3) or tricapped trigonal...
Fig. 49. Projection of the La$_3$Au$_4$In$_7$ structure on the $yz$ plane and coordination polyhedra of the atoms: La (a, b), Au (c–d), and In (e–h).

Fig. 50. Projection of the Eu$_2$Au$_3$In$_4$ structure on the $xy$ plane and coordination polyhedra of the atoms: Eu (a–c), Au (d–f), and In (g–j).
Fig. 51. Projection of the Pr$_5$Ni$_6$In$_{11}$ structure on the $xy$ plane and coordination polyhedra of the atoms: Pr (a, b), Ni (c, d), and In (e–i).

prisms (In4 with CN = 9). The shortest distances are Eu–Au (304.9 pm), Eu–In (337.3 pm), Au–In (266.5 pm), and In–In (313.7 pm).

4.31. Structure type Pr$_5$Ni$_6$In$_{11}$ (fig. 51). SG $Cmmm$; $Z = 2$; $a = 1457.9$ pm; $b = 1457.9$ pm; $c = 440.0(1)$ pm. 8Pr1: 8$p$ $x$ $y$ 0 ($x = 0.36301$, $y = 0.32683$); 2Pr2: 2$c$ 1/2 0 1/2; 8Ni1: 8$p$ ($x = 0.41618$, $y = 0.13737$); 4Ni2: 4$j$ 0 $y$ 1/2 ($y = 0.90655$); 8In1: 8$q$ $x$ $y$ 1/2 ($x = 0.30739$, $y = 0.15587$); 4In2: 4$g$ $x$ 0 0 ($x = 0.69544$); 4In3: 4$h$ $x$ 0 1/2 ($x = 0.16067$); 4In4: 4$j$ ($y = 0.72503$); 2In5: 2$a$ 0 0 0 (Kalychak et al., 1987). The Pr1 CP are face capped pentagonal prisms similar to the Ce2 CP in Ce$_4$Ni$_7$In$_8$ (CN = 17), Pr2 has face capped hexagonal prisms (CN = 20), Ni1 and Ni2 have tricapped (CN = 9) or tetracapped (CN = 10) trigonal prisms, and the In1–In5 atoms have distorted cuboctahedra (CN = 12). The shortest distances are Pr–Ni (286.8 pm), Pr–In (322.0 pm), Ni–Ni (244.3 pm), Ni–In (258.1 pm), and In–In (298.3 pm).

4.32. Structure type Ce$_8$Pd$_{24}$Sb (Gordon and DiSalvo, 1996) (fig. 52). SG $Pm\bar{3}m$; $Z = 1$; $a = 844.57$ pm for Ce$_8$Pd$_{24}$In; 8Ce: 8$g$ $x$ $x$ $x$ ($x = 0.25140$); 6Pd1: 6$f$ $x$ 1/2 1/2 ($x = 0.25552$); 6Pd2: 6$e$ 0 0 0 ($x = 0.31118$); 12Pd3: 12$h$ $x$ 1/2 0 ($x = 0.26675$); 1In: 1$a$ 0 0 0. (Gordon et al., 1996). The cerium CP have 19 vertices (CN = 19), the Pd1 and Pd3 CP are slightly distorted cuboctahedra (CN = 12), Pd2 has base capped tetragonal antiprisms (CN = 10), and the indium atoms have tetrahexahedra (CN = 14). The shortest distances are Ce–Ce (419.9 pm), Ce–Pd (297.0 pm), Ce–In (367.8 pm), Pd–Pd (276.0 pm), and Pd–In (262.8 pm).

4.33. Structure type PrCo$_2$Ga (Yarmolyuk and Krisyakevich, 1976) (fig. 53). SG $Pm\bar{3}m$; $Z = 2$; $a = 503.3(1)$, $b = 405.0(2)$, $c = 712.2(2)$ pm for TbCo$_2$In. 2Tb: 2$e$ 1/4 1/2 $z$
Fig. 52. Projection of the Ce$_8$Pd$_{24}$In structure on the $xy$ plane and coordination polyhedra of the atoms: Ce (a), Pd (b-d), and In (e).

Fig. 53. Projection of the TbCo$_2$In structure on the $xz$ plane and coordination polyhedra of the atoms: Tb (a), Co (b, c) and In (d).

(Z = 0.28813); 2Co1: 2b 0 0 0; 2Co2: 2e ($z = 0.9064$); 2In: 2f 1/4 0 $z$ ($z = 0.65061$) (Kalychak et al., 1993a). The TbCo$_2$In structure differs from the PrCo$_2$Ga structure, because the indium atoms occupy the Co1 position 2f instead of the gallium position 2e. The terbium CP are 17-vertices polyhedra, which are similar to the samarium CP in Sm$_2$Co$_9$In$_3$ (CN = 17). Co1 has a 12-vertices polyhedron, which is similar to the Cu2 CP in CaCu$_5$ (CN = 12) and Co2 has an 11-vertices polyhedron (CN = 11). The indium atoms have a distorted cuboctahedral coordination (CN = 12). Two more indium atoms at longer distances can be included in the coordination sphere, resulting in higher coordination numbers. These coordinations are similar to those in ZrNiAl and Lu$_3$Co$_{2-x}$In$_4$. The shortest distances are Tb–Co (272.8 pm), Tb–In (326.0 pm), Co–Co (247.5 pm), and Co–In (272.4 pm).

4.34. Structure type GdPt$_2$Sn (De Mooij and Buschow, 1984) (fig. 54). SG $P6_3/mmc$; $Z = 2$; $a = 464.45(7)$, $c = 935.4(2)$ pm for LaPd$_2$In. 2La: 2c 1/3 2/3 1/4; 4Pd: 4f 1/3 2/3 $z$ ($z = 0.5779$); 2In: 2a 0 0 0 (Xue et al., 1992). The lanthanum CP are 14-vertices polyhedra, which can be considered as distorted, base capped hexagonal prisms (CN = 14). The palladium coordination derives from an icosahedron, but with the reduced CN = 11. The indium atoms
have defective rhombododecahedra (CN = 12). The shortest interatomic distances are La–Pd (306.2 pm), La–In (355.8 pm), and Pd–In (277.7 pm). Similar atomic arrangements occur in LiCu$_2$Sn (Kripyakevich and Oleksiv, 1970) and ZrPt$_2$Al (Ferro et al., 1975). The series of RPt$_2$In ($R =$ Gd, Tb, Dy, Ho, Y; Dwight, 1987) has mistakenly been ascribed to the ZrPt$_2$Al type, in which the 2a position is occupied by the $R$ atoms and 2c by the indium atoms.

4.35. Structure type MnCu$_2$Al$_2$ (Heusler, 1934) (fig. 55). SG $Fm\bar{3}m$; $Z = 4$; $a = 684.5(1)$ pm for LaCu$_2$In. 4La: 4b 1/2 1/2 1/2; 8Cu: 4c 1/4 1/4 1/4; 4In: 4a 0 0 0 (Kalychak et al., 1984a). The CP of the three atomic sites are rhombododecahedra (CN = 14), if the atoms have similar radii. The CN14 CP occur for the larger size $R$ and indium atoms in the $RCu_2$In structures, while the copper atoms have only CN 8 in the form of a cube, because the six copper atoms at 342.2 pm may not be included into the nearest coordination sphere. The shortest distances are La–Cu (296.4 pm), La–In (342.2 pm), and Cu–In (296.4 pm). In a first publication (Marazza et al., 1975), the tungsten or CsCl types have been assigned to the $RAu_2$In compounds. The later work (Besnus et al., 1985) correctly refers to the MnCu$_2$Al$_2$ type.

4.36. Structure type MgCuAl$_2$ (Perlitz and Westgren, 1943) (fig. 56). SG $Cmcm$; $Z = 4$; $a = 432.8(2)$, $b = 1042.3(2)$, $c = 730.3(2)$ pm for TbNiIn$_2$. 4Tb: 4c 0 y 1/4 ($y = 0.0705$);
4Ni: $4c\ (y = 0.7844)$; 8In: $8f\ 0\ y\ z;\ (y = 0.3589, z = 0.0506)$ (Zaremba et al., 1987a).

The terbium CP are pentacapped pentagonal prisms (CN = 15). Two additional terbium atoms, which cap the prism bases, can be included in the coordination sphere of the terbium atoms. Their Tb–Tb distance corresponds to the lattice parameter $a$. The nickel CP are tricapped trigonal prisms (CN = 9); the indium atoms have a distorted cuboctahedral coordination (CN = 12). The shortest distances are Tb–Ni (299.1 pm), Tb–In (317.0 pm), Ni–In (265.6 pm), and In–In (290.7 pm).

4.37. Structure type PrNiIn$_2$ (fig. 57). SG Cmcm; $Z = 20;\ a = 440.0(2),\ b = 1833.9(6),\ c = 2164.6(5)$ pm. 8Pr1: $8f\ 0\ y\ z\ (y = 0.22883, z = 0.12764);\ 8Pr2: 8f\ (y = 0.43744, z = 0.07986);\ 4Pr3: 4c\ 0\ 1/4\ (y = 0.46930);\ 8Ni1: 8f\ (y = 0.01428, z = 0.15791);\ 8Ni2: 8f\ (y = 0.34429, z = 0.53222);\ 4Ni3: 4c\ (y = 0.31569);\ 8In1: 8f\ (y = 0.06702, z = 0.04296);\ 8In2: 8f\ (y = 0.13635, z = 0.67518);\ 8In3: 8f\ (y = 0.19974, z = 0.53323);\ 8In4: 8f\ (y = 0.59918, z = 0.14383);\ 4In5: 4c\ (y = 0.10966);\ 4In6: 4c\ (y = 0.72458)$ (Zaremba et al., 2000c). The Pr1–Pr3 CP are pentacapped pentagonal prisms (CN = 15), Ni1–Ni3 have tricapped trigonal prisms (CN = 9), and In1–In6 have distorted cuboctahedra (CN = 12). The shortest distances are Pr–Ni (281.1 pm), Pr–In (325.1 pm), Ni–In (265.6 pm), and In–In (315.6 pm).

4.38. Structure type HfNiGa$_2$ (Markiv et al., 1988; Schläuter et al., 2003) (fig. 58). SG I4mm; $Z = 20;\ a = 1397.7(1),\ b = 925.5(1)$ pm for PrPdIn$_2$. 8Pr1: $8d\ x\ 0\ z\ (x = 0.71950, z = 0.61820);\ 8Pr2: 8c\ x\ x\ z\ (x = 0.703249, z = 0.63088);\ 2Pr3: 2a\ 0\ 0\ z\ (z = 0.01276);\ 2Pr4: 2a\ (z = 0.54244);\ 8Pd1: 8d\ (x = 0.78208, z = 0.97650);\ 8Pd2: 8c\ (x = 0.66014, z = 0.96880);\ 4Pd3: 4b\ 0\ 1/2\ z\ (z = 0.88810);\ 16In1: 16e\ x\ y\ z\ (x = 0.85917, y = 0.64726, z = 0.88008);\ 8In2: 8d\ (x = 0.83413, z = 0.28644);\ 8In3: 8d\ (x = 0.61702, z = 0.14024);\ 8In4: 8c\ (x = 0.86651, z = 0.78597)$ for PrPdIn$_2$ (Zaremba et al., 2003c). The four crystallographically different praseodymium atoms have coordination numbers 16 (Pr4) and 17 (Pr1–Pr3) with quite irregular coordination polyhedra. This is also the case for the palladium atoms, which
Fig. 57. Projection of the PrNiIn$_2$ structure on the $yz$ plane and coordination polyhedra of the atoms: Pr (a–c), Ni (d–f), and In (g–l).

have CN 10 (Pd1 and Pd2) and CN 12 (Pd3). The coordination numbers for the In1–In4 atoms range from CN 13 to CN 15. All indium atoms have praseodymium, palladium, and indium neighbors. The shortest distances are Pr–Pd (306.4 pm), Pr–In (318.0 pm), Pd–In (276.0 pm), and In–In (304.5 pm).

4.39. Structure type Ho$_{10}$Ni$_9$In$_{20}$ (fig. 59). SG $P4/nmm$; $Z = 2$; $a = 1328.6(3)$, $c = 908.3(3)$ pm. 2Ho1: 2c 1/4 1/4 1/4 $z$ ($z = 0.6382$); 2Ho2: 2c ($z = 0.1696$); 8Ho3: 8i 1/4 $y$ $z$ ($y = 0.5277$, $z = 0.2324$); 8Ho4: 8j $x$ $x$ $x$ $z$ ($x = 0.4538$, $z = 0.7334$); 2Ni1: 2a 1/4 3/4 0; 8Ni2: 8i ($y = 0.0272$, $z = 0.5899$); 8Ni3: 8j ($x = 0.5948$, $z = 0.8997$); 8In1: 8g $x$ $y$ 0 ($x = 0.0964$, $y = 0.9036$); 8In2: 8h $x$ $y$ 1/2 ($x = 0.6207$, $y = 0.3793$); 8In3: 8i ($y = 0.0824$, $z = 0.9060$); 8In4: 8i ($y = 0.8654$, $z = 0.7659$); 8In5: 8j ($x = 0.3820$, $z = 0.4101$) (Zaremba et al., 1987b). The Ho1–Ho3 CP are pentagonal prisms with capped bases and four capped rectangular faces (CN = 16). The Ho4 atoms, however, have a completely face capped pen-
Fig. 58. Projection of the PrPdIn$_2$ structure on the $xy$ plane and coordination polyhedra of the atoms: Pr (a–d), Pd (e–g), and In (h–k).

tetragonal prism (CN = 17). Ni1 has a bisfenoid (CN = 8), Ni2 and Ni3 have distorted base capped tetragonal antiprisms (CN = 10). In1 and In2 have distorted cuboctahedra (CN = 12), while In3 and In5 have distorted cuboctahedra with an additional atom (CN = 13). In4 has a distorted icosahedral coordination (CN = 12). The shortest distances are Ho–Ni (288.7 pm), Ho–In (307.4 pm), Ni–In (262.1 pm), and In–In (296.9 pm).

4.40. **Structure type La$_6$Co$_{11}$Ga$_3$** (Sichevich et al., 1985) is identical to Nd$_6$Fe$_{13}$Si (Allemand et al., 1990) (fig. 60). SG I4/mcm; Z = 4; $a = 810.2$, $c = 2357.6$ pm for La$_6$Co$_{13}$In. 16La1: $16f$ $x$ 1/2 $+$ $x$ $z$ ($x = 0.1627$, $z = 0.1857$); 8La2: $8j$ 0 0 $z$ ($z = 0.8944$); 16Co1: $16l$ ($x = 0.1795$, $z = 0.0585$); 16Co2: $16k$ $x$ 0 ($x = 0.0659$, $y = 0.2103$); 16Co3: $16l$ ($x = 0.3856$, $z = 0.0936$); 4Co4: 4d 0 1/2 0; 4In: 4a 0 0 1/4. The atomic coordinates are taken from the isostructural compound with lead Pr$_6$Fe$_{13}$Pb (Weitzer et al., 1993). The La1 and La2 CP are pentacapped pentagonal prisms with one (CN = 16) and two (CN = 17) capped bases, Co1–Co4 have distorted icosahedra (CN = 12), and the indium atoms have base capped tetragonal antiprisms (CN = 10). The shortest distances are La–La (357.4 pm), La–Co (300.1 pm), La–In (339.2 pm), and Co–Co (240.7 pm).

4.41. **Structure types Fe$_2$P** (Rundqvist and Jellinek, 1959) and ZrNiAl (Kripyakevich et al., 1967) (fig. 61). These structures are related via their coloring of the atoms on the four Wyckoff positions. In the latter, the zirconium and aluminum atoms take the positions of the iron atoms in the Fe$_2$P structure. In the ternary indides of equiatomic composition RTIn ($T =$ Ni, Cu,
Fig. 59. Projection of the Ho$_{10}$Ni$_9$In$_{20}$ structure on the $xy$ plane and coordination polyhedra of the atoms: Ho (a–d), Ni (e–g), and In (h–l).

Fig. 60. Projection of the La$_6$Co$_{13}$In structure on the $yz$ plane and coordination polyhedra of the atoms: La (a, b), Co (c–f), and In (g).
Rh, Pd, Pt, Au) (Ferro et al., 1974a, 1974b; Buschow, 1975; Rossi et al., 1977) the transition metal and indium atoms are ordered. The structures can be ascribed to the ZrNiAl type; SG \( P6_2m; Z = 3; a = 740.8(1), c = 369.3(1) \) pm for LuNiIn. 3Lu: 3g x 0 1/2 (x = 0.5824); 1Ni1: 1b 0 0 1/2; 2Ni2: 2c 1/3 2/3 0; 3In: 3f x 0 0 (x = 0.2517) (Sysa et al., 1988). The CP of the lutetium atoms are pentagonal prisms with capped faces (CN = 17), the CP for the Ni1 and Ni2 atoms are tricapped trigonal prisms (CN = 9) and the indium atoms have distorted tetrahexahedra (CN = 14). The shortest distances are Lu–Ni (288.7 pm), Lu–In (306.3 pm), Ni–In (262.1 pm), and In–In (322.2 pm).

4.42. Structure type TiNiSi (Shoemaker and Shoemaker, 1965) (fig. 62). SG \( Pnma; Z = 4; a = 748.30(13), b = 447.20(8), c = 853.50(14) \) pm for EuPdIn. 4Eu: 4c x 1/4 z (x = 0.03494, z = 0.67695); 4Pd: 4c (x = 0.27153, z = 0.37356); 4In: 4c (x = 0.14172, z = 0.06247) (Pöttgen, 1996b). The CP of the europium atoms are pentagonal prisms with six additional equatorial atoms and two atoms capping the bases (CN = 18); the palladium atoms have tricapped trigonal prisms (CN = 9) and the CP of the indium atoms are distorted cuboc-
Fig. 63. Projection of the EuZnIn structure on the yz plane and coordination polyhedra of the atoms: Eu (a) and M (b) (M = Zn₀.₅₀In₀.₅₀).

tetrahedra (CN = 12). The shortest distances are Eu–In (314.6 pm), Eu–Pd (338.1 pm), and Pd–In (282.4 pm). The crystal structure of EuPdIn was also studied by Cirafici et al. (1985), however, the positions of the Pd and In atoms were not determined correctly.

4.43. Structure type CeCu₂ (Larson and Cromer, 1961) (i.e. KHg₂; Duwell and Baenziger, 1955) (SG Imma; Z = 4). For EuZnIn (fig. 63) a = 482.3(1), b = 780.7(2), c = 827.6(2) pm (Pöttgen, 1996a). The europium atoms take the positions of cerium (4e 0 1/4 z (z = 0.53763)), while the zinc and indium atoms (M) statistically occupy the copper positions (8h 0 y z (y = 0.0489, z = 0.1648)).

4.44. The structure type CaIn₂ (Iandelli, 1964) (SG P6₃/mmc; Z = 2) is characteristic for RZnIn (Mazzone et al., 1982) and RCDIn (Rossi et al., 1981). The R atoms take the positions of the calcium atoms, while the zinc (cadmium) and indium atoms statistically occupy the positions of the indium atoms of CaIn₂. For R(Ag₀.₂₅In₀.₇₅)₂ the 4c position is occupied by a mixture (M = Ag₀.₂₅In₀.₇₅) with z = 0.463 (fig. 64) (Sysa and Kalychak, 1992). The shortest distances for Ce(Ag₀.₂₅In₀.₇₅)₂ (a = 490.1(2), c = 759.2(6) pm) are Ce–M (326.5 pm) and M–M (288.5 pm).

4.45. Structure type Lu₃Co₁.₈₇In₄ (fig. 65). SG P6₅; Z = 1; a = 781.4(2), c = 352.1(1) pm. 3Lu: 3k x y 1/2 (x = 0.2949, y = 0.0432); 1Co1: 1a 0 0 0; 0.87Co2: 1d 1/3 2/3 1/2; 1In1: 1e 2/3 1/3 0; 3In2: 3 f x y 0 (x = 0.0744, y = 0.6650) (Zaremba et al., 1989). The CP

Fig. 64. Projection of the Ce(Ag₀.₂₅-In₀.₇₅)₂ structure on the xy plane and coordination polyhedra of the atoms: Ce (a) and M (b) (M = Ag₀.₂₅In₀.₇₅).
of the lutetium atoms are pentagonal prisms with three additional equatorial atoms and two atoms capping the bases (CN = 15). The CP for the Co1 and Co2 atoms are tricapped trigonal prisms (CN = 9). The CP for the In1 atoms are trigonal prisms with additional atoms capping all faces (CN = 11) and the In2 atoms have distorted tetrahedra (CN = 14). The shortest distances are Lu–Co (278.3 pm), Lu–In (311.6 pm), Lu–Lu (352.1 pm), Co–In (267.7 pm), and In–In (293.5 pm).

4.46. Structure type AlB₂ (Hofmann and Jäniche, 1935). SG P6/mmm; Z = 1; a = 481.8(1), c = 389.9(1) pm for Ce(Cu₀.₂₅In₀.₇₅)₂ (fig. 66). 1Ce: 1a 0 0 0; 2M: 2d 1/3 2/3 1/2 (M = Cu₀.₂₅In₀.₇₅) (Baranyak et al., 1988b). The CP of the cerium atoms are hexagonal prisms with capped bases (CN = 14), the M atoms (0.25Cu + 0.75In) have tricapped trigonal prisms (CN = 9). The shortest distances are Ce–M (339.6 pm) and M–M (278.2 pm). The AlB₂ structure of Tb₂CuIn₃ has been refined from neutron diffraction data (Siouris et al., 2001), i.e. Tb(Cu₀.₂₅In₀.₇₅)₂.

Fig. 65. Projection of the Lu₃Co₁.₈₇In₄ structure on the xy plane and coordination polyhedra of the atoms: Lu (a), Co (b, c), and In (d, e).

Fig. 66. Projection of the Ce(Cu₀.₂₅In₀.₇₅)₂ structure on the xy plane and coordination polyhedra of the atoms: Ce (a) and M (b) (M = Cu₀.₂₅In₀.₇₅).
4.47. **Structure type Mn$_2$AlB$_2$** (Becher et al., 1966). SG $Cmmm$; $Z = 2$; $a = 390.00(1)$, $b = 1418.6(2)$, $c = 369.4(1)$ pm for Y$_2$Ni$_2$In (fig. 67). 4Y: 4$j$ 0 $y$ 1/2 ($y = 0.36364$); 4Ni: 4$i$ 0 $y$ 0 ($y = 0.19892$); 2In: 2$a$ 0 0 0 (Zarembo et al., 1988). The CP of the yttrium atoms are pentacapped pentagonal prisms (CN = 15), the nickel atoms have tricapped trigonal prisms (CN = 9), and the indium atoms have distorted tetrahexahedra (CN = 14). The shortest distances are Y–Y (390.0 pm), Y–Ni (282.8 pm), Y–In (331.0 pm), Ni–Ni (243.0 pm), and In–In (282.2 pm).

4.48. **Structure type W$_2$CoB$_2$** (Rieger et al., 1966). SG $Immm$; $Z = 2$; $a = 438.8(5)$, $b = 574.30(6)$, $c = 873.37(9)$ pm for Yb$_2$Cu$_2$In (fig. 68). 4Yb: 4$j$ 1/2 0 $z$ ($z = 0.2962$); 4Cu: 4$h$ 0 $y$ 1/2 ($y = 0.2138$); 2In: 2$a$ 0 0 0 (Giovannini et al., 2001). The CP for the ytterbium atoms are pentacapped pentagonal prisms (CN = 15), for the copper atoms tricapped trigonal prisms (CN = 9), and distorted cuboctahedra (CN = 12) for the indium atoms. The shortest distances are Yb–Cu (306.5 pm), Yb–In (337.8 pm), and Cu–In (274.1 pm).

4.49. **Structure type U$_2$Pt$_2$Sn** (Gravereau et al., 1994). SG $P4_2/mnm$; $Z = 4$; $a = 779.1(1)$, $c = 734.8(2)$ pm for Tm$_2$Au$_2$In (fig. 69). 4Tm1 4$g$ $x$ $y$ 0 ($x = 0.3140$, $y = 0.6860$); 4Tm2 4$f$
The CP for both thulium atoms are pentagonal prisms with capped faces (CN = 17), tricapped trigonal prisms for the gold atoms (CN = 9), and distorted tetrahexahedra (CN = 14) for the indium atoms. The shortest distances are Tm–Tm (368.6 pm), Tm–Au (285.3 pm), Tm–In (338.5 pm), Au–Au (282.3 pm), Au–In (307.2 pm), and In–In (367.4 pm).

4.50. Structure type Mo2FeB2 (Rieger et al., 1964). SG P4/mbm; Z = 2; a = 749.9(2), c = 375.1(2) pm for Ce2Ni2In (fig. 70). 4Ce: 4h x 1/2 + x 1/2 (x = 0.1734); 4Ni: 4g x 1/2 + x 0 (x = 0.3768); 2In: 2a 0 0 0 (Kalychak et al., 1990). The CP for the cerium atoms are pentagonal prisms, which are capped on all faces (CN = 17), the nickel atoms have tricapped trigonal prisms (CN = 9), and the indium atoms have tetrahexahedra (CN = 14). The shortest distances are Ce–Ce (367.8 pm), Ce–Ni (285.9 pm), Ce–In (334.8 pm), Ni–Ni (261.2 pm), Ni–In (297.3 pm), and In–In (375.1 pm). The samarium and the yttrium compounds with Mo2FeB2 type show defects on the nickel sites leading to the composition R2Ni2−xIn (x = 0.22). For Gd2Ni1.88In the positions are: 4Gd: 4h (x = 0.17529); 3.56Ni: 4g (x = 0.37990); 2In: 2a 0 0 0 (Kalychak et al., 1990).

4.51. Structure type Lu3Ni2In4 (fig. 71). SG Pham; Z = 2; a = 1756.8(5), b = 779.8(2), c = 352.2(1) pm. 2Lu1: 2a 0 0 0; 4Lu2: 4g x y 0 (x = 0.2197, y = 0.2458); 4Lu3: 4g (x = 0.4137, y = 0.1170); 4Ni: 4h x y 1/2 (x = 0.3020, y = 0.0220); 4In1: 4h (x = 0.5676, y = 0.2071); 4In2: 4h (x = 0.8477, y = 0.0747) (Zaremba et al., 1991). The CP for the Lu1 and Lu3 atoms are pentagonal prisms with all faces capped (CN = 17), the Lu2 atoms have tetragonal prisms with capped faces (CN = 14), the nickel atoms have tricapped trigonal prisms (CN = 9), and the In1 and In2 atoms have distorted tetrahexahedra where some vertices...
Fig. 70. Projection of the Ce\textsubscript{2}Ni\textsubscript{2}In structure on the $xy$ plane and coordination polyhedra of the atoms: Ce (a), Ni (b), and In (c).

Fig. 71. Projection of the Lu\textsubscript{5}Ni\textsubscript{2}In\textsubscript{4} structure on the $xy$ plane and coordination polyhedra of the atoms: Lu (a–c), Ni (d), and In (e, f).

are not occupied (reduced CN = 12 and 13, respectively). The shortest distances are Lu–Lu (334.8 pm), Lu–Ni (273.9 pm), Lu–In (311.9 pm), Ni–In (273.6 pm), and In–In (323.0 pm).

4.52. The structure type CsCl is characteristic for the ternary indides $R$Ag\textsubscript{x}In\textsubscript{1–x} (Ihrg et al., 1973; Lal and Methfessel, 1981). The compounds belong to solid solutions of binary $R$Ag and $R$In, which both crystallize with the CsCl type structure (fig. 72).
4.53. Structure type LT–LaAg$_x$In$_{1-x}$ ($x = 0.75–0.89$). SG $I4/mmm$; $Z = 4$; $a \cong 755$, $c \cong 780$ pm for LaAg$_{0.75}$In$_{0.25}$ (fig. 73). 4La1: 4$d$ 0 1/2 1/4; 4La2: 4$e$ 0 0 1/4 $-\varepsilon$; 8(Ag$_x$In$_{1-x}$): 8$h$ 1/4 + $\delta$ 1/4 + $\delta$ 0; $\varepsilon$ and $\delta$ are the atoms shifts in the tetragonal versus the cubic phase (Maetz et al., 1980). The CP for La1 and La2 are rhombododecahedra (CN = 14) and the CP for the mixed site (Ag$_x$In$_{1-x}$) are cuboctahedra (CN = 12).

4.54. Structure type Mo$_5$B$_2$Si (Aronsson, 1958). SG $I4/mcm$; $Z = 4$; $a = 752.16(7)$, $c = 1315.9(1)$ pm for Tm$_{4.82}$Ni$_2$In$_{1.18}$ (fig. 74) (Lukachuk et al., 2002). 16Tm1: 16$l$ $x + 1/2$ $z$ ($x = 0.65841$, $z = 0.13672$); 4$M$: 4$c$ 0 0 0 ($M = \text{Tm}_{0.82}\text{Ni}_{0.18}$); 8Ni: 8$h$ $x + 1/2$ 0 ($x = 0.1298$); 4In2: 4$a$ 0 0 1/4. The CP of the Tm1 atoms are 16-vertices polyhedra (CN = 16). The CP of the $M$ atoms are tetragonal prisms with nickel capped rectangular faces and indium capped bases (CN = 14). The nickel atoms have tricapped trigonal prisms (CN = 9) and the In2 atoms have a base capped tetragonal antiprism (CN = 10) as coordination polyhedra. The shortest distances are Tm–Ni (282.5 pm), Tm–In (320.1 pm), and Ni–Ni (276.1 pm).
4.55. **Structure type Ce₁₂Pt₇In** (fig. 75). SG I4/mcm; Z = 4; \(a = 1210.2(1)\), \(c = 1454.2(2)\) pm. 32Ce1: 32m \(x y z\) (\(x = 0.0711\), \(y = 0.2082\), \(z = 0.14222\)), 8Ce2: 8g 0 1/2 0 (\(z = 0.1392\)), 8Ce3: 8h \(x x + 1/2 0\) (\(x = 0.1753\)), 16Pt1: 16l \(x x + 1/2 z\) (\(x = 0.17502\), \(z = 0.19565\)), 8Pt2: 8h (\(x = 0.6245\)), 4Pt3: 4a 0 0 1/4, 4In: 4c 0 0 0 (Galadzhun et al., 2003). The CP of the cerium atoms have 14 and 15 vertices (CN 14 and 15), the CP of Pt1 have nine vertices,
the CP of Pt2 are tricapped trigonal prisms (CN 9), the CP of Pt3 are tetragonal antiprisms (CN 8), and for the indium atoms the CP are cubes (CN 8). The shortest distances are Ce–Pt (284.5 and 293.9 pm), Ce–Ce (322.3 pm), and Ce–In (337.1 pm).

4.56. Structure type Sm$_{12}$Ni$_6$In (fig. 76). SG Im$ar{3}$; $Z = 2; a = 980.0(2)$ pm. 24Sm: 24g$_{xy}$ ($x = 0.6871, y = 0.1966$); 12Ni: 12e$_{00}1/2$ ($x = 0.8744$); 2In: 12. The CP for the samarium atom s are pentacapped pentagonal prisms (CN 15), the nickel atoms have tricapped trigonal prisms (CN 9), and the indium atoms have icosahedral coordination (CN 12). The shortest distances are Sm–Sm (362.3 pm), Sm–Ni (266.1 pm), Ni–Ni (246.3 pm), and Sm–In (349.3 pm).

4.57. Structure type Ho$_6$Co$_2$Ga (Gladyshevskii et al., 1983). SG Immm; $Z = 4; a = 934.8(6), b = 943.0(6), c = 990.6(9)$ pm for Ho$_6$Co$_{2+x}$In$_{1-x}$ ($x = 0.135$) (fig. 77). 8Ho1: 8n $x y 0 (x = 0.2855, y = 0.1874); 8$Ho2: 8m $x 0 z (x = 0.3006, z = 0.3155); 8$Ho3: 8f $0 y z (y = 0.2068, z = 0.2312); 4$Co1: 4j $1/2 1/2 0 (z = 0.1124); 4$Co2: 4g $1/2 0 1/2 (y = 0.1260); 2M: 20 0 0 0; 2In: 2c 0 0 1/2 ($M = Co_{0.27}In_{0.73}$) (Kalychak et al., 1993b). The CP of the Ho1 and Ho2 atoms are pentagonal prisms with five and four additional atoms on the rectangular faces (CN = 15 and 14), and the CP for the Ho3 atoms are tetragonal prisms with five additional atoms (CN = 13). The CP of the Co1 and Co2 atoms are tricapped trigonal prisms (CN = 9), the $M$ atoms have cubes (CN = 8), and the indium atoms center icosahedra (CN = 12). The shortest distances are Ho–Ho (336.7 pm), Ho–Co (274.2 pm), Ho–$M$ (300.8 pm), Ho–In (330.0 pm), and Co–Co (222.7 pm).

4.58. Structure type Lu$_{14}$Co$_2$In$_3$ (fig. 78). SG P4$_2$/nmc; $Z = 4; a = 933.3(2), c = 2263.3(4)$ pm. 4Lu1: 4c $3/4 1/4 0 (z = 0.1457); 4$Lu2: 4d $1/4 1/4 1/4 (z = 0.2127); 8Lu3:
Fig. 77. Projection of the Ho$_6$Co$_{2.135}$In$_{0.865}$ structure on the $yz$ plane and coordination polyhedra of the atoms: Ho (a–c), Co (d, e), In (f), and $M$ (g) ($M = $Co$_{0.27}$In$_{0.73}$).

Fig. 78. Projection of the Lu$_{14}$Co$_2$In$_3$ structure on the $yz$ plane and coordination polyhedra of the atoms: Lu (a–g), Co (h), and In (i, j).

8g 1/4 y z ($y = 0.5467, z = 0.3045$); 8Lu4: 8g ($y = 0.5595, z = 0.9945$); 8Lu5: 8f x y 1/4 ($x = 0.5612, y = 0.4388$); 8Lu6: 8g ($y = 0.4397, z = 0.4686$); 16Lu7: 16h x y z ($x = 0.4375, y = 0.4342, z = 0.1045$); 8Co: 8g ($y = 0.5354, z = 0.1886$); 4In1: 4c
The CP of the Lu1 and Lu4 atoms are pentagonal prisms with additional atoms capping the bases (CN = 12), the CP of the Lu2 and Lu5 atoms are pentagonal prisms with capped bases and two additional atoms capping the rectangular faces (CN = 14). The Lu3 atoms have pentacapped pentagonal prisms (CN = 15) and the CP of the Lu6 and Lu7 atoms have 13-vertices. Except for the Lu1 atom, all lutetium CP contain fragments of icosahedra. The CP for the cobalt atoms are tricapped trigonal prisms (CN = 9) and the CP for In1 and In2 are icosahedra (CN = 12). The shortest distances are Lu–Lu (326.0 pm), Lu–Co (262.5 pm), Lu–In (284.8 pm), and Co–In (295.2 pm). The structure of Gd14Co3In2.7 (SG P42/nmc; Z = 4; a = 961.9(2), c = 2329.1(4) pm) (Canepa et al., 2002b) is related to that of Lu14Co3In3. It differs from the latter by the fact that the Co1 atoms occupy the 8g position only by 86%, and additional cobalt atoms occupy the 4d position and in the 4c position a statistical mixture (Co0.31In0.69) occurs instead of a pure indium occupancy, resulting in a different composition.

5. Geometrical relations of some $R_x T_y In_z$ structures

A significant number of structural relationships between indides (table 12) are known in the literature. Principally, structural relations can be investigated by various methods. One very concise way is establishing a group–subgroup relations based on the Bärnighausen formalism (Bärnighausen, 1980; Bärnighausen and Müller, 1996). Here, the evolution of atomic parameters can be followed up in going from high symmetry (given by an aristotype) to
low symmetry (various superstructures). This is an excellent tool to analyze phase transitions. This concept, however, does not include structural similarities, in particular in the absence of a group–subgroup relation. Thus, it does not allow comparing and discussing those structures. Including atomic replacements widens the possibilities to compare structures (Kripyakevich, 1977). Another simplifying step is to look at the local environment in terms of slabs. The latter is known as chemical twinning of different structures (Aronsson et al., 1960; Andersson, 1983; Parthé and Chabot, 1984; Cenzual and Parthé, 1984; Parthé et al., 1985; Parthé, 1990; Parthé et al., 1993).

All ternary compounds of the rare earth metal–transition metal–indium systems with known crystal structure can be divided into three large groups depending on their position in the ternary system, i.e. on their composition. The first group comprises compounds within the triangle delineated by a transition metal, RM$_2$, and elemental indium (fig. 79). The compounds crystallizing with NaZn$_{13}$, ThMn$_{12}$, Th$_2$Ni$_{17}$, YNi$_5$In$_2$, EuAg$_4$In$_8$, CeNi$_5$Sn, CeCu$_{14}$In$_{1.62}$, MgCu$_4$Sn, Lu$_6$Co$_{26-x}$In$_{14}$, YbAg$_3$In$_4$, Nd$_6$Fe$_{13}$Si, Ce$_8$Pd$_2$Sb, and MnCu$_2$Al structure types belong to this group. In most cases, they have complex multilayer structures with at least one small lattice parameter of 480–500 pm. High values for the coordination numbers (CN) of all atom types and similar coordination polyhedra (CP) are typical for those structures.

The second range of compounds (fig. 79) is situated in the region with higher R content. It is limited by the compositions RM$_2$–R$_2$M–In. The triangle includes the structure types HoCoGa$_5$, Ho$_2$CoGa$_8$, YNiAl$_4$, MgCuAl$_2$, Ce$_4$Ni$_7$In$_5$, Pr$_5$Ni$_6$In$_{11}$, ZrNiAl, AlB$_2$, TiNiSi, Lu$_3$Co$_{1.87}$In$_4$, Mn$_2$AlB$_2$, and Mo$_2$FeB$_2$, as well as PrCo$_2$Ga and GdPt$_2$Sn, which lie on the

Fig. 79. Grouping of the various structure types within the systems R–M–In according to chemical composition.
boundary line. The shortest lattice parameter, ranging from \( \sim 360 \) to \( 420 \) pm, represents two layers of atoms and is typical for this group of compounds. The structure types Sm\(_2\)Co\(_9\)In\(_3\), HoNi\(_{2.6}\)Ga\(_{2.4}\), LaNi\(_3\)In\(_6\), and Lu\(_5\)Ni\(_2\)In\(_4\) also belong to this group as far as the shortest lattice parameter is of concern. CaIn\(_2\) and U\(_2\)Pt\(_2\)Sn, which are superstructures of AlB\(_2\) and Zr\(_3\)Al\(_2\), respectively, have such short lattice parameters in their subcells.

The third and the smallest group (fig. 79), which is limited by the points \( R_2 M – R – In \), comprises the structures of Sm\(_{12}\)Ni\(_6\)In, Ho\(_6\)Co\(_2\)Ga, and Lu\(_{14}\)Co\(_2\)In\(_3\). The Ho\(_{10}\)Ni\(_9\)In\(_{20}\) type also belongs to this group, however, according to its stoichiometry, it belongs to the second group. These types are complex multilayer structures with the value of the shortest lattice parameter of \( 900 \) pm or more. Additional types in this group are Lu\(_5\)Ni\(_2\)In\(_4\) and LaAgIn.

The structural relations between the types Lu\(_3\)Co\(_{1.87}\)In\(_4\) and ZrNiAl, CeCu\(_{4.38}\)In\(_1.62\) and CeCu\(_6\), YNi\(_9\)In\(_2\) and Ce(Mn\(_{0.45}\)Ni\(_{0.55}\))\(_{11}\), EuAg\(_4\)In\(_8\) and SmZn\(_{11}\) (SmZn\(_{12-x}\) with \( x = 1 \)), as well as YbCd\(_6\) and YbAg\(_2\)In\(_4\) will be discussed in this section.

As illustrated in fig. 80, the Lu\(_3\)Co\(_{1.87}\)In\(_4\) structure (Zaremba et al., 1989) is formed by substitution of one transition metal atom by an indium atom on a single site in the structure type ZrNiAl, i.e. \( RNiIn \), resulting in a change of the composition from “\( R_3 M_3 X_3 \)” to \( R_3 M_2 X_4 \). This requires a reduction of symmetry from \( P\bar{6}2m \) to \( P\bar{6} \) (Zumdick and Pöttgen, 1999). The coordination number of the indium atoms of this substituted site increases as compared with the CN of the \( M \) atoms in the ZrNiAl type structure from nine (tricapped trigonal prism) to 11 (pentacapped trigonal prism). Another related structure is Lu\(_3\)CoGa\(_5\). Grin and Gladyshevskii (1989) reported that the Lu\(_3\)CoGa\(_5\) structure type (fig. 80, right-hand side) does not occur in indium-based systems. In this structure type, the substitution of the transition metal by a \( p \)-element takes place according to the scheme \( R_3 M_3 X_3 \rightarrow R_3 M X_5 \), which allows preservation of the space group symmetry, \( P\bar{6}2m \).

The structure of CeCu\(_{4.38}\)In\(_1.62\) (Kalychak et al., 1988a) is a derivative of the CeCu\(_6\) type (SG \( Pnma \); \( a = 811.2, b = 510.2, c = 1016.2 \) pm) (Cromer et al., 1960). It consists of two unit cells of the latter compound, connected via a two-fold axis. As a result two lattice parameters of these structure types are similar \( (b_{\text{CeCu}_{4.38}}) \approx c_{\text{CeCu}_6} ; c_{\text{CeCu}_{4.38} \text{In}_1.62} \approx b_{\text{CeCu}_6} \) and the

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Fig. 80. Relations between the structure types ZrNiAl (\( R_3 M_3 X_3 \)) (\( a \)), Lu\(_3\)Co\(_{1.87}\)In\(_4\) (\( R_3 M_2 X_4 \)) (\( b \)), and Lu\(_3\)CoGa\(_5\) (\( R_3 M X_5 \)) (\( c \)). Note the origin shift of \( b \) in the original work (Zaremba et al., 1989).
Fig. 81. Relations between the structure types CeCu$_{4.8}$In$_{1.62}$ $(a)$ and CeCu$_6$ $(b)$.

third in the CeCu$_{4.38}$In$_{1.62}$ type is about twice as large as the corresponding one in the CeCu$_6$ structure ($a_{\text{CeCu}_{4.38}\text{In}_{1.62}} \approx 2a_{\text{CeCu}_6}$) (fig. 81).

The structure of YNi$_9$In$_2$ (Kalychak et al., 1984b) is a superstructure of the Ce(Mn$_{0.45}$-Ni$_{0.55}$)$_{11}$ type (SG $P4/mmbm$, $a = 834.8$, $c = 491.9$ pm) (Kalychak et al., 1975). In the latter the manganese and nickel atoms randomly occupy four Wyckoff positions. On the sites 2$c$, 8$k$, and 8$j$, the atoms have CN 12 and their CP are slightly distorted icosahedra. The atoms on the 4$g$ site have CN 15. In the YNi$_9$In$_2$ type, the large indium atoms occupy the site 4$g$ and have a high CN with a 15-vertex coordination polyhedron, while the nickel atoms occupy the 2$c$, 8$k$, and 8$j$ sites (fig. 26).

Due to their close relations we also review the structure types EuAg$_4$In$_8$ (fig. 24) (Sysa et al., 1994a) and SmZn$_{11}$ (SG $P6/mmm$, $a = 897.4$, $c = 891.8$ pm) (Mason et al., 1970). The similarity of the lattice parameters, the identity of the space group, and the occupation of the same atomic sites with close values of the coordinates allow us to consider the EuAg$_4$In$_8$ structure as a superstructure of the SmZn$_{11}$ type, where the silver and indium atoms are ordered on the sites of the zinc atoms. However, there is one significant difference concerning the composition. In the SmZn$_{11}$ structure, the samarium atoms occupy additionally the sites 2$c$ and 1$b$ at 15 and 4%, respectively, and the sites 2$e$ and 4$h$ are occupied by zinc atoms only at 96 and 85%, while in the EuAg$_4$In$_8$ structure the latter are occupied by indium atoms at 100%. The defects result in the composition $\sim$SmZn$_{11}$ as compared to SmZn$_{12}$ with full occupancies.

A similar relation is observed for the binary structure types YCd$_6$ (SG $Im\bar{3}$, $a = 1548.3$ pm) (Larson and Cromer, 1971) and YbCd$_6$ (SG $Im\bar{3}$, $a = 1565.8$ pm) (Palenzona, 1971), and ternary YbAg$_2$In$_4$ (Sysa et al., 1989). These structures are actually identical considering the lattice parameters, the space group symmetry, the sites, and the parameters of the atoms. In YbAg$_2$In$_4$ the smaller silver and indium atoms are ordered, thus, it represents a ternary ordered superstructure of the binary ones. However, there is a difference between the binary compounds: in YCd$_6$ one cadmium atom occupies the 24$g$ site by about one third, and in
YbCd₆ one cadmium atom occupies the 16f site by about one half. These inconsistencies might be due to a variable stoichiometry or different qualities of the structure solution. On the other hand, in the ternary compound YbAg₂In₄, the silver atoms occupy the site 48h by about one sixth. It should be mentioned, that the mixed site occupancies actually do not result in a change of the general structural motif. This manifests the view of the structure-superstructure relationship at least in the sense of Kripyakevich (1977). The YbAg₂In₄ structure has one more peculiarity: the sites 16f and 24g are occupied by the atoms Ag1, Ag2, Ag3, and Ag4, respectively, on split positions, but with a total occupancy of 100%.

5.1. Structures of multiple substitution

The formation of the superstructures, in particular those of the second kind following the definition of Kripyakevich (1977), occurs by ordered substitution of one kind of atoms in the parent structure by two different atom types in a way that the total number of atoms per unit cell remains the same, i.e. ternary ordered variants of binary structures. Another variant for the formation of more complex structures from simple ones is the substitution of single atoms by pairs of atoms of another kind. This way, a more complex structure is obtained by multiple substitutions of larger atoms by pairs of atoms with smaller size. The indides with the structure types Th₂Ni₁₇ (Sysa et al., 1989), EuAg₄In₈ (Sysa et al., 1994a), Th₂Ni₁₇ (Kalychak et al., 1997), and Sm₁₂Ni₆In (Kalychak et al., 1998) belong to this class of substitution structures for ternary indides. The first three structure types are obtained by multiple substitution of a part of the R atoms in the structure type CaCu₅ by pairs of smaller atoms. The geometrical relations take place according to the schemes described below (for simplification all atoms of smaller size are designated as X).

For the ThMn₁₂ type (Kripyakevich, 1977), (tetragonal symmetry), which is connected with the CaCu₅ type via aₜₜₜₜₜₜ = \sqrt[3]{3}a_{CaCu₅}, cₜₜₜₜₜₜ = a_{CaCu₅} and Vₜₜₜₜₜₜ = 4V_{CaCu₅}; (CaCu₅ type) 4RX₅ = R₄X₂₀ = R₂R′₂X₂₀ 2R = 4X R₂X₄X′₂₀ = R₂X₂₄ = 2RX₁₂ (ThMn₁₂ type). For the structure type Th₂Ni₁₇ (hexagonal symmetry), for which aₜₜₜₜₜₜₜ = \sqrt[3]{3}a_{CaCu₅}, cₜₜₜₜₜₜ = 2c_{CaCu₅} and Vₜₜₜₜₜₜ = 6V_{CaCu₅} we obtain: (CaCu₅ type) 6RX₅ = R₆X₃₀ = R₄R′₂X₃₀ 2R = 4X R₄X₄X′₃₀ = R₄X₃₄ = 2RX₁₇ (Th₂Ni₁₇ type). The structure type EuAg₄In₈, which also has hexagonal symmetry and lattice parameters similar to the Th₂Ni₁₇ type, has the same geometrical relation to the CaCu₅ type. However, the number of atoms of the R component, which are substituted by pairs of X atoms is different: (CaCu₅ type) 6RX₅ = R₆X₃₀ = R₃R′₃X₃₀ 3R = 6X R₃X₆X′₃₀ = R₃X₃₆ = 3RX₁₂ (EuAg₄In₈ type). Thus, for the formation of the ThMn₁₂ and EuAg₄In₈ types, half of the R atoms of the parent structure are replaced by pairs, and for the Th₂Ni₁₇ type only one third of the R atoms is replaced by X₂ pairs. Likewise, there is a close relationship between the types Th₂Ni₁₇ (SG P6₃/mmc, 38 atoms per unit cell) and EuAg₄In₈ (SG P6/mmm, 39 atoms per unit cell) with very close coordinates of the atoms and, accordingly, similar structural motifs. The EuAg₄In₈ structure is deduced from the Th₂Ni₁₇ type by a substitution of one of the two R atoms in site 2b 0 0 1/4 by two X atoms (in this case In) with the coordinates 0 0 z (z = 0.3434, site 2e) and a displacement of the other R atom to the
The geometrical scheme for the transformation is the following: (Th$_2$Ni$_{17}$ type) $2R_2X_{17} = R_4X_{34} = R_1R'_3X_{34} \xrightarrow{R=2X} R_3X_2X'_34 = R_3X_{36} = 3RX_{12}$ (EuAg$_4$I$_{34}$ type).

The structure type Sm$_{12}$Ni$_6$In (cubic symmetry, 38 atoms per unit cell) can be analogously obtained from the Ho$_6$Co$_2$Ga$_4$ type (orthorhombic pseudocubic symmetry, 36 atoms per unit cell) (Kalychak et al., 1993b; Grin and Gladyshevskii, 1989). The substitution involves two $X$ atoms and two pairs of transition metal atoms $M$ with $r_X > r_M$ and approximately identical volumes of the unit cells ($a_{Sm_{12}Ni_6In} \approx a_{Ho_6Co_2Ga} \approx b_{Ho_6Co_2Ga} \approx c_{Ho_6Co_2Ga}$). The geometrical scheme for the transformation is: (Ho$_6$Co$_2$Ga$_4$ type) $4R_6M_2X = R_24M_8X_4 = R_{24}M_8X_2X'_2 \xrightarrow{2X=4M} R_{24}M_8M'_4X_2 = R_{24}M_{12}X_2 = 2R_{12}M_6X$ (Sm$_{12}$Ni$_6$In type).

We have already mentioned that the EuAg$_4$I$_{34}$ structure can be considered as a superstructure of the SmZn$_{11}$ type with the latter having defects on the zinc sites. However, another interpretation is also possible: EuAg$_4$I$_{34}$ ($=RX_{12}$) can be considered as a filled version of SmZn$_{11}$. The sites 2$e$ and 4$h$ with the zinc defects are fully occupied by the atoms of smaller size, thus SmZn$_{11}$ can be considered as a defect derivative of the EuAg$_4$I$_{34}$ type neglecting the $R$ atoms on the sites 1$b$ and 2$c$, which are occupied only by 4 and 15%, respectively.

5.2. Structures of intergrowth of small slabs

A significant number of crystal structures occurring in binary and ternary indides can be considered as consisting of simple structural slabs in different ratios and arrangements. In most cases, these fragments are unit cells or fractions of unit cells of simple structures, which are formed in systems with indium or other metals, i.e. the slabs have real representatives among intermetallic compounds. In some cases, however, slabs are hypothetical and they occur only in combination with other slabs. Identical slabs, combined in different ratios, form homologous series of structure types. The so-called one-dimensional homologous series are the result of combining the slabs in one direction; two- and three-dimensional ones are obtained, if the combination of the slabs occurs within a plane or in all three directions, respectively. The slabs, which occur most often in the structures of indides, derive from the following structure types: tungsten ($\alpha$-Fe), CsCl, AuCu$_3$, AlB$_2$, CaCu$_5$ (and its derivatives), CeMg$_2$Si$_2$, Zr$_4$Al$_3$, and a few others (Kripyakevich, 1977).

The analysis of the structures shows, that the slabs of $\alpha$-Fe and AlB$_2$ are of special importance. The motif of tungsten ($\alpha$-Fe; a body-centered cube) is frequently distorted in real structures; the tetragonal deformation ($c/a > 1$) corresponds to the structure of elemental indium (SG $I4/mmm$, $a = 325.12$, $c = 494.67$ pm) (Donohue, 1974). Such slabs frequently occur in indium-rich binary and ternary compounds. In the case when the atoms within the slab and at the vertices are different, we obtain slabs derived from cubic CsCl or tetragonal AuCu, most often with the composition $RIn$. The phase diagrams of the binary systems $R$–In (Yatsenko et al., 1983) reveal that binary compounds $RIn$ with CsCl structure type mostly form directly from the melt and they have the highest melting points in these systems, i.e. they are thermodynamically very stable. This is confirmed by the experimental data for the heats of formation $\Delta H_{\text{form}}$. Their theoretical estimations are given by Colinet (1995). Considering this, the presence of such slabs in complex structures is not surprising. These cubic or tetragonally distorted fragments can form layers or isolated columns in more complex struc-
tures. Another very important structural element of binary and ternary indides is a trigonal prism as the main constituent of trigonal and hexagonal structures. Unfilled prisms exist with the composition \( E (E \text{ is a rare earth, } M, \text{ or } X \text{ element}) \) or centered by another atom with the composition \( EE' \) (which corresponds to the motif of the \( AlB_2 \) structure (two prisms connected by a rectangular face). In ternary compounds with different degrees of deformation, these two structural elements, a cube (or a tetragonal prism) and a trigonal prism (empty or filled), are the basic slabs for the geometrical construction of the crystal structures of indium-rich compounds.

For crystal structures of compounds, that are rich in transition metal (compounds of group I according to fig. 79) it is possible to isolate fragments of more simple structures. The structure types \( \text{ThMn}_{12}, \text{YNi}_9\text{In}_2, \text{CeNi}_5\text{Sn}, \text{CeCu}_{4.38}\text{In}_{1.62}, \text{and NdFe}_{13}\text{Si} \) are among those. In figs. 82–92 we illustrate some of the interesting examples where complex structures can be presented as various combinations of simple fragments. The unit cells are outlined by dashed lines and the fragments with solid lines. In these figures, the \( R, T(M), \) and \( \text{In} (X) \) atoms are presented as large gray, small black, and medium unfilled circles, respectively.

In the tetragonal types \( \text{ThMn}_{12} \) and \( \text{YNi}_9\text{In}_2 \) with very similar lattice parameters it is possible to isolate a unit cell of the \( \text{CeMg}_2\text{Si}_2 \) structure (SG \( P4/mmm, a = 425.0, c = 576.5 \text{ pm} \) (Kripyakevich, 1977) and in the first type, slabs of a hypothetical structure with a disordered arrangement of the atoms, and in the second type, a unit cell of the \( \text{Zr}_4\text{Al}_3 \) type

![Diagrams](image-url)
Fig. 83. Packing of slabs of the CaCu$_5$ type (c) and the hypothetical structures CeM$_2$In (d) and M$_3$In (e) in the structures of CeNi$_5$Sn (a) and CeCu$_{4.38}$In$_{1.62}$ (b).

(SG $P6/mmm$, $a = 543.3$, $c = 539.0$ pm) (Kripyakevich, 1977). In both structures, the ratio of the fragments is 2:2 (fig. 82). Then, the formulae of the structure types are described for the ThMn$_{12}$ type as follows: $(2$CeMg$_2$Si$_2$)$2R(M, X)_4 + 2(M, X)_8 = R_2(M, X)_8 + (M, X)_{16} = R_2(M, X)_{24} = 2R(M, X)_{12}$ and for the YNi$_9$In$_2$ type: $(2$CeMg$_2$Si$_2$)$2RM_2M'_2 + (2$Zr$_4$Al$_3$)$2M_5X_2 = R_2M_8 + M_{10}X_4 = R_2M_{18}X_4 = 2RM_9X_2$. The difference in the number of atoms in the Zr$_4$Al$_3$ type (7 atoms per unit cell) and in the hypothetical structure $(M, X)_8$ (8 atoms per unit cell) determines the difference in the compositions of ThMn$_{12}$ and YNi$_9$In$_2$.

The following two structure types, hexagonal CeNi$_5$Sn (Skolozdra and Komarovskaya, 1988) and orthorhombic CeCu$_{4.38}$In$_{1.62}$ (Kalychak et al., 1988a), also have similar metric ratios taking the orthohexagonal setting. Both types can be divided into identical fragments: a CaCu$_5$ related slab in the orthorhombic setting and two slabs of hypothetical compositions RM$_2X$ and M$_3X$, or, taking into account the presence of statistical occupancies of atoms
Fig. 84. Representation of the Nd₆Fe₁₃Si structure type (La₆Co₁₃In) (a) as a packing of fragments of the Cr₅B₃ (R₅M₂X) (b) and Ce(Mn₀.₅₅Ni₀.₄₅)₁₁ (RM₁₁) structures (c). The Cr₅B₃ structure (b) is an intergrowth of U₃Si₂ (R₃M₂) (d) and CuAl₂ (R₂X) (e) slabs. The structure of Ce(Mn₀.₅₅Ni₀.₄₅)₁₁ consists of CeMg₂Si₂ (RM₄) (f) and Zr₄Al₃ (M₇) (g) related slabs.

(in particular in the CeCu₄.₃₈In₁.₆₂ type), they can be designated as (M, X). One unit cell of each type contains one slab of the CaCu₅ type (R₂(M,X)₁₀) in the orthohexagonal setting and 4 slabs of the hypothetical composition: 2R₂(M,X)₁₀ + 1R₅M₂X = R₄(M,X)₂₀ + R₄(M,X)₁₂ + (M,X)₁₆ = R₈(M,X)₄₈ = 8R(M,X)₆. The main difference between the two types is that the arrangement of the slabs is linear (one-dimensional) for CeNi₅Sn and it is mosaic (two-dimensional) for CeCu₄.₃₈In₁.₆₂ (fig. 83).

The closely related structure types Nd₆Fe₁₃Si and La₆Co₁₁Ga₃ (Weitzer et al., 1993; Sichevich et al., 1985) can be considered as intergrowth structures, which consist of Ce(Mn₀.₅₅Ni₀.₄₅)₁₁ (= RM₁₁) (SG P4/mmb, a ≈ 830, c ≈ 490 pm) (Kalychak et al., 1975) (in systems with indium the YNi₉In₂ structure corresponds to it) and Cr₅B₃ (SG I₄/mcm, a ≈ 760, c ≈ 1400 pm for Ho₅Ga₃) slabs with composition R₅M₂X (Grin and Gladyshevskii, 1989) in the ratio 2:1 (fig. 84). The composition of the compound is deduced as follows: 2R₂M₁₁ + 1R₅M₂X = R₄M₄₄ + R₂₀M₈X₄ = R₄₄M₅₂X₄ = R₆₁₃X₄ (= La₆Co₁₃In).

As mentioned above, the RM₁₁ structure is an intergrowth of the more simple types Zr₄Al₃ and CeMg₂Si₂ in the ratio 2:2, and the structure of Cr₅B₃ (Kuz'ma, 1983) is an intergrowth of the types CuAl₂ and U₃Si₂ in the ratio 1:2. The U₃Si₂ type structure itself consists of CsCl and AlB₂ related slabs (see below).
Considering the packing of slabs in the group II compounds (fig. 79), they can be divided into several subgroups with close structural relations. One subgroup consists of the types MgCuAl\(_2\) (ternary indides \(RNiIn_2\)), YNiAl\(_4\) (\(RNiIn_4\)), and LaNi\(_3\)In\(_6\) (\(RNiIn_6\)) (Kalychak et al., 1985). All have orthorhombic symmetry and two approximately identical lattice parameters, which are close to \(\sim 440\) and \(\sim 730\) pm.

The structure of MgCuAl\(_2\) is the simplest one of them; half of its unit cell is a basic building block for the formation of two other structures, YNiAl\(_4\) and LaNi\(_3\)In\(_6\). In the YNiAl\(_4\) type two of such fragments are separated by a chain of indium atoms and in the LaNi\(_3\)In\(_6\) type the building block is sandwiched by two fragments of a hypothetical structure Ni\(_2\)In\(_4\) in the form of distorted, empty octahedra (fig. 85).

![Fig. 85. Relations between the structures of MgCuAl\(_2\) (a), YNiAl\(_4\) (b), and LaNi\(_3\)In\(_6\) (c). In each structure half of a MgCuAl\(_2\) unit cell is shaded. The dotted lines outline the motif of the CaCu\(_5\) type structure.](image)
The next subgroup consists of tetragonal structure types with very similar $a$ parameters: the HoCoGa$_5$ (Grin et al., 1979a) type for $RCoIn_5$ (Kalychak et al., 1989a) and the Ho$_2$CoGa$_8$ (Grin et al., 1979a) type for $R_2CoIn_8$ (Kalychak et al., 1989a). They are the members of a one-dimensional homologous series, which is based on AuCu$_3$ and PtHg$_2$ type slabs (Kripyakevich, 1977) and is described by the general formula $R_mM_{n+2}X_3$, where $m$ and $n$ are the number of AuCu$_3$ (composition $RX_3$) and PtHg$_2$ (composition $MX_2$) fragments per unit cell. For HoCoGa$_5$, $m = n = 1$, and for Ho$_2$CoGa$_8$, $m = 2$ and $n = 1$ (fig. 86).

The orthorhombic structures Pr$_5$Ni$_6$In$_{11}$ (Kalychak et al., 1987) and Ce$_4$Ni$_7$In$_8$ (Baranyak et al., 1988a) also have two approximately identical lattice parameters and in contrast to other structures of this group, they contain slabs of the CeMg$_2$Si$_2$ type (compounds of group $I$ such as ThMn$_{12}$ and YNi$_3In_2$ contain these fragments), AuCu$_3$, CsCl, and variations of the CaCu$_5$ structure. The CaCu$_5$ slab in Pr$_5$Ni$_6$In$_{11}$ is ordered as in the superstructure of CeCo$_3$B$_2$ (SG $P6/mmm$, $a = 505.7$, $c = 303.6$ pm (Kuz’ma, 1983). Ce$_4$Ni$_7$In$_8$ contains derivatives of the HoNi$_2.6$Ga$_2.4$ type (Grin and Gladyshevskii, 1989), which has representatives in the systems $R$–Ni–In. The intergrowth of these slabs is presented in fig. 87. The Pr$_5$Ni$_6$In$_{11}$ structure contains: (4CeMg$_2$Si$_2$) 4$RNi_2In_2$ + (2AuCu$_3$)2$RIN_3$ + (2CsCl)2$RIn$ + (2CeCo$_3$B$_2$)2$RNI_2In_3 = R_{10}Ni_{12}In_{22} = 2R_3Ni_6In_{11}$. The Ce$_4$Ni$_7$In$_8$ structure consists of: (4CeMg$_2$Si$_2$) 4$RNi_2In_2$ + (2AuCu$_3$)2$RIN_3$ + (2CsCl)2$RIn$ + (structural fragments of the HoNi$_2.6$Ga$_2.4$ type) 2$RNI_4In$(I) + 2$RNi_2In_3$(II) + 4$RNi_2In_3$(III) = $R_{16}Ni_{28}In_{32} = 4R_4Ni_7In_8$ (= Ce$_4$Ni$_7$In$_8$).

The two orthorhombic structure types Pr$_2$Co$_2$Ga (Grin and Gladyshevskii, 1989) and Sm$_2$Co$_9$In$_3$ (Baranyak et al., 1993) have two similar lattice parameters and similar fragments (fig. 88 a, b). $RNi_2In$ and $RCO_2In$ (Kalychak et al., 1993c; Kalychak and Zaremba, 1994) crystallize with the Pr$_2$Co$_2$Ga type. Both types have a larger transition metal content than the structures described above. They contain CaCu$_5$ type slabs (fig. 88 d) and fragments of the
Fig. 87. The structures of Pr$_5$Ni$_6$In$_{11}$ (a) and Ce$_4$Ni$_7$In$_8$ (b). The various slabs derive from CeMg$_2$Si$_2$ (c), AuCu$_3$ (d), CsCl (e), and CeCo$_3$B$_2$ (f) in Pr$_5$Ni$_6$In$_{11}$. A larger slab of Ce$_4$Ni$_7$In$_8$ corresponds to the structure of HoNi$_{2.6}$Ga$_{2.4}$ (g).

CsCl type (fig. 88 c). The more complex structure of Sm$_2$Co$_9$In$_3$ (fig. 88 e) contains two unit cells of the HoCo$_2$Ga structure and a Co$_5$In slab with the motif of the CaCu$_5$ type. The compositions of the compounds are described by the formulæ: for the HoCo$_2$Ga type (compound TbCo$_2$In): (1CsCl) TbIn + (1CaCu$_5$) TbCo$_4$In = Tb$_2$Co$_4$In$_2 = 2$TbCo$_2$In and for the Sm$_2$Co$_9$In$_3$ type: (2CsCl) 2SmIn + (2CaCu$_5$) 2Sm$_0.5$Co$_4$In$_{1.5} + (2$CaCu$_5$) 2Sm$_0.5$Co$_5$In$_{0.5}$ = Sm$_4$Co$_{18}$In$_6 = 2$Sm$_2$Co$_9$In$_3$.

Orthorhombic Mn$_2$AlB$_2$ (Becher et al., 1966) for R$_2$Ni$_2$In (Zaremba et al., 1988), tetragonal Mo$_2$FeB$_2$ (Rieger et al., 1964) for R$_2$Cu$_2$In and R$_2$Ni$_2$In (Kalychak et al., 1990), and orthorhombic Lu$_5$Ni$_2$In$_4$ for R$_5$Ni$_2$In$_4$ (Zaremba et al., 1991) contain only CsCl and AlB$_2$
The TbCo$_2$In (g) slabs in the Sm$_2$Co$_9$In$_3$ structure (f) are outlined at the bottom part of the figure.

related slabs. These structures have a similar value for the $c$ lattice parameter, i.e. $\sim 360$–370 pm.

In addition, Mo$_2$FeB$_2$ and Lu$_5$Ni$_2$In$_4$ have a similar value for the $b$ lattice parameter ($\sim 700$ pm). Together, the structure types Mn$_2$AlB$_2$ and Cr$_3$AlB$_4$ (SG Pmmm, $a = 295.2$, $b = 809.1$, $c = 298.9$ pm (Kuzma, 1983) form a one-dimensional homologous series based on CsCl ($RX$) and AlB$_2$ ($RM_2$) slabs as described by the general formula $R_{m+n}M_{2n}X_m$, where $m$ and $n$ are the numbers of CsCl and AlB$_2$ related slabs, respectively. For the Mn$_2$AlB$_2$ type, $m = n = 2$, and for Cr$_3$AlB$_4$, $m = 1$ and $n = 2$ (Kuz’ma, 1983) (fig. 89 a, b). The Cr$_3$AlB$_4$ stacking sequence, however, is not known in the indium-based structures.

The structures of Mo$_2$FeB$_2$ and Lu$_5$Ni$_2$In$_4$ are the representatives of a two-dimensional homologous series formed by the same types and with the same general formula. For Mo$_2$FeB$_2$,
Fig. 89. The structure types Mn$_2$AlB$_2$ (Y$_2$Ni$_2$In) (a) and Cr$_3$AlB$_4$ (b) represent a one-dimensional series. The types Mo$_2$FeB$_2$ (c) and Lu$_5$Ni$_2$In$_4$ (d) represent a two-dimensional homologous series $R_m+nM_{2n}X_m$, based on CsCl (e) and AlB$_2$ (f) related slabs. AlB$_2$ slabs occur also in Yb$_2$Cu$_2$In (g).

$m = n = 2$ and for Lu$_5$Ni$_2$In$_4$, $m = 8$, $n = 2$ (fig. 89 c, d). U$_2$Pt$_2$Sn (Gravereau et al., 1994) is a ternary ordered version of the Zr$_3$Al$_2$ type (Kripyakevich, 1977). This latter structure is a representative of a three-dimensional homologous series of structures, based on the same slabs.
Among binary compounds in the systems $R$–In, the structure of EuIn$_4$ (Fornasini and Cirafici, 1990) consists of BaAl$_4$ ($RX_4$), AuCu$_3$ ($RX_3$), and $\alpha$-Fe ($X_2$) related slabs. The formula of the compound is described by $R_{l+m}X_{4l+3m+n}$, where $l$, $m$, and $n$ are the numbers of the respective slabs (fig. 90).

As mentioned above, a significant number of structure types, in which binary and ternary indides of rare earth elements crystallize, contain tetragonal motifs of the indium, CsCl, or AuCu structures and filled or empty trigonal prisms. In particular, binary rare earth metal indides crystallize with the structure types AuCu$_3$, CaIn$_2$, CeCu$_2$, Ni$_2$In, Co$_2$Si, and EuIn$_4$, and also some ternary indides, i.e. the structure types HoCoGa$_5$, Ho$_2$CoGa$_8$, YNiAl$_4$, MgCuAl$_2$, TiNiSi, AlB$_2$, and to some degree GdPt$_2$Sn. From a purely geometrical point of view, all these structures consist of two different layers, which are stacked along the shortest lattice periods. Often these layers coincide with mirror planes in the structures. Depending on the nature of the rare earth element and the structure type, the shortest lattice parameters have values between 470 and 550 pm. The hexagonal structures of CaIn$_2$, AlB$_2$, or Ni$_2$In are viewed in the orthohexagonal settings. In fig. 91 (a–d) the geometrical elements of the structures are presented, and in fig. 91 (e–k) their packing in the real structures is shown.

The structure type of AuCu$_3$ with $a \sim 326$, $c \sim 461$ pm for GdIn$_3$ (fig. 87 d) can be considered as a packing of slabs of elemental indium (we remind that elemental indium has $a = 325$, $c = 494$ pm) and AuCu with composition $R$In according to $R$In$_3 = R$In + In$_2$. In the hexagonal CaIn$_2$, AlB$_2$, and Ni$_2$In types, orthorhombic CeCu$_2$ and Co$_2$Si types, and monoclinic EuIn$_4$ type structures of binary indides of rare earth elements, depending on their composition, one can see tetragons of composition $R$In (or $R_1.5$In$_{0.5}$ for compounds with a high rare earth metal content) and empty trigonal prisms of compositions In or $R_0.5$In$_{0.5}$ with a different degree of distortion. The formulæ of the binary structure types are described by the sums of their constituents: AlB$_2$ ($2R$In + 2In = 2$R$In$_2$); CaIn$_2$ ($4R$In + 4In = 4$R$In$_2$); CeCu$_2$ ($4R$In + 4In = 4$R$In$_2$); Ni$_2$In ($4R_{1.5}$In$_{0.5}$ + 4$R_0.5$In$_{0.5}$ = 4$R_2$In$_{0.5}$); Co$_2$Si ($4R_{1.5}$In$_{0.5}$ + 4$R_0.5$In$_{0.5}$ = 4$R_2$In$_{0.5}$).

In the ternary structure types HoCoGa$_5$ and Ho$_2$CoGa$_8$, formed only by filled tetragonal prisms In$_2$ and $R$In, also empty tetragonal prisms of composition In and transition metal filled
Fig. 91. Geometrical motifs of the structures of binary indides (a–d) and their packing in some structure types of binary compounds of the systems R–In (e–k).

tetragonal prisms exist (fig. 92 a, b). The formulae of these structure types are: HoCoGa$_5$ (1RIn + 1In$_2$ + 1CoIn + 1In = RCoIn$_5$); Ho$_2$CoGa$_8$ (2RIn + 2In$_2$ + 1CoIn + 1In = R$_2$CoGa$_8$).

The structure types MgCuAl$_2$ and YNiAl$_4$ (fig. 92 c and e) can be considered as transition metal filled versions of binary compounds RIn$_2$ (Hoffmann and Pöttgen, 2001). In contrast to binary structures of composition RIn$_2$, only half of the trigonal prisms are filled by transition metal atoms. The difference between these two structure types is that in the MgCuAl$_2$ type a slab of tetragonal prisms RIn alternates with a slab of trigonal prisms, NiIn$_{0.5}$ and EIn$_{0.5}$ in the ratio 2:2 (E is an empty site). In the YNiAl$_4$ type, two slabs of tetragonal prisms In$_2$ and RIn alternate with one slab of trigonal prisms, NiIn$_{0.5}$ and EIn$_{0.5}$ (a ratio of 4:2 per unit cell) (fig. 92 c, d). The formulae of the structure types are the following: YNiAl$_4$ (4RIn + 4In$_2$...
Fig. 92. Tetragonal and trigonal prisms in the structures of the ternary indides HoCoGa$_5$ (a), Ho$_2$CoGa$_8$ (b), MgCuAl$_2$ (c), PrNiIn$_2$ (d), YNiAl$_4$ (e), LaCoIn$_4$ (f), LaNi$_3$In$_6$ (g), and Ce$_2$Au$_3$In$_5$ (h).

$4\text{NiIn}_{0.5} + 4\text{EIn}_{0.5} = R_4\text{Ni}_4\text{In}_{16} = 4\text{RNiIn}_4$; MgCuAl$_2$ $(4\text{RIn} + 4\text{NiIn}_{0.5} + 4\text{EIn}_{0.5} = R_4\text{Ni}_4\text{In}_8 = 4\text{RNIIn}_2)$.

The LaNi$_3$In$_6$ type is formed by the geometrical elements $R\text{In}$, $\text{In}_2$, NiIn$_{0.5}$, and $R\text{In}_{0.5}$. Furthermore, this structure type contains empty octahedra formed by nickel and indium atoms (fig. 92 g).

Ce$_2$Au$_3$In$_5$ (fig. 92 h) contains alternating slabs of LaCoAl$_4$ (fig. 92 f) and cutouts of MgCuAl$_2$ (fig. 92 c).

6. Chemical bonding in rare earth transition metal indides

The nature of chemical bonding in the various rare earth transition metal indides strongly depends on the composition. In fig. 93 we present the different structural varieties that occur
in the ternary systems. As shown in fig. 93, two large families of compounds, drawn as islands within a generic ternary system, can be identified.

The most complicated structures occur in the region with high rare earth metal content. As an example we present the Tm$_{13.5}$Ni$_3$In$_{3.5}$ structure at the left-hand side of fig. 93. Due to the large size of the rare earth metal atoms, these structures are dominated by high coordination numbers. Depending on the rare earth metal content, the coordination numbers vary.

With the coordination numbers CN 12, CN14, CN15, and CN16, sometimes slightly distorted Frank–Kasper polyhedra occur (Frank and Kasper, 1958, 1959). In many cases, the packing of the different polyhedra is very complex. Mostly we observe interpenetrating polyhedra. Chemical bonding in these compounds is more or less isotropic. The shortest interatomic distances in such structures occur between the transition metal and indium atoms. These distances mostly are close to the sum of the covalent radii (Emsley, 1999) and one can assume strong $T$–In bonding. Nevertheless, in $R$-rich compounds, $T$–In interactions play only a subordinate role.

The second family of compounds has a larger transition metal and/or indium content. In all of these compounds $T$–In bonding is much more important than in the $R$-rich compounds. The transition metal atoms have broadly varying indium coordination. In compounds like Ce$_8$Pd$_{24}$In (Gordon et al., 1996), only one indium atom is bonded to palladium. Such small fragments also occur in the rare earth metal rich compounds. Normally, the lowest coordination number for a transition metal by indium is three. In fig. 94 we present the various RhIn$_x$, PdIn$_x$, and PtIn$_x$ monomeric units in the ternary indides with rhodium, palladium, and platinum as transition metal component. With the other late transition metals similar
units occur. The highest coordination number of the transition metal by indium is CN 9. The monomeric units are condensed via the indium atoms, which are shared by neighboring units. This way one gets two- or three-dimensional $T_x\text{In}_y$ polyanionic networks. As examples, we have drawn the polyanions, [Ni$_6$In$_{11}$] and [Rh$_2$In$_8$], of the Nd$_5$Ni$_6$In$_{11}$ (Pöttgen et al., 1999a) and EuRh$_2$In$_8$ (Pöttgen and Kußmann, 2001) structures in the upper right-hand part of fig. 93. The rare earth atoms are located in cavities or channels of the three-dimensional polyanions, while they separate layers in the case of two-dimensional networks.
There is not a uniform coloring scheme for the polyanions. The degree of $T$–$T$, $T$–In, and In–In contacts does not necessarily depend on the composition. In GdRhIn$_2$ (Hoffmann et al., 2000) there occur only Rh–In and In–In interactions within the three-dimensional [RhIn$_2$] polyanion, while several Ni$_2$ dumb-bells (244 and 273 pm) occur in Nd$_5$Ni$_6$In$_{11}$ with quite similar composition.

The first very simple parameter that gives information about a potential charge transfer is the electronegativity. For most indides discussed in the present review, the rare earth element is the most electropositive and the late transition metal the most electronegative component, while the indium atoms play a mediate role. These differences in electronegativity lead to a significant filling of the transition metal $d$ bands. The position of the transition metal $d$ block is well below the Fermi level. Thus, the late transition metal atoms become negatively charged and consequently one obtains nickelides, aurides, and related species rather than indides.

We will discuss the situation of chemical bonding for the indides $RT$In$_2$ and for GdAu$_{0.44}$In$_{1.56}$ in more detail. With the alkaline earth elements, divalent europium, and divalent ytterbium, the Zintl phases $AE$In$_2$ ($AE$ = Ca, Sr, Ba), EuIn$_2$, and YbIn$_2$ exist (Iandelli, 1964). Chemical bonding in the binaries is relatively easy to understand. The more electropositive elements lose their two valence electrons and one consequently obtains In$^-$ with a formal charge of $-$1. The indium atoms thus acquire the electron configuration of the fourth group main element and indeed they form a network of slightly distorted, condensed tetrahedra similar to lonsdaleite (Donohue, 1974). An ionic formula splitting leads to an electron precise formulation Eu$^{2+}$($\text{In}^-$)$_2$, etc. In ternary indides like EuPdIn$_2$, EuAuIn$_2$, or YbPdIn$_2$, the indium substructure of binary EuIn$_2$ and YbIn$_2$ is retained in an orthorhombically distorted arrangement (fig. 95).

The transition metal atoms in the ternaries fill some of the triangles formed by the alkaline earth, europium, or ytterbium atoms. Since the late transition metal atoms are by far the most electronegative component of the $AET$In$_2$, EuTiIn$_2$, and YbTiIn$_2$ compounds, they tend to fill their $d$ bands. The europium atoms have already completely transferred their valence electrons to the indium network. The electrons filling the transition metal $d$ band consequently originate from an oxidation of the indium network, resulting in longer In–In distances in the ternary indides (fig. 95). The MgCuAl$_2$ type structure (Perlitz and Westgren, 1943) of these indides is somewhat flexible with respect to the electron count. Such indides also exist with a trivalent rare earth metal, i.e. the series $RR$In$_2$ ($R$ = La, Ce, Pr, Nd, Sm) (Zaremba et al., 2002a) and $RN$In$_2$ ($R$ = Eu–Dy) (Zaremba et al., 1987a; Kalychak et al., 1997). The crystal orbital overlap populations have been calculated for BaRhIn$_2$ (Hoffmann and Pöttgen, 2001). The Rh–In and In–In COOP curves show all bonding states above the Fermi level. Assuming a rigid band model, we can thus explain the existence of the $RR$Rh$_2$ series. For BaPtIn$_2$, the Fermi energy already separates In–In bonding from antibonding states. So far, no $RP$In$_2$ indides are known.

Another possibility to obtain compounds with a trivalent rare earth element with CaIn$_2$ structure is the direct substitution of indium atoms by largely electronegative transition metal atoms. While a binary indide “GdIn$_2$” with CaIn$_2$ structure is not known, such a structural arrangement is realized for GdAu$_{0.44}$In$_{1.56}$ (Pöttgen et al., 1998).
In all rare earth metal–transition metal–indides with polyanionic networks there is a smaller range for the $T$–In than for the In–In distances. The $T$–In distances are mostly close to the sum of the covalent radii (Emsley, 1999). Many of the In–In distances compare well with those in tetragonal body-centered elemental indium, where each indium atom has four nearest indium neighbors at 325 pm and eight further neighbors at 338 pm (Donohue, 1974). Sometimes there are also longer In–In contacts, which still belong to the coordination spheres. Based on these distance criteria and the course of the crystal orbital overlap populations (Pöttgen and Dronskowski, 1996; Pöttgen et al., 1998), chemical bonding within the transition metal–indium networks is governed by strong $T$–In and In–In interactions (Landrum et al., 1998). The rare earth metal atoms have at least one short contact to the network and they are essentially electrostatically (ionically) bonded to the polyanion. This has been demonstrated for various europium and ytterbium intermetallics (Pöttgen and Johrendt, 2000; Pöttgen et al., 2001).

A special situation occurs for some of the cerium and ytterbium based compounds. Here, the $4f^{13} \text{Yb}^{3+}$ configuration is the hole analogue of $4f^1 \text{Ce}^{3+}$. The degree of $4f$ state contribution at the Fermi energy has a drastic influence on the physical properties of these materials, leading to valence instabilities or heavy fermion behavior.

Two special situations of chemical bonding occur in the compounds with $T$–In polyanions, i.e. (i) clustering of the $T$ atoms when the transition metal content is high (lower left-hand side
of fig. 93) and (ii) the formation of indium cubes at high indium content (lower right-hand side of fig. 93). A prominent example for such a cluster compound is LaNi$_7$In$_6$ (Kalychak et al., 2001) (fig. 96). The nickel $d$ block in this cluster compound stays sharply localized well below the Fermi energy. The course of the gross Mulliken charges, La: +1.81, Ni: −0.94, and In: +0.80 characterizes LaNi$_7$In$_6$ as a nickelide. The Ni–Ni bonding and antibonding states are filled formally leading to a closed-shell interaction. The Ni–In and In–In interactions within the cluster chain are strongly bonding, while La–Ni and La–In interactions are much weaker. The small overlap populations and the course of the Mulliken charges fully justify the ionic formula splitting La$_3^+$(Ni$_7$In$_6$)$_3^-$ as a first approximation.

With increasing indium content, the indium substructures within the polyanionic networks gradually more resemble the structure of elemental indium. As emphasized at the lower right-hand part of fig. 93, these indium sub-units are more or less distorted bcc indium cubes with In–In distances in the range of elemental indium.
A very interesting situation is the comparison of chemical bonding in the ZrNiAl type indides LaNiIn, CeNiIn, and NdNiIn with the corresponding hydrides $R_3Ni_3In_3H_4$, i.e. $RNiInH_{1.33}$ (Vajeeston et al., 2003). When hydrogen is introduced in the ZrNiAl type substructure, there occurs an anisotropic contraction of the structure along [001]. Force minimization, volume, and $c/a$ optimizations have been performed by first-principles calculations. The theoretical data are in good agreement with the experimental ones. The hydrogen atoms are located in slightly distorted tetrahedral sites formed by three rare earth metal and one nickel atom, with short Ni–H distances. Since the tetrahedra are alternating face and corner sharing along the $c$ axis, there occur very short H–H distances within the three hydrides (156–164 pm). The electronic structure calculations revealed significant ionic bonding for the Ni–H contacts and weak metallic bonding for the short H–H contacts. The electron distribution at the hydrogen atoms is polarized towards the rare earth metal and indium atoms, reducing the repulsion between the negatively charged hydrogen atoms. This further explains the short H–H distances. The density of states curves reveal metallic behavior for the three hydrides. Roughly speaking, there is an ionically bonded Ni–H species within a metallic matrix, a situation of chemical bonding that is very similar to the alkaline earth metal subnitrides (Simon and Steinbrenner, 1996).

The calculated bulk modulus for NdNiIn increases upon hydrogenation, leading to an enhancement of the overall bond strength in this hydride. The integrated crystal orbital Hamilton populations ICOHP’s (Dronskowski and Blöchl, 1993) give a direct indication for the bond strength. For the ternary LaNiIn, the strongest bonding interactions were found for the Ni–In contacts followed by La–Ni. This situation is different for the hydride, where the Ni–H interactions are by far the strongest ones, followed by Ni–In. In comparison with LaNiIn, one observes a slight weakening of Ni–In and La–Ni bonding in the hydride LaNiInH$_{1.33}$.

7. Chemical and physical properties

In this section we discuss the chemical and various physical properties of rare earth–transition metal–indides. Most of these indides are light gray in polycrystalline form. Single crystals have metallic luster. Although the rare earth metals are relatively electropositive elements similar to the alkaline earth metals, there is a significant difference in the stability of the ternary compounds. Most rare earth–transition metal–indides are stable in moist air over several months. There are only few exceptions, e.g. EuZnIn (Pöttgen, 1996a). The stability of the indides, however, differs from that of the corresponding rare earth–transition metal–stannides. The latter can often be synthesized from a tin flux and the excess tin can be dissolved with diluted hydrochloric acid while the ternary stannide is not attacked. Several indides are not stable under such conditions. In the following subsections we first discuss the properties of the extended families of equiatomic indides $RTIn$ and those with the ordered $U_3Si_2$ or $Zr_3Al_2$, HoCoGa$_5$, or Ho$_2$CoGa$_8$, and MgCu$_4$Sn type structures. These compounds have intensively been investigated with respect to their magnetic and electrical properties since they have comparably simple crystal structures. This facilitates the calculation of the electronic structure and various properties.
7.1. Ternary equiatomic indides RTIn

7.1.1. Magnetic and electronic properties

Short overviews on the unusual low-temperature properties of equiatomic CeTx compounds were given by Adroja and Malik (1991) and Fujita et al. (1992). These microreviews focus on valence fluctuation and heavy fermion behavior. The CeTIn indides are discussed together with the CeTGe germanides and CeTSn stannides. Those and most recent data on the indides are summarized here in the following paragraphs.

Single crystal magnetic susceptibility and electrical resistivity data reveal valence-fluctuating behavior for CeNiIn with a significantly anisotropic Kondo effect, which arises from anisotropic $c$–$f$ mixing along the $a$ and $c$ axes (Fujii et al., 1989). At 2 K, $C/T$ reaches a value of 60 mJ/mol K$^2$, which is ten times larger than that in LaNiIn without 4$f$ electrons. The physical properties of CePdIn and CeNiIn have been reviewed and compared to those of UPdIn (Fujii et al., 1992).

The magnetic ordering in the Kondo-lattice compound CePdIn has been investigated in detail by Brück et al. (1988). A transition to an antiferromagnetically ordered state has been observed at 1.65 K. In the temperature range 200 to 300 K, CePdIn is a Curie–Weiss paramagnet and the experimental moment (table 13) indicated essentially trivalent cerium. Two maxima occur in the electrical resistivity curve and the Hall effect is anomalously large. Specific heat measurements show an electronic specific heat coefficient $\gamma = 123$ mJ/mol K$^2$. All these features are reminiscent of low $T_SF$ spin-fluctuating systems. Isotypic LaPdIn becomes superconducting at 1.6 K. Single crystal magnetic susceptibility data of CePdIn (Fujii et al., 1990) reveal a small peak at the Néel temperature in the $c$ axis magnetic susceptibility, while no anomaly occurs for the $a$ axis. The difference in the paramagnetic Curie temperatures $\Theta_a$ and $\Theta_c$ is 22 K. In contrast to the magnetic susceptibility data, both the $a$ and $c$ axis electrical resistivity show a double peak structure characteristic for a Kondo lattice system.

In contrast to the antiferromagnetic ordering in CePdIn, no sign for a magnetic phase transition is evident for the dense Kondo compound CePtIn down to 50 mK (Fujita et al., 1987; Maeno et al., 1987; Satoh et al., 1988, 1990). Specific heat data confirmed that CePtIn is a heavy fermion system with $C/T \geq 0.5$ J/mol K$^2$ below 1 K. Thus, the magnetic ground state of the CeTIn compounds varies dramatically with $T$. This variation most likely arises from a change in the coupling constant $J$ between the localized 4$f$ electrons and the conduction electrons. The temperature dependence of the electrical resistivity of CeTIn ($T = \text{Ni, Pd, Pt}$) was compared with those of LaTIn ($T = \text{Ni, Pd, Pt}$). Subtracting the electrical resistivities of the lanthanum compounds, it was possible to extract the part of magnetic scattering for the cerium compounds (Fujii et al., 1987). The temperature dependence of the thermoelectric power is similar for the three indides. CeNiIn has the highest absolute values with 50 $\mu$V/K near 120 K (Yamaguchi et al., 1990).

CeRhIn (Adroja et al., 1989; Malik et al., 1990a) is a mixed-valent indide with ZrNiAl type structure. The magnetic susceptibility shows a broad shoulder around 150 K which is a characteristic feature of intermediate-valent cerium compounds. The magnetic susceptibility behavior can be expressed via the Sales and Wohlleben (1975) interconfiguration fluctuation model. The fit resulted in a spin fluctuation temperature of about 500 K. Due to strong magnetic scattering, also the electrical resistivity of CeRhIn shows a broad maximum at about
The local magnetic susceptibility is highly reduced at low temperatures compared to that of perturbed angular correlation method (Nyayate et al., 1990). The measurements reveal that of 140 Ce in YRhIn, LaRhIn, CeRhIn, and GdRhIn was investigated using the time differential 140 K. In contrast, LaRhIn behaves like a classical metal. The local magnetic susceptibility of 140 Ce in YRhIn, LaRhIn, CeRhIn, and GdRhIn was investigated using the time differential perturbed angular correlation method (Nyayate et al., 1990). The measurements reveal that the local magnetic susceptibility is highly reduced at low temperatures compared to that of the free Ce$^{3+}$ ion.

Brück et al. (1993) studied the electronic properties of the solid solution CePd$_x$Rh$_{1-x}$In for $x = 0, 0.2, 0.4, 0.6, 0.8, 0.85, 0.95$, and 1. The properties continuously change from CeRhIn with unstable 4$f$ moments towards CePdIn with localized 4$f$ moments. This behavior was studied by magnetic susceptibility and magnetization measurements. At 4.2 K the magnetization at 35 T continuously increases from $x = 0$ to $x = 1$. The electronic specific heat coefficient $\gamma$ increases with increasing palladium content and reaches a maximum before magnetic ordering sets in at $x = 0.8$. Thus, $\gamma$ is 40 mJ/molK$^2$ for CeRhIn and it is consid-

<table>
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<th>$\theta$/K</th>
<th>$\mu_{\text{sf}}/\mu_B$</th>
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<td>--</td>
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<tr>
<td>TbPdIn</td>
<td>F</td>
<td>10.4</td>
<td>70, 27, 6</td>
<td>6</td>
<td>3.8</td>
<td>Balanda et al., 2002</td>
</tr>
<tr>
<td>TbPdIn</td>
<td>F</td>
<td>66</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>Nishigori et al., 1998</td>
</tr>
<tr>
<td>DyPdIn</td>
<td>F</td>
<td>11.0</td>
<td>34, 14</td>
<td>5.2</td>
<td>6.7</td>
<td>Balanda et al., 2002</td>
</tr>
<tr>
<td>DyPdIn</td>
<td>F</td>
<td>31</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>Nishigori et al., 1998</td>
</tr>
<tr>
<td>DyPdIn</td>
<td>F</td>
<td>10.5</td>
<td>38, 23</td>
<td>29</td>
<td>--</td>
<td>Li et al., 2002</td>
</tr>
<tr>
<td>HoPdIn</td>
<td>F</td>
<td>10.8</td>
<td>25, 6</td>
<td>7</td>
<td>7.7</td>
<td>Balanda et al., 2002</td>
</tr>
<tr>
<td>ErPdIn</td>
<td>F</td>
<td>9.7</td>
<td>12.3</td>
<td>1.6</td>
<td>6.0</td>
<td>Balanda et al., 2002</td>
</tr>
<tr>
<td>PrRhIn</td>
<td>F</td>
<td>3.69(5)</td>
<td>5.8(6)</td>
<td>--2.7(4)</td>
<td>1.60(5)</td>
<td>Zaremba et al., 2000c</td>
</tr>
<tr>
<td>EuRhIn</td>
<td>F</td>
<td>7.9(1)</td>
<td>22.0(5)</td>
<td>34(1)</td>
<td>6.7(1)</td>
<td>Pöttgen et al., 1999c</td>
</tr>
<tr>
<td>PrPn</td>
<td>P</td>
<td>3.53</td>
<td>--</td>
<td>--</td>
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<td>--</td>
</tr>
<tr>
<td>EuPn</td>
<td>MM</td>
<td>8.0(1)</td>
<td>16.0(5)</td>
<td>20(2)</td>
<td>7.0(1)</td>
<td>Müllmann et al., 1998</td>
</tr>
<tr>
<td>YbPn</td>
<td>AF</td>
<td>4.21</td>
<td>3.1</td>
<td>--16</td>
<td>1.6</td>
<td>Kaczorowski et al., 2000b</td>
</tr>
<tr>
<td>CeAuIn</td>
<td>AF</td>
<td>2.50</td>
<td>5.7</td>
<td>--10</td>
<td>--</td>
<td>Pleger et al., 1987</td>
</tr>
<tr>
<td>EuAuIn</td>
<td>MM</td>
<td>7.5(1)</td>
<td>21.0(5)</td>
<td>22(1)</td>
<td>5.9</td>
<td>Pöttgen, 1996b</td>
</tr>
<tr>
<td>GdAuIn</td>
<td>AF</td>
<td>8.2(1)</td>
<td>12.5(5)</td>
<td>--17(1)</td>
<td>2.0(1)</td>
<td>Pöttgen et al., 1998</td>
</tr>
<tr>
<td>TbAuIn</td>
<td>AF</td>
<td>9.8</td>
<td>35</td>
<td>--17</td>
<td>1.5</td>
<td>Szytuła et al., 2002</td>
</tr>
<tr>
<td>DyAuIn</td>
<td>AF</td>
<td>10.3</td>
<td>11</td>
<td>--6</td>
<td>3.2</td>
<td>Szytuła et al., 2002</td>
</tr>
<tr>
<td>HoAuIn</td>
<td>AF</td>
<td>10.1</td>
<td>4.8</td>
<td>--9</td>
<td>5.0</td>
<td>Szytuła et al., 2002</td>
</tr>
</tbody>
</table>
erably enhanced for CePd$_{0.2}$Rh$_{0.8}$In (78 mJ/mol K$^2$). For $x = 0.4$ and $x = 0.6$, pronounced upturns are observed in $C/T$ at low temperatures, yielding extrapolated values of about 280 and 700 mJ/mol K$^2$. CePd$_{0.6}$Rh$_{0.4}$In is a heavy fermion compound showing no sign of magnetic ordering above 300 mK.

CeCuIn remains paramagnetic down to 4.2 K (Malik et al., 1990a). Curie–Weiss behavior is observed above 40 K with an experimental magnetic moment of 2.50 $\mu_B$/Ce atom and $\Theta = 18$ K. The temperature dependence of the electrical resistivity shows metallic behavior with a linear part between 100 and 300 K and a curvature below 100 K.

The electronic structures of mixed-valent CeNiIn and the 6.2 K antiferromagnet CeAuIn have experimentally been investigated via XPS measurements (Gondek et al., 2003). The nickel 3$d$ and cerium 4$f$ states are close to the Fermi level in CeNiIn, while only the Ce 4$f$ states stay close to $E_F$ in CeAuIn. The gold 5$d$ band is presented by two peaks in the region between 3 and 5 eV below the Fermi energy. Interpretation of the cerium 3$d$ spectra according to the Gunnarsson–Schönhammer (1983) theory reveal that the hybridization energy between the cerium 4$f$ band and the conduction band is 144 meV in CeAuIn and 205 meV in CeNiIn. An overview on L$_{III}$ X-ray absorption spectroscopy, including data on CeAuIn, is given by Wohlfleben and Röhler (1984).

The electrical resistivities of the dense Kondo systems CeNiIn, CePdIn, and CePtIn have been measured under hydrostatic pressures up to 19 kbar (Kurisu et al., 1990). The Kondo temperature of CeNiIn and CePtIn shifts linearly with pressure to higher temperatures at rates of 2.3 and 1.5 K/kbar, respectively. For CePdIn, the pressures were not high enough to reach the CePtIn or CeNiIn state. Measurements of the elastic properties of CePdIn reveal that all elastic constants exhibit softening at low temperatures due to the crystal electric field effect and the antiferromagnetic ordering (Suzuki et al., 1990).

CeAuIn orders antiferromagnetically at 5.7 K as is evident from magnetic susceptibility and specific heat data (Pleger et al., 1987). The electronic specific heat coefficient $\gamma$ is 30 mJ/mol K$^2$. The temperature dependence of the electrical resistivity of CeAuIn shows a magnetic scattering contribution. Thermal conductivity measurements show values of 50 and 60 mW/cm K for LaAuIn and CeAuIn, respectively.

The magnetic properties of $R$PdIn with hexagonal ZrNiAl type structure have intensively been investigated. The compounds with Gd through Er as rare earth metal component show ferro- or ferrimagnetic ordering at comparatively high ordering temperatures (Oesterreicher, 1977; Nishigori et al., 1998). The recent investigations by Bałanda et al. (2002) are very detailed. These studies revealed additional magnetic phase transitions for the terbium, dysprosium, and holmium compound. The magnetization curves classify the $R$PdIn indides as soft ferromagnets with almost negligible hysteresis. Some of the $R$PdIn compounds are not easily magnetizable. Large magnetic anisotropy leads to different critical fields for this metamagnet. This behavior has been investigated in more detail for TbPdIn and DyPdIn in external fields up to 23 T. The critical fields are 5.2 and 15 T for TbPdIn and 3.2 T for DyPdIn at 4.2 K.

The magnetic structures of TbPdIn and DyPdIn were solved from powder neutron diffraction data (Javorský et al., 1998, 2000). TbPdIn is a non-collinear $k = (0, 0, 0)$ ferromagnet with the terbium magnetic moments within the basal plane. For DyPdIn two different magnetic phases are observed. In the high-temperature phase, $15 K < T < T_{ord}$, the dysprosium
magnetic moments order ferromagnetically along the $c$ axis. Below 15 K, basal-plane components of the dysprosium moments show antiferromagnetic ordering with a propagation vector $\mathbf{k} = (1/2, 0, 1/2)$.

An independent investigation on TbPdIn and DyPdIn by Li et al. (2002) revealed different ordering temperatures as compared to the study by Balanda et al. (2002). The ordering temperatures are slightly higher for both compounds and the second transition temperature for DyPdIn differs. The nature of these discrepancies remains unclear. One possibility is a difference in the sample preparation or a variable stoichiometry. The investigations by Li et al. (2002) further revealed that for both compounds the ground state is not a collinear ferromagnet, but some canted one with an antiferromagnetic component. The observed metastable magnetic properties are discussed in terms of the domain-wall pinning model. The AC magnetic susceptibility measurements on TbPdIn and DyPdIn by Nishigori et al. (1998) revealed spin-glass behavior. The spin-glass state arises from spin frustration due to the rare earth atoms forming a deformed Kagomé network.

The magnetic properties of SmPdIn with ZrNiAl structure were studied on large single crystals (Ito et al., 1995). SmPdIn orders ferromagnetically at $T_C = 54$ K. Isothermal magnetizations were measured at 4.5 K along the $a$ and the $c$ axis up to external fields of 5.5 T. The $a$ axis is the easy magnetization axis, however, at 5.5 T the magnetization has reached only a value of 0.21 $\mu_B$/Sm atom, roughly 30% of the saturation magnetization. Specific heat measurements on a polycrystalline sample revealed a magnetic entropy that is only 80% of the theoretical value, suggesting that also ferromagnetic ordering might be possible for SmPdIn.

The magnetic hyperfine interactions in GdNiIn, GdCuIn, and GdPdIn have been studied by $^{155}$Gd Mössbauer spectroscopy (de Vries et al., 1985). Full hyperfine field splitting is observed at 4.2 K with hyperfine fields between 23.3 and 26.2 T at the gadolinium nuclei. The fitting parameters are listed in Table 14.

The lattice parameters of TbPdIn have been refined from X-ray powder diffraction data as a function of temperature (Daniš et al., 2002). These experiments revealed a large anisotropy for the evolution of the $a$ and $c$ lattice parameter. A pronounced minimum occurs for the $a$ parameter around 80 K, slightly higher than the magnetic ordering temperature of 66 K. The $c$ lattice parameter shows a constant increase from low to high temperatures, but with a steeper increase below the ordering temperature. These studies agree well with the magnetization data which revealed the $a$ axis as the easy magnetization axis.

The magnetic susceptibility of EuPdIn and YbPdIn has been measured in the temperature range 100 to 1000 K (Cirafici et al., 1985). While the europium atoms in EuPdIn are in a stable divalent oxidation state over the whole temperature range, a continuous increase of

### Table 14

<table>
<thead>
<tr>
<th>Compound</th>
<th>$B_{hf}$/T</th>
<th>$\Delta E_{Q}$/mms$^{-1}$</th>
<th>$I'$/mms$^{-1}$</th>
<th>$\delta$/mms$^{-1}$</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>GdNiIn</td>
<td>23.3(1)</td>
<td>0.53(8)</td>
<td>0.76(4)</td>
<td>0.39(1)</td>
<td>de Vries et al., 1985</td>
</tr>
<tr>
<td>GdPdIn</td>
<td>24.0(1)</td>
<td>0.56(1)</td>
<td>0.68(4)</td>
<td>0.36(1)</td>
<td>de Vries et al., 1985</td>
</tr>
<tr>
<td>GdCuIn</td>
<td>26.2(12)</td>
<td>1.06(14)</td>
<td>0.71(7)</td>
<td>0.48(1)</td>
<td>de Vries et al., 1985</td>
</tr>
<tr>
<td>GdAuIn</td>
<td>18.4(2)</td>
<td>$-0.707(4)$</td>
<td>$-$</td>
<td>0.355(4)</td>
<td>Pöttgen et al., 1998</td>
</tr>
</tbody>
</table>
the ytterbium valence from 2.05 at 100 K to 2.55 at 1000 K is observed for YbPdIn. The intermediate-valent character of ytterbium is also evident from the specific heat data.

Single crystal magnetization data confirm the Néel temperature of 13 K for EuPdIn with TiNiSi type structure (Ito et al., 1998). The isothermal magnetizations along the \( a, b, \) and \( c \) axis exhibit three anomalies that are associated with metamagnetic transitions. The magnetic phase diagrams for all three axes have been established. According to the experimental data, EuPdIn has only a small magnetic anisotropy, in contrast to the RPdIn indides with hexagonal ZrNiAl structure.

The influence of thorium substitution in YPdIn and GdPdIn has been studied by Buschow (1975). The lattice parameters correlate with the substitution. In the solid solution \( \text{Gd}_x \text{Th}_{1-x} \text{PdIn} \), the \( c \) parameter decreases while \( a \) increases with increasing \( x \). The thorium substitution in YPdIn changes the Pauli susceptibility. In ferromagnetic GdPdIn the magnetic coupling drastically changes upon thorium substitution, resulting in a significant decrease of the paramagnetic Curie temperature.

YbPtIn is a Kondo lattice system, which orders antiferromagnetically at 3.1 K (Kaczorowski et al., 2000a, 2000b; Trovarelli et al., 2000). Successive spin reorientations at lower temperatures are evident from the specific heat and electrical resistivity data. According to the large electronic specific heat coefficient of \( \gamma = 750 \text{ mJ/mol K}^2 \), YbPtIn can be classified as a heavy Fermion system. The complex magnetic behavior of YbPtIn arises from a topological frustration in the ytterbium atom substructure with trigonal symmetry. At 1.7 K and in an external field of 5 T, the magnetization tends towards saturation. The magnetic moment of 1.6 \( \mu_B / \text{Yb atoms} \), however, is significantly smaller than the free ion value of 4 \( \mu_B \) for Yb\(^{3+} \).

The Eu\( T \)In (\( T = \text{Zn, Rh, Pd, Pt, Au} \)) indides have intensively been studied by magnetization measurements, electrical resistivity data, and \(^{151}\text{Eu} \) Mössbauer spectroscopy with respect to their magnetic hyperfine interactions at low temperatures (Müllmann et al., 1998; Pöttgen, 1996a, 1996b; Pöttgen et al., 1999c). The basic magnetic data and the fitting parameters for the Mössbauer spectra are listed in tables 13 and 15, respectively. While EuZnIn is a simple antiferromagnet that is stable even in an external field of 5.5 T, metamagnetic transitions are observed for EuPdIn, EuPtIn, and EuAuIn. The \(^{151}\text{Eu} \) isomer shifts of the Eu\( T \)In indides show a linear correlation with the shortest Eu–Eu distance (Pöttgen et al., 1999c). By comparing the Eu\( T \)In indides with similar gallides, stannides, and germanides Müllmann et al. (2001) observed that the isomer shift indeed depends on the valence electron concentration.

The metallic indide PrRhIn (105 \( \pm \) \( \mu \Omega \text{ cm at room temperature} \)) orders ferromagnetically at 5.8(6) K with a saturation magnetic moment of 1.60(5) \( \mu_B / \text{Pr} \) at 2 K and 5.5 T (Zaremba et al.,

<table>
<thead>
<tr>
<th>Compound</th>
<th>( B_{hf} / T )</th>
<th>( \Delta E_0 / \text{mms}^{-1} )</th>
<th>( \Gamma / \text{mms}^{-1} )</th>
<th>( \delta / \text{mms}^{-1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>EuZnIn</td>
<td>20.1(3)</td>
<td>5</td>
<td>3.8(3)</td>
<td>−10.27(9)</td>
</tr>
<tr>
<td>EuRhIn</td>
<td>22.6(2)</td>
<td>8(1)</td>
<td>2.4</td>
<td>−7.9(1)</td>
</tr>
<tr>
<td>EuPdIn</td>
<td>22.2(1)</td>
<td>−2(1)</td>
<td>2.4(1)</td>
<td>−8.8(1)</td>
</tr>
<tr>
<td>EuPtIn</td>
<td>21.5(2)</td>
<td>8(1)</td>
<td>2.3</td>
<td>−9.7(4)</td>
</tr>
<tr>
<td>EuAuIn</td>
<td>23.1(2)</td>
<td>10(1)</td>
<td>2.3</td>
<td>−8.66(7)</td>
</tr>
</tbody>
</table>

Table 15

Hyperfine parameters derived from fitting of \(^{151}\text{Eu} \) Mössbauer spectra of various Eu\( T \)In compounds (4.2 K data). From Müllmann et al. (1998)
In contrast, PrPtIn remains paramagnetic down to 1.7 K (Zaremba et al., 2001). The largely negative paramagnetic Curie temperature of $-29 \text{ K}$ may indicate antiferromagnetic ordering at much lower temperatures. SmPtIn orders ferromagnetically at 25 K. The saturation magnetization of 0.20 $\mu_B$/Sm at 1.7 K and 5 T is much smaller than expected for the free Sm$^{3+}$ ion (0.71 $\mu_B$). PrRhIn and SmPtIn may be classified as soft ferromagnets.

The compounds $R$MnIn ($R = \text{Y, Nd, Gd, Dy, Er}$) are the first known manganese containing indides (Dhar et al., 2002a, 2002b). They crystallize with the structures of the cubic Laves phase MgCu$_2$ ($R = \text{Nd}$) or the hexagonal Laves phase MgZn$_2$ ($R = \text{Y, Gd, Dy, Er}$). Since with a composition $R$MnIn an ordered distribution of manganese and indium atoms in the corresponding sites is not possible in the structures of the Laves phases, we observe Mn/In statistics leading to spin-glass behavior. The manganese atoms carry a substantial magnetic moment, but no signature of long-range magnetic ordering is observed. This is supported by specific heat data. After subtraction of the free ion values of $R^{3+}$, magnetic moments of 5.3 and 1.7 $\mu_B$ per manganese atom for GdMnIn and DyMnIn result. The spin-glass behavior is evident from differences in the zero-field cooling and field cooling data. Magnetization measurements reveal substantial coercive fields of 0.072 and 0.22 T for GdMnIn and DyMnIn, respectively. The magnetizations for GdMnIn and DyMnIn are smaller than those for the binary Laves phases GdMn$_2$ and DyMn$_2$. Similar magnetic properties have been observed for ThMnIn with the cubic Laves phase structure. The latter indide shows a sharp ferromagnetic-like transition around 270 K, which may be ascribed to Mn–Mn interactions (Manfrinetti et al., 2000). The magnetic ground state is not clear. There are indications for possible ferromagnetic ordering or even a reentrant spin-glass like state, which most likely results from the Mn/In statistics.

The antiferromagnetic ground state of GdAuIn is stable at least up to 5.5 T (Pöttgen et al., 1998). The magnetization at that field and 4.2 K is 2.1 $\mu_B$/Gd atom. The $^{155}$Gd Mössbauer spectra show one signal that is subject to quadrupole splitting. The ordering temperatures determined from the magnetic susceptibility measurements and the Mössbauer spectra agree well. Full magnetic hyperfine field splitting is detected at 4.2 K. The fitting parameters are listed in table 14.

The magnetic structures of $R$AuIn ($R = \text{Tb, Dy, Ho}$) were determined from neutron powder diffraction data (Sztyuka et al., 2002). Antiferromagnetic ordering is evident from the temperature dependent magnetic susceptibility data (table 13). The magnetic ordering in TbAuIn and DyAuIn is described by the propagation vector $\mathbf{k} = (0, 0, 1/2)$, while two propagation vectors $\mathbf{k}_1 = (0, 1/2, 1/2)$ and $\mathbf{k}_2 = (0, 0, 1/2)$ are needed for HoAuIn. The magnetization isotherms at 1.9 K reveal metamagnetic transitions at critical fields of 2.05 T (TbAuIn), 1.5 T (DyAuIn), and 0.8 T (HoAuIn). For TbAuIn, spin-glass behavior occurs between the Néel temperature $T_N = 35 \text{ K}$ and the spin-freezing temperature $T_{SG}$ of 58.5 K. TbAuIn is paramagnetic above 58 K. Susceptibility data of YbAuIn and YbPdIn indicate intermediate ytterbium valence (Zell et al., 1981). The valence/pressure dependence of both indides and of YbAu$_2$In has also been determined.

7.1.2. Hydrogenation behavior

Many of the equiatomic indides have intensively been studied with respect to their hydrogenation behavior. In the $R$NiIn series, only the indides with lanthanum, cerium, praseodymium,
and neodymium form hydrides, while no hydrogen incorporation is observed for those with yttrium and the heavy rare earth metals (Bulyk et al., 1999). The hydrogen insertion into LaNiIn, CeNiIn, and NdNiIn causes a pronounced anisotropic expansion of the unit cell along the c axis and results in volume increases up to 9%. The structures of LaNiInH$_{2.0}$, CeNiInH$_{1.8}$, and NdNiInH$_{1.7}$ have been investigated by X-ray powder diffraction and the hydrogen desorption behavior was studied in detail. High-resolution neutron diffraction data led to a clear localization of the hydrogen atoms and they revealed extraordinary short H–H separations around 160 pm (Yartys et al., 2002). The chemical bonding situation in $R_3Ni_3In_3H_4$ (Vajeeston et al., 2003) was discussed in sect. 6 Chemical Bonding.

The hydrides CeNiInH$_{1.0}$ and CeNiInH$_{1.6}$ have been investigated by $^1$H nuclear magnetic resonance (Ghoshray et al., 1993). The spectra show a narrow single line near 300 K but a Pake doublet at lower temperature. Fitting of the spectrum yields a proton–proton separation of 148(2) pm. Similar results have been obtained for PrNiInH$_{1.29}$ (Sen et al., 1994; Sen et al., 1996).

Hydrogenation of intermediate-valent CeNiIn induces a ferromagnetic transition at $T_C = 6.8$ K (Chevalier et al., 2002). Thus, the insertion of hydrogen forces a cerium valence transition from intermediate-valent to trivalent. CeNiInH$_{1.8(1)}$ is a Curie–Weiss paramagnet above 100 K with an experimental magnetic moment of 2.61 $\mu_B$/Ce atom and $\Theta = 20$ K. In the ferromagnetically ordered state at 2 K and 2 T, the saturation magnetization of 1.31 $\mu_B$ is smaller than that of the free Ce$^{3+}$ ion (2.14 $\mu_B$), suggesting magnetic anisotropy. The first systematic investigations on the hydrogenation behavior of CeNiIn, however, have been performed by Sen et al. already in 1994. Quaternary hydrides of compositions CeNiInH$_x$ ($x = 0.5, 0.6, 0.67, 0.75, 1.0, 1.6$) showed an increase of the magnetic moment on the cerium atoms from 1.73 $\mu_B$/Ce for $x = 0$ to 2.36 $\mu_B$/Ce for $x = 1.6$.

Mixed-valent CeRhIn forms a higher hydride CeRhInH$_{1.55}$, which is quite unstable and transforms to the lower hydride CeRhInH$_{0.55}$ (Raj et al., 2002). The hexagonal ZrNiAl related host structure remains for the hydride in a somewhat expanded form. According to magnetic susceptibility measurements, cerium remains in a mixed-valent state also in the hydride; however, a definite shift towards Ce$^{3+}$ is detected, as is evident from a reduction of the Kondo temperature. The hydride formation results in an increasing occupancy of the rhodium $d$-bands.

### 7.2. Indides $R_2T_2In$ with ordered $U_3Si_2$ or $Zr_3Al_2$ type structures

The indides with the diamagnetic rare earth elements scandium, yttrium, lanthanum, and lutetium show Pauli paramagnetism, in agreement with the metallic conductivity. Plots of the magnetic data have so far only been published for Sc$_2$Ni$_2$In (Pöttgen and Dronskowskii, 1996).

If a magnetic rare earth metal is introduced into these indides, various kinds of magnetic interactions occur. The basic magnetic data of these intermetallics are listed in table 16. The palladium and copper based cerium compounds Ce$_2$Pd$_2$In and Ce$_2$Cu$_2$In show extended homogeneity ranges Ce$_2$Pd$_{2+x}$In$_{1-x}$ and Ce$_2$Cu$_{2+x}$In$_{1-x}$ (Sampathkumaran and Mallik, 1996). The essential features of thermoelectric power ($S$) measurements of Ce$_2$Pd$_{2.05}$In$_{0.95}$ resemble those of a heavy fermion system (Pinto et al., 1997). Magnetoresistivity data underline the important role of the Kondo effect.
### Table 16: Magnetic properties of selected $R_2T_2X$ intermetallics

<table>
<thead>
<tr>
<th>Compound</th>
<th>Magnetism</th>
<th>$\mu_{exp}/\mu_B$</th>
<th>$T_N, T_C/K$</th>
<th>$\Theta/K$</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ce$_2$Cu$_2$In</td>
<td>MM</td>
<td>2.48</td>
<td>5.5</td>
<td>-7.7</td>
<td>Kaczorowski et al., 1996a</td>
</tr>
<tr>
<td>Gd$_2$Cu$_2$In</td>
<td>F</td>
<td>-</td>
<td>85.5</td>
<td>90</td>
<td>Fisher et al., 1999</td>
</tr>
<tr>
<td>Tb$_2$Cu$_2$In</td>
<td>F</td>
<td>-</td>
<td>81</td>
<td>77.2</td>
<td>Fisher et al., 1999</td>
</tr>
<tr>
<td>Dy$_2$Cu$_2$In</td>
<td>F/ AF</td>
<td>-</td>
<td>45.5; 22</td>
<td>46</td>
<td>Fisher et al., 1999</td>
</tr>
<tr>
<td>Ho$_2$Cu$_2$In</td>
<td>F</td>
<td>-</td>
<td>26.7</td>
<td>29</td>
<td>Fisher et al., 1999</td>
</tr>
<tr>
<td>Er$_2$Cu$_2$In</td>
<td>F</td>
<td>-</td>
<td>37</td>
<td>30</td>
<td>Fisher et al., 1999</td>
</tr>
<tr>
<td>Tm$_2$Cu$_2$In</td>
<td>F</td>
<td>-</td>
<td>29.5</td>
<td>21</td>
<td>Fisher et al., 1999</td>
</tr>
<tr>
<td>Ce$_2$Pd$_2$In</td>
<td>AF</td>
<td>2.9</td>
<td>4.3</td>
<td>12.3</td>
<td>Kaczorowski et al., 1996b</td>
</tr>
<tr>
<td>Ce$_2$Pd$_2$In</td>
<td>AF/F</td>
<td>2.48(3)</td>
<td>~ 4.2</td>
<td>18(2)</td>
<td>Gordon et al., 1995</td>
</tr>
<tr>
<td>Ce$_2$Pd$_2$In</td>
<td>F</td>
<td>1.7</td>
<td>4</td>
<td>4</td>
<td>Hulliger and Xue, 1994</td>
</tr>
<tr>
<td>Pr$_2$Pd$_2$In</td>
<td>AF</td>
<td>3.55</td>
<td>5</td>
<td>-17</td>
<td>Giovannini et al., 1998</td>
</tr>
<tr>
<td>Pr$_2$Pd$_2$In</td>
<td>AF</td>
<td>3.54</td>
<td>5</td>
<td>1</td>
<td>Giovannini et al., 1994</td>
</tr>
<tr>
<td>Nd$_2$Pd$_2$In</td>
<td>AF</td>
<td>3.64</td>
<td>8</td>
<td>3</td>
<td>Hulliger and Xue, 1994</td>
</tr>
<tr>
<td>Nd$_2$Pd$_2$In</td>
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<td>3.57</td>
<td>7.5</td>
<td>4</td>
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<tr>
<td>Sm$_2$Pd$_2$In</td>
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<td>0.58</td>
<td>12; 9</td>
<td>-12</td>
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</tr>
<tr>
<td>Gd$_2$Pd$_2$In</td>
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<td>20</td>
<td>-9</td>
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</tr>
<tr>
<td>Gd$_2$Pd$_2$In</td>
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<td>7.95</td>
<td>21</td>
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</tr>
<tr>
<td>Tb$_2$Pd$_2$In</td>
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<td>32</td>
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<tr>
<td>Tb$_2$Pd$_2$In</td>
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<td>31</td>
<td>-5</td>
<td>Hulliger and Xue, 1994</td>
</tr>
<tr>
<td>Dy$_2$Pd$_2$In</td>
<td>AF</td>
<td>10.47</td>
<td>12</td>
<td>-20</td>
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</tr>
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<td>Dy$_2$Pd$_2$In</td>
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<td>10.45</td>
<td>12</td>
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<td>Hulliger and Xue, 1994</td>
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<td>Ho$_2$Pd$_2$In</td>
<td>AF</td>
<td>10.4</td>
<td>8.5</td>
<td>-2</td>
<td>Giovannini et al., 1998</td>
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<td>8</td>
<td>-2</td>
<td>Hulliger and Xue, 1994</td>
</tr>
<tr>
<td>Er$_2$Pd$_2$In</td>
<td>AF</td>
<td>9.55</td>
<td>5.5</td>
<td>-5</td>
<td>Giovannini et al., 1998</td>
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<td>AF</td>
<td>9.65</td>
<td>5</td>
<td>-5</td>
<td>Hulliger and Xue, 1994</td>
</tr>
<tr>
<td>Tm$_2$Pd$_2$In</td>
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<td>7.32</td>
<td>3.8; 3.2</td>
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<td>4</td>
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<td>Hulliger and Xue, 1994</td>
</tr>
<tr>
<td>Yb$_2$Pd$_2$In</td>
<td>IV</td>
<td>4.05</td>
<td>-</td>
<td>-104</td>
<td>Giovannini et al., 1998</td>
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<tr>
<td>Ce$_2$Pt$_2$In</td>
<td>P</td>
<td>2.49</td>
<td>-</td>
<td>-8.4</td>
<td>Kaczorowski et al., 1996a</td>
</tr>
<tr>
<td>Ce$_2$Au$_2$In</td>
<td>MM</td>
<td>2.48</td>
<td>3.2</td>
<td>-15.7</td>
<td>Kaczorowski et al., 1996a</td>
</tr>
<tr>
<td>Pr$_2$Au$_2$In</td>
<td>F</td>
<td>3.51</td>
<td>&lt; 4</td>
<td>-7</td>
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<tr>
<td>Nd$_2$Au$_2$In</td>
<td>F</td>
<td>3.50</td>
<td>~ 36</td>
<td>11</td>
<td>Hulliger, 1996</td>
</tr>
<tr>
<td>Gd$_2$Au$_2$In</td>
<td>F</td>
<td>7.80</td>
<td>~ 83</td>
<td>85</td>
<td>Hulliger, 1996</td>
</tr>
<tr>
<td>Tb$_2$Au$_2$In</td>
<td>F</td>
<td>9.31</td>
<td>~ 73</td>
<td>76</td>
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<tr>
<td>Dy$_2$Au$_2$In</td>
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<td>24</td>
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</tr>
<tr>
<td>Ho$_2$Au$_2$In</td>
<td>F</td>
<td>10.3</td>
<td>~ 20</td>
<td>6</td>
<td>Hulliger, 1996</td>
</tr>
<tr>
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<td>9.47</td>
<td>19</td>
<td>7</td>
<td>Hulliger, 1996</td>
</tr>
<tr>
<td>Tm$_2$Au$_2$In</td>
<td>AF</td>
<td>7.50</td>
<td>6.5</td>
<td>-3</td>
<td>Hulliger, 1996</td>
</tr>
</tbody>
</table>

The antiferromagnetic structures of Pr$_2$Pd$_2$In and Nd$_2$Pd$_2$In have been refined from neutron powder diffraction data (Fischer et al., 2000). A modulated ordering of the praseodymium and neodymium magnetic moments with propagation vectors $\mathbf{k} = [0, 0, 1/2]$ and $[1/4, 1/4, 0]$ below the Néel temperatures $T_N = 5$ K and 8 K is evident for Pr$_2$Pd$_2$In and Nd$_2$Pd$_2$In, respectively.
Depending on the size of the transition metal, the ytterbium compounds Yb$_2$T$_2$In ($T = \text{Cu, Pd, Au}$) crystallize with two different structure types, i.e. the tetragonal Mo$_2$FeB$_2$ type for the gold and palladium compound and the orthorhombic W$_2$CoB$_2$ type for Yb$_2$Cu$_2$In (Giovannini et al., 2001). Susceptibility, electrical resistivity, and heat capacity measurements revealed no magnetic order for any of these compounds. Yb$_2$Cu$_2$In and Yb$_2$Au$_2$In show stable divalent ytterbium while an intermediate ytterbium valence is detected for Yb$_2$Pd$_2$In. Divalent ytterbium in Yb$_2$Au$_2$In was also reported by Hulliger (1996). Yb$_2$Cu$_2$In has independently been investigated by Tsujii et al. (2001). This group prepared Yb$_2$Cu$_2$In under high-pressure (3.5 GPa)–high-temperature (870 K) conditions and the properties determined for their sample are in good agreement with the results by Giovannini et al. (2001). A $^{170}$Yb Mössbauer spectrum of Yb$_2$Pd$_2$In at 4.2 K and 40 mK shows a single signal subject to quadrupole splitting, reflecting the non-cubic site symmetry of ytterbium (Bauer et al., 2003). The quadrupolar coupling parameter is 1.85 mm/s.

A very detailed investigation of the magnetic properties was performed for the series $R_2$Pd$_2$In (Giovannini et al., 1998). The basic magnetic data are listed in table 16. As expected, the compounds with the diamagnetic lanthanum and lutetium atoms show Pauli-paramagnetism. Interesting results are the wide homogeneity range for Ce$_2$Pd$_{2+x}$In$_{1-x}$, which is comparable to the tin compound, Ce$_2$Pd$_{2+x}$Sn$_{1-x}$. The intermediate-valent behavior of Yb$_2$Pd$_2$In is discussed in detail by Giovannini et al. (2001). Magnetization measurements show metamagnetic transitions for many of the $R_2$Pd$_2$In compounds with critical fields around 1 T. Strong crystal field splitting occurs for Ce$_2$Pd$_2$In (Kaczorowski et al., 1996b). The pressure dependence of the electrical resistivity and the heat capacity was investigated in detail.

The solid solution Ce$_2$Ni$_{2-x}$Pd$_{2+x}$In was studied for $x = 0–1$ (Ijiri and DiSalvo, 1996). The magnetic properties change gradually from ferromagnetic Ce$_2$Pd$_2$In with $T_C = 4$ K to the temperature-independent paramagnet Ce$_2$Ni$_2$In. These changes are also evident from the behavior of the unit cell parameters and the evolution of the electrical resistivity behavior as functions of $x$.

Preliminary magnetic data for the series of $R_2$Rh$_2$In compounds have been reported by Hulliger (1995b). The behavior of the unit cell volumes and the magnetic susceptibility data confirmed tetravalent cerium in Ce$_2$Rh$_2$In. For Pr$_2$Rh$_2$In (3.60 $\mu_B$ and $\Theta = 14$ K) and Nd$_2$Rh$_2$In (3.55 $\mu_B$) ferromagnetic transitions at $T_C = 13.5$ and $\sim 10$ K have been observed. The platinum based indides Pr$_2$Pt$_2$In and Nd$_2$Pt$_2$In order ferromagnetically at $T_C = 9.5$ and 17 K, respectively (Hulliger, 1995a). Tb$_2$Pt$_2$In is an antiferromagnet ($T_N = 40$ K, 9.75 $\mu_B$ and $\Theta = 33$ K). A metamagnetic transition occurs at 1.8 T and $T = 2.4$ K. This is also the case for Ho$_2$Pt$_2$In ($T_N = 8$ K, 10.3 $\mu_B$, $\Theta = 3$ K, $B_C = 0.4$ T, and $T = 2$ K). All these values are only preliminary data. As stated by Hulliger (1995a), the magnetic measurements were carried out on impurity-containing samples. The magnetic data of the $R_2$Au$_2$In series have also been investigated (Hulliger, 1996). They are listed in table 16. Ce$_2$Au$_2$In shows strong crystal field effects and the samarium sample probably contained a ferromagnetic impurity with a Curie temperature of 44 K.

The whole series of Ce$_2$T$_2$In ($T = \text{Ni, Cu, Rh, Pd, Pt, Au}$) indides has been investigated by Kaczorowski et al. (1996a, 1996b). The authors report structure refinements, magnetization measurements, and a detailed study of the electrical resistivity behavior. The magnetic
contribution to the electrical resistivity was evaluated by subtractions of the diamagnetic part derived from the isostructural lanthanum compounds. The detailed investigations led to a better understanding of the electronic structures of these interesting materials. Magnetization and specific heat measurements of Ce$_2$T$_2$In ($T =$ Cu, Au, Pt) at extremely low temperatures gave further interesting results (Mock et al., 1997). Ce$_2$Cu$_2$In and Ce$_2$Au$_2$In show clear metamagnetic transition at 0.3 T. Spin-glass freezing below $\sim 0.5$ K is observed for Ce$_2$Pt$_2$In.

Cerium substitution by diamagnetic lanthanum, yttrium, or lutetium in Ce$_2$Rh$_2$In and Ce$_2$Pd$_2$In has been investigated by Mallik et al. (1997). Cerium L$_{III}$ spectra revealed cerium valences of 3.07 in Ce$_2$Pd$_2$In, 3.07 in Y$_{1.6}$Ce$_{0.4}$Pd$_2$In, 3.0 in Lu$_{1.6}$Ce$_{0.4}$Pd$_2$In, 3.18 in Ce$_2$Rh$_2$In, and 3.20 in CeLaRh$_2$In. The large $\gamma$ values (70–100 mJ/mol K$^2$ for Y$_{1.6}$Ce$_{0.4}$Pd$_2$In and $\sim 150$ mJ/mol K$^2$ for Lu$_{1.6}$Ce$_{0.4}$Pd$_2$In) are attributable to heavy fermion behavior.

All $R_2$Cu$_2$In indides order ferromagnetically with Curie temperatures between 26.7 and 85.5 K. Strong crystal field anisotropies result in $T_C$ values that do not scale with the de Gennes function of the rare earth elements (Fisher et al., 1999). Dy$_2$Cu$_2$In shows a second phase transition to an antiferromagnetic ground state at $T_N = 22$ K.

All $R_2$Pd$_2$In indides are metallic conductors (Fischer et al., 2000). Anomalies at low temperatures indicate the onset of magnetic ordering. The electrical resistivity of Ce$_2$Cu$_2$In, Ce$_2$Au$_2$In, and Ce$_2$Pt$_2$In has been measured as a function of temperature and pressure (Hauser et al., 1997). For the copper compound, the Néel temperature increases with increasing pressure, while for Ce$_2$Au$_2$In $T_N$ stays nearly constant in the pressure range investigated. Thus, the AF transition is hardly affected by the external pressure. For Ce$_2$Pt$_2$In a rapid reduction of $N(E_F)$ with increasing pressure is observed. Additionally, the specific heat behavior of Ce$_2$Cu$_2$In, Ce$_2$Au$_2$In, and Ce$_2$Pt$_2$In at various external fields has been measured. The high $\gamma$ value of 500 mJ/mol K$^2$ for Ce$_2$Pt$_2$In classifies this indide as a heavy-fermion system.

A detailed study of the electrical resistivity behavior of the La$_2$T$_2$In and Ce$_2$T$_2$In ($T =$ Ni, Cu, Rh, Pd, Pt, Au) indides was performed by Kaczorowski et al. (1996a, 1996b). They evaluated the electrical resistivity data according to the modified Bloch–Grüneisen relation and derived the Debye temperatures (table 17).

7.3. **Indium-rich indides with HoCoGa$_5$ or Ho$_2$CoGa$_8$ structures**

The crystallographic data of CeCoIn$_5$ and Ce$_2$CoIn$_8$ have been published in 1987 by Kalychak et al. The isotypic rhodium and iridium indides have been synthesized in the form of large

<table>
<thead>
<tr>
<th>Compound</th>
<th>$\rho_0$ $\mu\Omega$ cm</th>
<th>$\theta_D$/K</th>
<th>$R$ $\mu\Omega$ cm K$^{-1}$</th>
<th>$K/10^{-2}$ $\mu\Omega$ cm K$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>La$_2$Ni$_2$In</td>
<td>52.5</td>
<td>118</td>
<td>0.578</td>
<td>9.9</td>
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<tr>
<td>La$_2$Cu$_2$In</td>
<td>25.9</td>
<td>114</td>
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</tr>
<tr>
<td>La$_2$Rh$_2$In</td>
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<td>119</td>
<td>0.268</td>
<td>5.1</td>
</tr>
<tr>
<td>La$_2$Pd$_2$In</td>
<td>91.8</td>
<td>130</td>
<td>0.964</td>
<td>12.5</td>
</tr>
<tr>
<td>La$_2$Pt$_2$In</td>
<td>42.4</td>
<td>104</td>
<td>0.707</td>
<td>14.3</td>
</tr>
<tr>
<td>La$_2$Au$_2$In</td>
<td>20.7</td>
<td>98</td>
<td>0.376</td>
<td>8.7</td>
</tr>
</tbody>
</table>
single crystals from indium fluxes in the last three years. Discovery of the heavy fermion state and superconductivity in these materials (Petrovic et al., 2001a, 2001b; Thompson et al., 2001) triggered large research programs. In the following paragraphs we summarize the recent results. The data are certainly not complete since several new publications already appeared after the completion of this manuscript.

Among the indides with HoCoGa$_5$ type structure, CeRhIn$_5$ (Hegger et al., 2000) has most intensively been investigated with respect to the magnetic and electrical properties. CeRhIn$_5$ is a heavy-fermion antiferromagnet with an ordering temperature of 3.8 K. Christianson et al. (2002a, 2002b) give a detailed report on the magnetotransport properties of CeRhIn$_5$. Resistivity measurements revealed an anisotropy in the Kondo-derived magnetic scattering. The magnetoresistance of CeRhIn$_5$ is positive. Application of hydrostatic pressure induces a first-order-like transition from an unconventional antiferromagnetic to a superconducting state with $T_C = 2.1$ K (Hegger et al., 2000). The CeRhIn$_5$ structure contains a tetragonally distorted CeIn$_3$ slab. Taking the bulk modulus $B = 650$ kbar for binary CeIn$_3$ with Cu$_3$Au structure, the difference in the lattice parameters between cubic CeIn$_3$ at atmospheric pressure and the tetragonally distorted slab in CeRhIn$_5$ experiences a chemical pressure of approximately 14 kbar. As yttrium is substituted for cerium in the solid solution Ce$_{1-x}$Y$_x$RhIn$_5$, the Néel temperature is suppressed, yielding a quantum critical point at an yttrium concentration of $x = 0.38$ (Zapf et al., 2003).

The cerium compound CeRhIn$_5$ forms a solid solution Ce$_x$La$_{1-x}$RhIn$_5$ with the lanthanum compound LaRhIn$_5$ where the paramagnetic cerium atoms are substituted by diamagnetic lanthanum (Alver et al., 2001). Measurements of the de Haas–van Alphen effect for indides with $x = 0–1$ reveal that the cerium 4$f$ electrons remain localized. Such a behavior is typical of antiferromagnetic heavy fermion compounds. A detailed study of cerium site dilution by lanthanum was presented by Pagliuso et al. (2002) for CeRhIn$_5$ and Ce$_2$RhIn$_8$. Magnetic susceptibility and low-temperature specific heat data are consistent with a smooth suppression of the Néel temperature by lanthanum doping. This is also the case within the solid solution Ce$_x$La$_{1-x}$CoIn$_5$ (Petrovic et al., 2002).

High pulsed magnetic fields up to 50 T have been used to observe the de Haas–van Alphen oscillations in ternary CeRhIn$_5$ (Cornelius et al., 2000). Specific heat data revealed an enhanced value of the electronic specific heat coefficient $\gamma = 420$ mJ/mol K$^2$. The electronic and magnetic properties of CeRhIn$_5$ are anisotropic due to the quasi-two-dimensional character of the crystal structure. In addition, in isotypic CeIrIn$_5$ the de Haas–van Alphen oscillation has been observed. Due to the tetragonal structure, the topology of the Fermi surface is found to be nearly cylindrical (Haga et al., 2001; Shishido et al., 2002). Electronic structure calculations and the de Haas–van Alphen effect have been used to study the angular-dependent extrema areas and effective masses of CeRhIn$_5$ (Hall et al., 2001a). Detailed comparison between theory and experiment was given. The density of states shows a predominance of cerium 4$f$ states at the Fermi energy. The DOS at $E_F$ is quite large but does not indicate a significant structural instability. Time-of-flight neutron scattering data on large high quality single crystals of CeRhIn$_5$ gave evidence for two crystal field excitations at 6.9 and 23.6 meV (Christianson et al., 2002b).
Specific heat measurements on CeCoIn$_5$ and CeIrIn$_5$ in magnetic fields up to 32 T show only a weak field dependence suggesting that the large Sommerfeld coefficients of both indides are due to correspondingly large effective electron masses (Kim et al., 2001).

The compound Ce$_2$RhIn$_8$ is a heavy-fermion antiferromagnet (Cornelius et al., 2001) with a $\gamma$ value around 400 mJ/mol K$^2$. The specific heat data at zero applied magnetic field are consistent with the existence of an anisotropic spin-density wave opening a gap in the Fermi surface of CeRhIn$_5$. In contrast, Ce$_2$RhIn$_8$ shows behavior consistent with a simple antiferromagnetic magnon. The complex magnetic phase diagrams for CeRhIn$_5$ and Ce$_2$RhIn$_8$ studied by Cornelius et al. (2001) are shown in fig. 97. Furthermore, the complex magnetic behavior in the solid solution CeRh$_{1-x}$Ir$_x$In$_5$ and the pressure–temperature phase diagram of Ce$_2$RhIn$_8$ have intensively been studied (Nicklas et al., 2003a, 2003b; Pagliuso et al., 2001a, 2001b). Susceptibility measurements show experimental moments of 2.6 and 2.3 $\mu_B$/Ce atom for Ce$_2$RhIn$_8$ and Ce$_2$IrIn$_8$, respectively (Ohara et al., 2003; Nicklas et al., 2003b), in agreement with trivalent cerium. The $\Theta$ values of $-15$ K and $-85$ K for $H||c$ and $H||a$ have been observed for Ce$_2$RhIn$_8$ and Ce$_2$IrIn$_8$, indicating similar crystal field effects in both compounds. The isotypic praseodymium indides show magnetic moments close to 3.58 $\mu_B$ as expected for Pr$^{3+}$. The indides $R_2$RhIn$_8$ and $R_2$IrIn$_8$ ($R = $ La, Ce, Pr)

![Fig. 97. Cumulative phase diagrams for CeRhIn$_5$ and Ce$_2$RhIn$_8$ for various applied fields along the $a$ axis. From Cornelius et al. (2001). I, II, and III are different antiferromagnetic phases. Filled rhombi denote $T_N$.](image)
are metallic conductors. The temperature dependence for the electrical resistivity of La$_2$RhIn$_8$ is somewhat peculiar and a quantitative understanding has not been achieved yet. The magnetic structure of Ce$_2$RhIn$_8$ has been solved from neutron powder diffraction data (Wei et al., 2001). Ce$_2$RhIn$_8$ is a collinear antiferromagnet with a propagation vector of $(1/2, 1/2, 0)$ and a staggered magnetic moment of 0.55(6) $\mu_B$ per cerium atom at 1.6 K tilted by 38(2)$^\circ$ from the tetragonal $c$ axis.

The $^{115}$In nuclear quadrupole resonance (NQR) measurements of CeRhIn$_5$ in the paramagnetic and in the antiferromagnetic states gave evidence for a spiral magnetic order of the cerium magnetic moments, which are incommensurate with the nuclear structure (Curro et al., 2000; Zheng et al., 2003). According to the $^{115}$In NQR data of Kohori et al. (2000, 2001), non-$s$-wave superconductivity occurs in CeRhIn$_5$. The magnetic dynamic indicates possible two-dimensional behavior. NQR is an ideal probe for the local magnetic fields as well as the dynamic behavior of the electronic system. Most investigations on the Ce$T$In$_5$ and Ce$_2$TIn$_5$ intermetallics have been performed on single crystalline materials. Nevertheless, for the NQR measurements on CeRhIn$_5$, the crystals were ground to a fine powder in order to facilitate the penetration of the rf fields. Pressure-dependent $^{115}$In NQR measurements of CeRhIn$_5$ were performed by Kawasaki et al. (2001, 2003) and Mito et al. (2001). The Néel temperature is reduced at $p \geq 1.23$ GPa with an emerging pseudogap behavior. Corresponding $^{115}$In NQR data for CeIrIn$_5$ are available from Zheng et al. (2001).

Neutron powder diffraction data confirm the incommensurate magnetic structure of CeRhIn$_5$ (Wei et al., 2000). The magnetic propagation vector is $\mathbf{k} = (1/2, 1/2, 0.297)$ and temperature independent up to the Néel temperature. At 1.4 K a staggered magnetic moment of 0.264(4) $\mu_B$ resides on the cerium atoms and spirals transversely along the $c$ axis. The magnetic moments of the nearest cerium neighbors in the tetragonal basal plane show antiferromagnetic alignment. Magnetic neutron scattering data reveal anisotropic three-dimensional antiferromagnetic fluctuations with an energy scale of less than 1.7 meV for temperatures as high as 3$T_C$ (Wei et al., 2002).

CeCoIn$_5$ has been investigated by means of $^{115}$In and $^{59}$Co nuclear magnetic resonance measurements above and below the superconducting state (Curro et al., 2001). The hyperfine field couplings of both $^{115}$In and $^{59}$Co are anisotropic and they exhibit a large change below 50 K arising from changes in the crystal field populations of the cerium atoms indicating localized $f$ electrons. Measurements of the de Haas–van Alphen effect in CeCoIn$_5$ have been performed in the temperature range 20 to 500 mK and in applied fields up to 18 T. According to the angular-dependent measurements of the external Fermi surface areas, CeCoIn$_5$ has a more extreme two-dimensional surface than is found in isotypic CeRhIn$_5$ and CeIrIn$_5$ (Hall et al., 2001a, 2001b). The unconventional superconducting properties of CeCoIn$_5$ are strongly supported by point-contact spectra (Goll et al., 2003). The thermal expansion of CeCoIn$_5$ in zero field shrinks for the [100] direction in the superconducting state, while it expands for [001] (Takeuchi et al., 2002).

The thermal conductivity of CeCoIn$_5$ has been studied in a magnetic field rotating within the 2D planes (Izawa et al., 2001). These data clearly reveal fourfold symmetry for the thermal conductivity and indicate that the symmetry most likely belongs to $dx^2-y^2$ implying that the anisotropic antiferromagnetic fluctuation is relevant to the superconductivity. Independent
thermal conductivity studies were reported by Movshovich et al. (2001). DC magnetization measurements of a high-quality single crystal of CeCoIn$_5$ down to 50 mK showed a sharp magnetization jump with a small hysteresis at an upper critical field $H_{C2}$ for both the $a$ and $c$ directions (Tayama et al., 2002; Sakakibara et al., 2003). These data revealed that the transition to the normal state is of first order. The magnetic phase diagram under magnetic field parallel to the $a$ and $c$ axes are presented. Low-temperature specific heat data in magnetic fields up to 8 T are presented by Ikeda et al. (2001). The solid solution CeRh$_{1-x}$Co$_x$In$_5$ was recently investigated with respect to the magnetic and superconducting properties (Zapf et al., 2001; Majumdar et al., 2003). $T_C$ increases and $T_N$ decreases with increasing $x$.

High-field magnetization data revealed weak metamagnetism for CeIrIn$_5$ for the field along the [001] direction (Takeuchi et al., 2001). The critical field is around 42 T. In the [110] direction, the magnetization increases gradually up to 50 T. For CeRhIn$_5$, a two-step metamagnetic transition occurs at 2 and 50 T along [001]. Together with detailed specific heat measurements, the magnetic phase diagram of CeRhIn$_5$ has been constructed. The anisotropy in the magnetization data for both compounds is paralleled by the thermal expansion data along [100] and [001]. All data have been analyzed based on the CEF model. The splitting energies from the first ground state to the first and second excited states are 61 K and 300 K for the iridium and 68 and 330 K for the rhodium compound.

Besides the detailed investigations on the lanthanum and cerium compounds within this structural family, some magnetic data on the isotypic neodymium (Pagliuso et al., 2000) and gadolinium compounds (Pagliuso et al., 2001a) were reported. The basic magnetic data of these intermetallics are listed in table 18. For the neodymium compounds, good agreement has been observed for the transition temperatures detected from the magnetic susceptibility and specific heat measurements.

### 7.4. Cubic indides with MgCu$_4$Sn structure

The ternary indide YbCu$_4$In has a phase transition at 40 K (Felner et al., 1987; Kojima et al., 1990; Lawrence et al., 2001). This indide belongs to a series of Yb$_x$In$_{1-x}$Cu$_2$ compounds

<table>
<thead>
<tr>
<th>Compound</th>
<th>Magnetism</th>
<th>$\mu_{exp}/\mu_B$</th>
<th>$T_N$</th>
<th>$T_C/K$</th>
<th>$\Theta/K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NdRhIn$_5$</td>
<td>AF</td>
<td>3.66(3)</td>
<td>11</td>
<td>17</td>
<td>-</td>
</tr>
<tr>
<td>Nd$_2$RhIn$_8$</td>
<td>AF</td>
<td>3.57(3)</td>
<td>10.7</td>
<td>14</td>
<td>-</td>
</tr>
<tr>
<td>NdIrIn$_5$</td>
<td>AF</td>
<td>3.58(3)</td>
<td>13.75</td>
<td>18</td>
<td>-</td>
</tr>
<tr>
<td>Nd$_2$IrIn$_8$</td>
<td>AF</td>
<td>3.60(3)</td>
<td>12.30</td>
<td>13</td>
<td>-</td>
</tr>
<tr>
<td>SmRhIn$_5$</td>
<td>AF</td>
<td>-</td>
<td>15</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Sm$_2$RhIn$_8$</td>
<td>AF</td>
<td>-</td>
<td>15</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>SmIrIn$_5$</td>
<td>AF</td>
<td>-</td>
<td>14.3</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Sm$_2$IrIn$_8$</td>
<td>AF</td>
<td>-</td>
<td>14.2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>GdRhIn$_5$</td>
<td>AF</td>
<td>8.0(1)</td>
<td>40</td>
<td>-69</td>
<td>-</td>
</tr>
<tr>
<td>Gd$_2$RhIn$_8$</td>
<td>AF</td>
<td>7.4(1)</td>
<td>40</td>
<td>-73</td>
<td>-</td>
</tr>
<tr>
<td>GdIrIn$_5$</td>
<td>AF</td>
<td>7.9(1)</td>
<td>42</td>
<td>-64</td>
<td>-</td>
</tr>
<tr>
<td>Gd$_2$IrIn$_8$</td>
<td>AF</td>
<td>8.2(1)</td>
<td>41</td>
<td>-75</td>
<td>-</td>
</tr>
</tbody>
</table>
with structures derived from the cubic Laves phase MgCu$_2$. A short overview on many experimental results and a comparison with other Yb$X$Cu$_4$ ($X =$ semi-metal or transition metal) intermetallics was given recently by Sarrao (1999).

The valence transitions in these indides are accompanied by jumps in the unit cell size. Measurements of $^{119}$Sn and $^{170}$Yb Mössbauer spectra, neutron diffraction, magnetic susceptibility data, and X-ray absorption measurements at the ytterbium L$_{III}$ edge were used for the study of the valence transition. The transition temperature changes with increasing applied pressure (De Teresa et al., 1996; Hedo et al., 2003). From the volume anomaly at the phase transition, the valence change of ytterbium is found to decrease at the rate $\Delta_v \approx 0.0045$ kbar$^{-1}$. It has been speculated that the first-order transition in YbCu$_4$In is a Mott transition between a high-temperature phase, in which localized moments are stabilized by the entropy term of the free energy, and a band-like diamagnetic ground state of the $f$ electrons (Dzero et al., 2000).

The magnetoresistance of YbCu$_4$In has been investigated in magnetic fields as high as 30 T (Immer et al., 1997). Applied pressure shifts the valence transition to lower temperatures. The studies revealed that a single energy scale is associated with the valence transition. Filamentary superconductivity was observed above 0.74 GPa and below $\sim 1.2$ K (Hedo et al., 2003).

Analyses of the $^{63,65}$Cu NQR resonance frequency and the spin-lattice relaxation time microscopically prove the transition from the local moment state with stable trivalent ytterbium to the Fermi liquid state at the valence transition temperature of 40–50 K (Nakamura et al., 1990).

A time-of-flight neutron scattering study of YbCu$_4$In was performed to understand the diamagnetic scattering (Lawrence et al., 1999). The key result of the investigation is that the spin dynamics in the mixed valent phase are well described over a broad range of energy transfer ($10 < \Delta E < 150$ meV) and incident energies ($35 < E_i < 150$ meV) by a Lorentzian power spectrum, centered at $E_0 = 40$ meV. The scattering for YbIn$_{0.3}$Ag$_{0.7}$Cu$_4$ is inelastic at low temperatures. The crystal field parameters were derived from inelastic neutron scattering data (Severing et al., 1990). The crystal field schemes of YbCu$_4$In and YbNi$_4$In are compared.

Neutron powder diffraction data of YbCu$_4$In revealed that the first order phase transition at 40 K is indeed isomorphic (Lawrence et al., 1996). No change in the crystal symmetry could be detected. An important point concerns the site disorder in the YbCu$_4$In structure. The diffraction line widths decrease systematically on going from polycrystalline samples with two transitions (40 and 70 K) to polycrystals with a single transition at 40 K to flux-grown single crystals with a sharp transition at 40 K. This is an important result, which clearly shows the influence of the sample type on the long-range ordering. The latter certainly influences the physical properties. An X-ray diffraction study of YbCu$_4$In up to 13 GPa gave no indication for a structural change (Oomi et al., 1994).

The series Yb$_x$In$_{1-x}$Cu$_2$ has intensively been studied in detail with respect to the ytterbium valence transition (Oesterreicher and Parker, 1977; Felner and Nowik, 1986; Felner et al., 1987; Sampathkumaran et al., 1987; Nowik et al., 1988; Kojima et al., 1989). For the solid solution $0.30 \leq x \leq 0.65$, a sharp Yb$^{2+} \rightarrow$ Yb$^{3+}$ valence transition occurs at temperatures between 40 and 80 K. For comparison, the authors have also measured the magnetic susceptibility of Gd$_{0.4}$In$_{0.6}$Cu$_2$, Lu$_{0.4}$In$_{0.6}$Cu$_2$, and Yb$_{0.5}$In$_{0.5}$Ni$_2$. The gadolinium and ytterbium
compounds show paramagnetism down to 2 and 4.2 K, respectively, with stable trivalent rare earth atoms. High-field magnetization data on Yb$_x$In$_{1-x}$Cu$_2$ reveal metamagnetic transitions at 30 T (Shimizu et al., 1988; Yoshimura et al., 1988).

Ultraviolet and X-ray photoemission spectra of the solid solution Yb$_x$In$_{1-x}$Cu$_2$ at 293, 80, and 10 K show a valence transition of ytterbium between 293 and 80 K, much higher than the bulk transition temperature of about 40 K estimated from the magnetic susceptibility data (Ogawa et al., 1988). After subtraction of the Yb(II) surface contribution, the valence was estimated to be 2.63 at 293 K and 2.38 at 80 and 10 K, still much smaller than the bulk data. These measurements suggest that a surface region at least as thick as the XPS probing depth is strongly modified versus the bulk. A high-resolution photoemission study of YbCu$_4$In showed that the ytterbium 4$f$ peak is located just below the Fermi level (Susaki et al., 2001). Temperature dependent measurements show a shift of the photoemission peak from $-35$ meV at 75 K to $-40$ meV at 7 K. The bare 4$f$ level, which is close to the Fermi energy, is shifted downwards and the hybridization strength decreases from the high-temperature to the low-temperature phase. This picture of chemical bonding is supported by electronic band structure calculations for YbCu$_4$In and LuCu$_4$In (Takegahara and Kasuya, 1990).

Yb$_{0.4}$In$_{0.6}$Cu$_2$ exhibits a sharp valence transition at 50 K (Felner et al., 1987). Neutron diffraction data prove the absence of magnetic order down to 10 K. Specific heat data show a characteristic increasing of $C_P$ at the transition temperature. Ytterbium L$_{III}$ X-ray absorption spectra show a sudden change in the 4$f$ electron occupancy. Based on these data, the ytterbium valence changes from 2.9 to 2.8 below the transition temperature. Furthermore, the authors investigated the influence of ytterbium/indium substitution by lanthanum, europium, and tin. The pressure effect on the valence transition of Yb$_{0.4}$In$_{0.6}$Cu$_2$ (electrical resistivity data up to 10 kbar) was measured by Kojima et al. (1988). The valence transition temperature decreases at a rate of $-1.7$ K/kbar.

Several other indides with the same structure, such as YbIn$_{1-x}$Ag$_x$Cu$_4$, have been investigated. Alloying YbInCu$_4$ with silver increases the temperature of the first-order isomorphic phase transition (Cornelius et al., 1997; Yoshimura et al., 1990). Within the solid solution YbIn$_{1-x}$Ag$_x$Cu$_4$, the valence phase transition temperature decreases with increasing pressure at a rate of $-1.7$ K/kbar for $x = 0$, $-1.8$ K/kbar for $x = 0.1$, and $-1.9$ K/kbar for $x = 0.2$ (Matsumoto et al., 1992). The crossover from a first order valence transition in YbInCu$_4$ to the dense Kondo lattice YbAgCu$_4$ was also monitored via the concentration dependence of the electronic specific heat coefficient, by electrical resistivity, magnetic susceptibility, and X-ray powder diffraction measurements (Pillmayr et al., 1992; Sarrao et al., 1996). $^{63}$Cu NQR studies on YbInCu$_4$, YbIn$_{0.9}$Ag$_{0.1}$Cu$_4$, and YbIn$_{0.9}$Au$_{0.1}$Cu$_4$ show a correlation between the increase of the valence transition temperature and a decrease of the NQR frequency (Kojima et al., 1992). The decrease of the frequency comes mainly from the electronic contribution, suggesting a change in the hybridization of the ytterbium 4$f$ and the conduction electrons. The nickel-based indide YbNi$_4$In is an intermediate-valent compound as is evident from electrical resistivity, thermopower, and ytterbium L$_{III}$ data (Koterlyn et al., 1991).
External pressure of \( p \geq 0.8 \) GPa suppresses the valence transition in \( \text{Yb}_{0.8}\text{Y}_{0.2}\text{InCu}_4 \) and induces weak ferromagnetism at the low Curie temperature of 1.7 K (Mitsuda et al., 2002). Although the ytterbium atoms are in the trivalent state, magnetization measurements reveal an extremely small ordered moment of 0.05 \( \mu_B/\text{Yb} \) atom, much smaller than the theoretical value of 4 \( \mu_B \) for \( \text{Yb}^{3+} \). The whole system \( \text{Yb}_{1-x}\text{Y}_x\text{InCu}_4 \) (\( 0 \leq x \leq 1 \)) was studied with the copper nuclear quadrupole resonance technique, in order to investigate the valence transition and 4\( f \) electron spin dynamics in the diluted system (Nakamura and Shiga, 1995). The valence transition temperature decreases with increasing \( x \) and the NQR frequency at low temperatures is sensitively affected. Analyses of the nuclear spin-lattice relaxation time indicate that the stability of the low-temperature Fermi liquid state is related to the intersite coherence between the ytterbium 4\( f \) electrons. Magnetization and electrical resistivity measurements on \( \text{YC}_{u4}\text{In} \) and \( \text{LuCu}_4\text{In} \) with the diamagnetic rare earth metals indicate semimetal behavior (Nakamura et al., 1993b, 1994b).

Alloying of \( \text{YbCu}_4\text{In} \) in the series \( \text{Yb}_{1-x}\text{R}_x\text{InCu}_4 \) (\( \text{R} = \text{Y, La, Ce, Lu} \)) has been studied for various \( x \) values (Mushnikov et al., 2002). Correlating with the size of the rare earth elements, the lattice parameters increase with \( x \) for \( \text{R} = \text{Y, La, and Ce} \), while they decrease for lutetium. The influence of alloying on the valence transition temperature is discussed. Zhang et al. (2002) investigated the ytterbium substitution by yttrium, lutetium, and zirconium.

\( \text{GdCu}_4\text{In} \) is a Curie–Weiss paramagnet with an experimental magnetic moment of 8.15 \( \mu_B/\text{Gd} \) atom and a paramagnetic Curie temperature of \(-45 \) K (Nakamura et al., 1993a). Although the large negative \( \Theta \) indicates strong antiferromagnetic correlations, the Néel temperature is depressed down to 6.9 K. From the low-temperature specific heat data a typical lambda-type transition is evident at 6.9 K, however, the magnetic entropy reaches only a value of about 60%. Thus, short-range magnetic correlations persist at temperatures higher than \( T_N \). These data can be explained in terms of magnetic frustration. The magnetization vs external field dependence increases monotonically and approaches a moment of 6.5 \( \mu_B/\text{Gd} \) atom at 4.2 K and 3.5 T, close to the saturation magnetization of 7 \( \mu_B \) for \( \text{Gd}^{3+} \). The antiferromagnetic short-range correlations above the Néel temperature have also been studied in detail for the solid solution series \( \text{Gd}_{1-x}\text{Lu}_x\text{Cu}_4\text{In} \), where part of the magnetic gadolinium atoms is substituted by diamagnetic lutetium (Nakamura et al., 1994a).

The nickel-based compound \( \text{PrNi}_4\text{In} \) has a diamagnetic ground state (Suzuki et al., 2003). A small upward curvature of the inverse magnetic susceptibility vs temperature dependence indicates a relatively large crystal electric field splitting. Magnetization data at 2 K in magnetic fields up to 30 T show a field-induced transition and a tendency to saturation above 20 T at a value of \( \sim 2.4 \) \( \mu_B \), smaller than the expected moment of 3.2 \( \mu_B \) for \( \text{Pr}^{3+} \).

Most of the platinum based compounds \( \text{RPt}_4\text{In} \) remain paramagnetic down to 4.2 K (Malik et al., 1990b). The magnetic moments are close to the free ion values of the respective \( \text{R}^{3+} \) cations (table 19). Ferromagnetic ordering is detected for the europium and gadolinium compounds. In the magnetically ordered state, \( \text{EuPt}_4\text{In} \) and \( \text{GdPt}_4\text{In} \) exhibit full parallel spin alignment. Deviations from the trivalent oxidation states are evident for \( \text{CePt}_4\text{In} \) and \( \text{EuPt}_4\text{In} \). The latter has a stable 4\( f \)\(^7 \) configuration for the europium atoms, while the strongly negative paramagnetic Curie temperature of \( \text{CePt}_4\text{In} \) is indicative for intermediate valence or Kondo type interactions.
### Table 19
Magnetic $R$Pt$_4$In indides with MgCu$_4$Sn structure (Malik et al., 1990b)

<table>
<thead>
<tr>
<th>Compound</th>
<th>Magnetism</th>
<th>$\mu_{\text{exp}}/\mu_B$</th>
<th>$T_N$, $T_C$/K</th>
<th>$\Theta$/K</th>
<th>$\mu_{\text{sm}}/\mu_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LaPt$_4$In</td>
<td>P</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>CePt$_4$In</td>
<td>P</td>
<td>2.54</td>
<td>–</td>
<td>–255</td>
<td>–</td>
</tr>
<tr>
<td>PrPt$_4$In</td>
<td>P</td>
<td>3.63</td>
<td>–</td>
<td>–6</td>
<td>–</td>
</tr>
<tr>
<td>NdPt$_4$In</td>
<td>P</td>
<td>3.78</td>
<td>–</td>
<td>–8</td>
<td>–</td>
</tr>
<tr>
<td>SmPt$_4$In</td>
<td>NCW</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>EuPt$_4$In</td>
<td>P</td>
<td>7.56</td>
<td>53</td>
<td>66</td>
<td>7.3</td>
</tr>
<tr>
<td>GdPt$_4$In</td>
<td>P</td>
<td>8.03</td>
<td>20</td>
<td>16</td>
<td>6.9</td>
</tr>
<tr>
<td>TbPt$_4$In</td>
<td>P</td>
<td>10.13</td>
<td>–</td>
<td>5</td>
<td>–</td>
</tr>
<tr>
<td>DyPt$_4$In</td>
<td>P</td>
<td>10.83</td>
<td>–</td>
<td>4</td>
<td>–</td>
</tr>
<tr>
<td>HoPt$_4$In</td>
<td>P</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>ErPt$_4$In</td>
<td>P</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>TmPt$_4$In</td>
<td>P</td>
<td>7.85</td>
<td>–</td>
<td>0</td>
<td>–</td>
</tr>
</tbody>
</table>

### 7.5. Ternary indides with CsCl superstructures or Heusler phases

The binary intermetallics GdAg and GdZn form solid solutions GdAg$_{1-x}$In$_x$ and GdZn$_{1-x}$In$_x$ (Buschow et al., 1972; Lal, 1982; Larica and Guimarães, 1976; Oppelt et al., 1972; Alfieri et al., 1966). These compounds show ferromagnetic ordering above 100 K. The Curie temperatures within the solid solutions vary with the valence electron concentration. With the diamagnetic rare earth elements yttrium and lanthanum, the Pauli susceptibility also varies with the number of conduction electrons. A cubic-to-tetragonal phase transition is observed within the solid solution LaAg$_x$In$_{1-x}$ (Ihrig et al., 1973). The LaAg$_x$In$_{1-x}$ alloys have intensively been investigated by X-ray powder diffraction and magnetic susceptibility measurements. According to the experimental data, the phase transformation arises from a band Jahn–Teller effect. The magnetic data of the whole series of $R$Ag$_{0.5}$In$_{0.5}$ ($R$ = Gd, Tb, Dy, Ho, Er, Tm, Yb) compounds are listed in table 20 (Lal and Methfessel, 1981).

The series of terbium compounds TbAg$_{1-x}$In$_x$ has been investigated in detail with respect of the influence of the valence electron concentration on the magnetic structure (Cable et al., 1965; Yagasaki et al., 1978a). The Néel temperatures for TbAg$_{0.8}$In$_{0.2}$, TbAg$_{0.6}$In$_{0.4}$, TbAg$_{0.5}$In$_{0.5}$, and TbAg$_{0.4}$In$_{0.6}$ are 90, 57, 64, and 77 K, respectively. The magnetic structures were determined from neutron diffraction data. Also in the pseudo-binary system GdAg$_{1-x}$In$_x$, the type of magnetic ordering depends on the valence electron concentration (Sekizawa and Yasukōchi, 1966). With $x = 0.1$, 0.8, and 0.9 antiferromagnetic ordering occurs, while samples with $x = 0.3$, 0.5, and 0.7 order ferromagnetically. The basic magnetic data are summarized in table 20. Under hydrostatic pressure conditions, the Néel temperatures do not change with pressure within the experimental error, while the Curie temperatures shifting to higher temperatures with increasing pressure (Yagasaki et al., 1978b).

The structural and magnetic properties of the whole series of Heusler type compounds $RAu_2$In ($R$ = Y, La–Lu) has been investigated with great care by Besnus et al. (1985, 1986). The indides with lanthanum, cerium, praseodymium, and neodymium show a crystallographic transition from cubic to tetragonal as is evident from X-ray powder data. The transition temperature decreases from 355 K for LaAu$_2$In to 70 K for the neodymium compound. This
gamma and ytterbium compound show non-Curie–Weiss behavior. Specific heat data revealed a high change in the slope), but no observable effect on the magnetic data (table 20). The samarium structural transition has a drastic effect on the electrical behavior of these indides (abrupt change in the slope), but no observable effect on the magnetic data (table 20). The samarium and ytterbium compound show non-Curie–Weiss behavior. Specific heat data revealed a high γ coefficient of 130 mJ/mol K² for the cerium compound (Pleger et al., 1987).

The electrical resistivity of the intermediate-valent ytterbium compound YbAu₂In has been investigated under high-pressure conditions (Alami-Yadri et al., 1998) up to 8 GPa. The Kondo temperature decreases with increasing pressure, while the residual electrical resistiv-
ity increases in the same range. A minimum in $\rho(T)$ is observed at low temperatures. The pressure dependence can be ascribed to a Kondo hole effect.

The crystal field (CF) level scheme of YbPd$_2$In and HoPd$_2$In has been determined by inelastic neutron scattering experiments (Babateen et al., 1995): $W = 0.02673; X = 0.3543$ for HoPd$_2$In and $W = -0.5194; X = -0.7485$ for YbPd$_2$In. For the magnetization curves, satisfactory agreement is observed for the calculated and measured data.

The solid solution LaAg$_x$In$_{1-x}$ crystallizes with a CsCl type structure (Balster et al., 1975). This pseudobinary system undergoes a martensitic (displacive) crystal structure transition. At low temperature the X-ray powder patterns of polycrystalline samples show line splitting corresponding to a cubic-to-tetragonal transformation. The indium concentration $1 - x$ and thus the electron count per formula unit has a large influence on the transition temperature. This structural phase transition is revealed also in the temperature dependence of the electrical resistivity. For indium concentrations above 5% the curves show a pronounced hysteresis behavior.

The magnetic properties in the whole series of $R$Ag$_2$In indides, except europium and ytterbium, have been investigated by Galera et al. (1984). The magnetic data are summarized in table 20. SmAg$_2$In shows Van Vleck behavior, while the other compounds behave as Curie–Weiss paramagnets, ordering antiferromagnetically at low temperatures. Most structural investigations were done by X-ray powder diffraction. Since silver and indium differ only by two electrons, the ordering between these atoms is hardly detectable by X-rays. The authors additionally performed neutron diffraction experiments for the cerium and neodymium compounds and found indications for at least partially ordered Ag/In sites. The $R$Ag$_2$In indides are metallic conductors. Compounds with a rare earth element that orders magnetically show a steeper decrease of the electrical resistivity below the ordering temperature. The precise crystal field parameters for the cerium, praseodymium, and neodymium compounds have been determined from inelastic neutron scattering experiments.

The magnetic hyperfine interactions of the gadolinium magnetic moments in GdAg$_2$In have been investigated by magnetic susceptibility data (table 20) and $^{155}$Gd Mössbauer spectroscopy (de Vries et al., 1985). The hyperfine parameters at 4.2 K are $\delta = 0.52(3)$ mm/s, $\Gamma = 0.66(5)$ mm/s and $B_{hf} = 24.8(8)$ T.

CeCu$_2$In shows interesting pressure dependent physical properties (Kagayama et al., 1994). A crossover from the heavy fermion state to an itinerant 4$f$ state is induced by applying a pressure of 2 GPa. This change is associated with a large enhancement of the Kondo temperature. The data of the cerium compound are compared to those of LaCu$_2$In with the diamagnetic lanthanum atoms and binary CeCu$_6$. The Heusler phases CeCu$_2$In and CeAg$_2$In form a solid solution CeAg$_{2-x}$Cu$_x$In (Lahiouel et al., 1987). In this series, the hybridization continuously increases from the antiferromagnetic Kondo lattice CeAg$_2$In to the heavy fermion compound CeCu$_2$In. The electrical resistivities of diluted solutions La$_{1-x}$Ce$_x$InCu$_2$ and Y$_{1-x}$Ce$_x$InCu$_2$ can be decomposed into a single ion term plus a pair interaction term, which follows the Curie–Weiss law (Najib et al., 1988). Magnetization and specific heat measurements indicate antiferromagnetic correlations. Various single crystals of CeCu$_2$In have been grown (Onuki et al., 1987). Crystals with a slightly higher cerium content, Ce$_{1.06}$Cu$_2$In, show slightly varying magnetic and electrical properties. Thermal expansion measurements of single crystalline
CeCu₂In show isotropic behavior in the [100] and [110] directions as expected for a cubic material (Oomi et al., 1990). The thermal expansion coefficient of CeCu₂In below 100 K, however, is larger compared with that of LaCu₂In, which has no 4f electrons.

The lattice parameters of LaCu₂In and CeCu₂In have been measured under pressures up to 14 GPa by X-ray powder diffraction (Kagayama et al., 1990). Fine powders were prepared from Czochralski grown single crystals in an alcohol atmosphere in order to avoid oxidation of the material. In contrast to the differences reported for single crystals by Oomi et al. (1990), almost similar values for the bulk modulus and the expansion coefficient are reported for the LaCu₂In and CeCu₂In powder data.

### 7.6. Other indides

Most indides with a diamagnetic rare earth element, i.e. scandium, yttrium, lanthanum and lutetium, are Pauli paramagnetic. An example is the metallic conductor Sc₂Ni₂In (Pöttgen and Drönskowski, 1996).

Susceptibility measurements revealed intermediate-valent behavior for CeNi₉In₂ (Moze et al., 1995). The magnetic susceptibility is nearly temperature independent over the whole temperature range indicating that the cerium atoms do not carry a localized moment. This is also evident from the magnetization vs field dependence at 1.6 K, where only a moment of 0.06 µB per formula unit is detected at 12 T. The electrical resistivity data show metallic behavior and the absence of magnetic ordering at low temperatures. Isotypic GdNi₉In₂ (µexp = 8.35 µB and Θ = −3.5 K) orders presumably ferromagnetically below 2 K (Mulder et al., 1998). The values of the magnetization at 2 K are slightly higher than the corresponding value of the Brillouin function. The ¹⁵⁵Gd spectrum at 4.2 K shows a well resolved doublet at an isomer shift of 0.35 mm/s and a Vzz component of 6.9 · 10²¹ V/m², reflecting the non-cubic site symmetry of gadolinium.

The properties of the solid solution Ce_{1−x}La_xCu₅In have been compared to the pure cerium compound which exhibits Kondo lattice properties with a specific heat coefficient γ = 200 mJ/mol K² (Tchoula Tchokonte et al., 2003). Susceptibility measurements gave no hint for magnetic ordering in the Ce_{1−x}La_xCu₅In indides. Resistivity data enabled the extraction of the Kondo temperature for the different compositions.

The palladium-rich hexagonal indide CePd₂In (Bianchi et al., 1995) orders antiferromagnetically at TN = 1.23 K. A further spin rearrangement has been detected around 0.6 K. The temperature dependence of the magnetic susceptibility does not show significant crystal field effects. LaPd₂In with the diamagnetic lanthanum atoms was investigated for comparison. The electronic specific heat of the cerium compound is enhanced by a factor of five with respect to the lanthanum compound. LaPd₂In behaves like a classical metal while CePd₂In shows a Kondo minimum and a steep drop in electrical resistivity at the Néel temperature. The indium-rich compound CePdIn₂ with MgCuAl₂ structure orders ferromagnetically at 10 K (Ijiri et al., 1996). The saturation magnetization at 4.2 K is 1.6 µB/Ce atom, slightly reduced as compared to the expected gJ value of 2.14 µB for Ce³⁺ in a J = 5/2 state.

The heavy fermion antiferromagnet Ce₃Pt₄In₁₃ (TN = 0.95 K) exhibits unusual Kondo behavior (Hundley et al., 2001) and it has an unusually large electronic contribution to the specific heat of about 1 J/mol K² at the Néel temperature. Thermoelectric power and Hall effect
measurements suggest that the RKKY and Kondo type interactions in Ce$_3$Pt$_4$In$_{13}$ are closely balanced. The isotypic compound La$_3$Pt$_4$In$_{13}$ with diamagnetic lanthanum is a conventional superconductor with $T_C = 3.3$ K.

The rare earth metal atoms in EuPdIn$_2$ and YbPdIn$_2$ are divalent (Galadzhun et al., 1999a). YbPdIn$_2$ shows a diamagnetic signal over the whole temperature range. The europium compound shows Curie–Weiss behavior (table 21) and orders ferromagnetically at 14.5(5) K. EuPdIn$_2$ shows negligible coercivity and remanent magnetization, classifying this indide as a soft ferromagnet. Both compounds are metallic conductors.

Eu$_2$Au$_3$In$_4$ with anti-Hf$_2$Co$_4$P$_3$ structure (Hoffmann et al., 1999) orders antiferromagnetically (table 21). Magnetization measurements at 2 K indicate a metamagnetic phase transition at a critical field of 0.5(2) T and a saturation magnetization of 5.95 $\mu_B$/Eu atom at 5.5 T. Eu$_2$Au$_3$In$_4$ is a metallic conductor. The divalent character of europium in this indide was confirmed by $^{151}$Eu Mössbauer data that show only one signal at an isomer shift of $-10.5(5)$ K. Hyperfine field splitting is observed below 10 K, however, due to the various europium sites in the structure, a fit of the spectra was not possible.

Gd$_6$Co$_{2.2}$In$_{0.8}$ with orthorhombic Ho$_6$Co$_2$Ga structure shows complex magnetic behavior (Canepa et al., 2002a). Specific heat and magnetic susceptibility data reveal two magnetic transitions. At 34 K, the antiferromagnetic coupling changes to ferromagnetic and at 66 K a transition to a paramagnetic state occurs. The low magnetic moment of 6.8 $\mu_B$ in the paramagnetic state can be explained within the RKKY theory by a strong coupling of the conduction

<table>
<thead>
<tr>
<th>Compound</th>
<th>Magnetism</th>
<th>$\mu_{\text{exp}}/\mu_B$</th>
<th>$T_N, T_C/K$</th>
<th>$\Theta/K$</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ce$_2$CuIn$_3$</td>
<td>P</td>
<td>2.49</td>
<td>–</td>
<td>–8</td>
<td>Siouris et al., 2000b</td>
</tr>
<tr>
<td>Pr$_2$CuIn$_3$</td>
<td>P</td>
<td>3.48</td>
<td>–</td>
<td>–3</td>
<td>Siouris et al., 2000b</td>
</tr>
<tr>
<td>Nd$_2$CuIn$_3$</td>
<td>AF</td>
<td>3.75</td>
<td>12</td>
<td>–12</td>
<td>Siouris et al., 2000b</td>
</tr>
<tr>
<td>Tb$_2$CuIn$_3$</td>
<td>AF</td>
<td>10.3</td>
<td>30</td>
<td>–72</td>
<td>Siouris et al., 2000b</td>
</tr>
<tr>
<td>Tb$_2$CuIn$_3$</td>
<td>AF</td>
<td>10.3</td>
<td>33(1)</td>
<td>–72</td>
<td>Siouris et al., 2001</td>
</tr>
<tr>
<td>Dy$_2$CuIn$_3$</td>
<td>AF</td>
<td>10.5</td>
<td>22</td>
<td>–30</td>
<td>Siouris et al., 2000b</td>
</tr>
<tr>
<td>Ho$_2$CuIn$_3$</td>
<td>AF</td>
<td>10.47</td>
<td>8</td>
<td>2</td>
<td>Siouris et al., 2000b</td>
</tr>
<tr>
<td>Tm$_2$CuIn$_3$</td>
<td>AF</td>
<td>10.55</td>
<td>9</td>
<td>23</td>
<td>Siouris et al., 2000b</td>
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<tr>
<td>CeNi$_2$In$_4$</td>
<td>P</td>
<td>2.6(1)</td>
<td>–</td>
<td>4(1)</td>
<td>Pöttgen, 1995</td>
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<tr>
<td>Ce$_5$Ni$<em>5$In$</em>{11}$</td>
<td>AF</td>
<td>2.57(5)</td>
<td>0.63; 1.10</td>
<td>–8</td>
<td>Tang et al., 1995</td>
</tr>
<tr>
<td>CePd$_2$In</td>
<td>AF</td>
<td>2.69</td>
<td>1.23</td>
<td>–3</td>
<td>Bianchi et al., 1995</td>
</tr>
<tr>
<td>CePdIn$_2$</td>
<td>F</td>
<td>2.47(2)</td>
<td>10</td>
<td>16.5(15)</td>
<td>Ijiri et al., 1996</td>
</tr>
<tr>
<td>Ce$_3$Pt$<em>4$In$</em>{13}$</td>
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<td>2.64</td>
<td>0.95</td>
<td>–36</td>
<td>Hundley et al., 2001</td>
</tr>
<tr>
<td>Ce$_2$Au$_3$In$_5$</td>
<td>P</td>
<td>2.51(2)</td>
<td>–</td>
<td>–16(1)</td>
<td>Galadzhun et al., 1999b</td>
</tr>
<tr>
<td>Nd$_5$Ni$<em>6$In$</em>{11}$</td>
<td>AF</td>
<td>3.60(5)</td>
<td>11.4(1)</td>
<td>–18(1)</td>
<td>Pöttgen et al., 1999a</td>
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<tr>
<td>EuNi$_4$</td>
<td>AF</td>
<td>7.86(5)</td>
<td>16(1)</td>
<td>–8(1)</td>
<td>Pöttgen et al., 1996</td>
</tr>
<tr>
<td>EuPdIn$_2$</td>
<td>F</td>
<td>7.8(1)</td>
<td>14.5(5)</td>
<td>22(1)</td>
<td>Galadzhun et al., 1999a</td>
</tr>
<tr>
<td>Eu$_2$Au$_3$In$_4$</td>
<td>AF</td>
<td>7.88(5)</td>
<td>9.5(5)</td>
<td>4.0(5)</td>
<td>Hoffmann et al., 1999</td>
</tr>
<tr>
<td>Gd$<em>6$Co$</em>{2.2}$In$_{0.8}$</td>
<td>AF / F</td>
<td>6.82</td>
<td>34, 66</td>
<td>80.3</td>
<td>Canepa et al., 2002a</td>
</tr>
<tr>
<td>Gd$_{14}$Co$<em>3$In$</em>{2.7}$</td>
<td>AF</td>
<td>7.34</td>
<td>37</td>
<td>50</td>
<td>Canepa et al., 2002b</td>
</tr>
<tr>
<td>GdAu$<em>{0.44}$In$</em>{1.56}$</td>
<td>AF</td>
<td>7.8(1)</td>
<td>21.0(5)</td>
<td>–30(1)</td>
<td>Pöttgen et al., 1998</td>
</tr>
</tbody>
</table>
electrons with the localized magnetic moments. In the ferromagnetically ordered state, the magnetic moment of $4.81 \mu_B$ is far from the saturation value for the Gd$^{3+}$ ion of $7 \mu_B$.

The complex metal-rich indide Gd$_{14}$Co$_3$In$_2.7$ orders antiferromagnetically at $T_N = 37$ K (Canepa et al., 2002b) and undergoes a metamagnetic phase transition at 1.4 T. The $2.65 \mu_B$ per gadolinium atom at the highest obtainable field of 9 T is even smaller than that of Gd$_6$Co$_{2.2}$In$_{0.8}$.

CeNiIn$_4$ shows Curie-Weiss behavior over the whole investigated temperature range (Pöttgen, 1995). No magnetic ordering could be detected down to 2 K. CeNiIn$_4$ is a good metallic conductor. In the solid solution CeNiAl$_{4-x}$In$_x$, the thermopower is significantly higher than in the parent compound (Poduska et al., 2000). Also, EuNiIn$_4$ is a Curie-Weiss paramagnet (table 21) with a Néel temperature of 16 K (Pöttgen et al., 1996). Magnetic ordering is already evident at 32 K in the $^{151}$Eu Mössbauer spectra. In the region between 16 and 32 K the $^{151}$Eu spectra can be fit as a superposition of a magnetically split and an unsplit component, while full magnetic hyperfine field splitting is detected below $T_N$ with a hyperfine field of 26.4 T at 4.2 K at the europium nuclei.

GdAu$_{0.44}$In$_{1.56}$ with CaIn$_2$ structure orders antiferromagnetically 21.0(5) K (Pöttgen et al., 1998). The antiferromagnetic ground state is stable in high magnetic fields. The magnetization at 5 K and 5.5 T is 2.6(1) $\mu_B$/Gd atom. The $^{155}$Gd Mössbauer spectrum shows a single signal at an isomer shift of 0.452 mm/s subject to quadrupole splitting of $-0.842$ mm/s. At 4.2 K a hyperfine field of 19.8 T is detected at the gadolinium nuclei.

Ce$_5$Ni$_6$In$_{11}$ is an intermediate heavy fermion material with a high electronic specific heat coefficient of $\gamma = 145 \pm 20$ mJ/mol K$^2$ (Tang et al., 1995). The heat capacity measurements reveal two magnetic transitions, both being of antiferromagnetic nature. Most likely the cerium atoms on the 2$c$ Wyckoff position order at $T_{N1}$ and those on the 8$p$ position at $T_{N2}$. There is also evidence for weak magnetic coupling between the two cerium sites. Nd$_5$Ni$_6$In$_{11}$ is a 11.4 K antiferromagnet (Pöttgen et al., 1999a). Magnetization and specific heat measurements indicate a complex magnetic phase diagram with spin reorientations into three antiferro- and one ferromagnetic phase. Nd$_5$Ni$_6$In$_{11}$ is a metallic conductor with a specific electrical resistivity of 120 $\mu\Omega$ cm at room temperature.

The magnetic ground states of the silver-based indides $R_2$AgIn$_3$ ($R =$ Pr, Nd, Tb, Ho, Er) have been investigated (Siouris et al., 2000a). The silver and indium atoms in these indides are randomly distributed on the indium network of the CaIn$_2$ structure. Consequently, due to the structural disorder, many of these compounds show spin-glass behavior. Except for the terbium compound, neutron diffraction data revealed the absence of long-range magnetic ordering. The antiferromagnetic structure ($T_N = 42$ K) of Tb$_2$AgIn$_3$ was determined from neutron powder diffraction data. The magnetic unit cell corresponds to the orthohexagonal setting of the hexagonal nuclear structure. The magnetic moment of the terbium atoms is 5.9 $\mu_B$ at 4.2 K (Semitelou et al., 1999).

The rare earth atoms are in a stable trivalent state in all $R_2$CuIn$_3$ compounds (Siouris et al., 2000b). The basic magnetic data are listed in table 21. No magnetic ordering has been observed for the cerium and praseodymium compounds, while antiferromagnetism is detected for the remaining indides. The magnetic structure of the collinear antiferromagnet Tb$_2$CuIn$_3$ was refined from neutron powder diffraction data (Siouris et al., 2001), with a magnetic unit
cell similar to Tb$_2$AgIn$_3$. The terbium magnetic moments (6.8 $\mu_B$ at 3.8 K) are oriented along the $c$ axis. According to the small almost linear increase of the magnetization isotherm at 5 K, the antiferromagnetic structure is stable in high external fields.

YbAg$_2$In$_4$ follows the Curie–Weiss law above 90 K with an experimental magnetic moment of 2.3 $\mu_B$/Yb atom, much smaller than the free ion value of 4.45 $\mu_B$ for Yb$^{3+}$ (Sysa et al., 1998). The low magnetic moment and the large negative paramagnetic Curie temperature ($\Theta = -86$ K) are both indicative for intermediate-valent ytterbium. When ytterbium is substituted by calcium, the compound becomes Pauli paramagnetic with a low magnetic susceptibility value of $0.8 \cdot 10^{-6}$ cm$^3$/g. Both indides are metallic conductors.

X-ray absorption L$_{III}$ spectra indicated stable divalent europium in EuAg$_4$In$_8$, while signals for Eu$^{II}$ and Eu$^{III}$ were observed for EuCuIn$_4$ and EuCu$_{6.5}$In$_{6.5}$ (Sysa et al., 1994a). A variety of indides $R$Ag$_6$In$_6$ with site occupancy variants of the tetragonal ThMn$_{12}$ type have been analyzed with respect to their magnetic and electrical properties (Zaremba et al., 1999). CeAg$_{6.4}$In$_{5.6}$ and PrAg$_6$In$_6$ are paramagnets with experimental magnetic moments of 2.32 $\mu_B$/Ce ($\Theta = 3.84$ K) and 3.41 $\mu_B$/Pr ($\Theta = -2.83$ K), close to the free ion values of Ce$^{3+}$ and Pr$^{3+}$. NdAg$_6$In$_6$ (3.43 $\mu_B$/Nd and $\Theta = -2.15$ K) orders antiferromagnetically at 5 K. The magnetization curves increase almost linearly up to 5 T. These silver compounds are metallic conductors.

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References


Ferro, R., Marazza, R., Rambaldi, G., 1974a. Z. Metallkd. 65, 37.


Chapter 219

UNCONVENTIONAL SUPERCONDUCTIVITY AND MAGNETISM IN LANTHANIDE AND ACTINIDE INTERMETALLIC COMPOUNDS

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7. Summary and outlook

Acknowledgements

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\( \alpha(T) \) \hspace{1cm} \text{ultrasonic attenuation} \hspace{1cm} m_b \hspace{1cm} \text{conduction electron band mass}

\( \alpha_{0}, \beta, \gamma_0 \) \hspace{1cm} \text{Landau free energy parameters} \hspace{1cm} m^* \hspace{1cm} \text{effective quasiparticle mass}

\( B \) \hspace{1cm} \text{magnetic induction} \hspace{1cm} M \hspace{1cm} \text{magnetization}

\( c, c^\dagger \) \hspace{1cm} \text{conduction electron operators} \hspace{1cm} M_Q \hspace{1cm} \text{staggered magnetization}

\( C(T) \) \hspace{1cm} \text{specific heat} \hspace{1cm} N(E) \hspace{1cm} \text{quasiparticle DOS}

\( \delta \) \hspace{1cm} \text{CEF splitting energy} \hspace{1cm} N_n \hspace{1cm} \text{normal state quasiparticle DOS at } E_F

\( \Delta(k) \) \hspace{1cm} \text{gap function (SC, SDW, etc.; singlet, triplet, etc.)} \hspace{1cm} \rho \hspace{1cm} \text{pressure}

\( d(k) \) \hspace{1cm} \text{vector of triplet gap functions} \hspace{1cm} Q \hspace{1cm} \text{nesting or ordering wave vector}

\( \varepsilon_f \) \hspace{1cm} \text{f-level position} \hspace{1cm} \rho(T) \hspace{1cm} \text{electrical resistivity}

\( \varepsilon_k \) \hspace{1cm} \text{quasiparticle energy in the normal state} \hspace{1cm} \rho_Q \hspace{1cm} \text{conduction electron charge density}

\( E_k \) \hspace{1cm} \text{quasiparticle energy in the ordered state (SC, SDW, etc.)} \hspace{1cm} s_Q \hspace{1cm} \text{conduction electron spin density}

\( E_F \) \hspace{1cm} \text{Fermi energy} \hspace{1cm} \sigma \hspace{1cm} \text{vector of Pauli matrices}

\( f(k) \) \hspace{1cm} \text{form factor of gap functions} \hspace{1cm} \theta, \phi \hspace{1cm} \text{polar angles of } \mathbf{H}

\( f(E_k) \) \hspace{1cm} \text{Fermi function} \hspace{1cm} \theta, \varphi \hspace{1cm} \text{polar angles of } \mathbf{k}

\( f_{L}, f_{GL} \) \hspace{1cm} \text{Landau and Ginzburg–Landau free energy density} \hspace{1cm} \tau \hspace{1cm} \text{next nearest neighbor hopping matrix element}

\( \gamma \) \hspace{1cm} \text{linear specific heat (Sommerfeld) coefficient} \hspace{1cm} T \hspace{1cm} \text{temperature}

\( \Gamma \) \hspace{1cm} \text{quasiparticle scattering rate} \hspace{1cm} T^{*} \hspace{1cm} \text{characteristic or ‘Kondo’ temperature of HF compound}

\( |\Gamma| \) \hspace{1cm} \text{CEF state} \hspace{1cm} T_N \hspace{1cm} \text{Néel temperature}

\( \eta \) \hspace{1cm} \text{SC vector order parameter} \hspace{1cm} T_c \hspace{1cm} \text{superconducting transition temperature}

\( H \) \hspace{1cm} \text{magnetic field} \hspace{1cm} T^{-1}_1 \hspace{1cm} \text{NMR relaxation rate}

\( H_{c2} \) \hspace{1cm} \text{upper critical field of the superconductor} \hspace{1cm} U \hspace{1cm} \text{bare on-site Coulomb interaction}

\( I \) \hspace{1cm} \text{residual on-site quasiparticle repulsion or contact exchange} \hspace{1cm} u(\omega) \hspace{1cm} \text{local CEF susceptibility}

\( J(q) \) \hspace{1cm} \text{effective interaction between quasiparticles or local moments} \hspace{1cm} v_F \hspace{1cm} \text{Fermi velocity}

\( J \) \hspace{1cm} \text{total angular momentum of f-shell} \hspace{1cm} v_s(\mathbf{r}) \hspace{1cm} \text{superfluid velocity field}

\( \kappa(T) \) \hspace{1cm} \text{thermal conductivity} \hspace{1cm} V_l \hspace{1cm} \text{effective quasiparticle interaction \((l = \text{angular momentum channel})\)}

\( k \) \hspace{1cm} \text{Boltzmann constant} \hspace{1cm} \bar{v} \hspace{1cm} \text{average Fermi velocity in uniaxial crystal}

\( k \) \hspace{1cm} \text{conduction electron wave vector} \hspace{1cm} \chi_{N}, \chi_s(T) \hspace{1cm} \text{static homogeneous \((q = 0)\) conduction electron spin susceptibility in normal and SC state}

\( k_F \) \hspace{1cm} \text{Fermi wave number} \hspace{1cm} \chi(q, \omega) \hspace{1cm} \text{magnetic susceptibilities for conduction electrons or localised moments}

\( \lambda \) \hspace{1cm} \text{electron–phonon interaction} \hspace{1cm} \chi_{N}, \chi_s(T) \hspace{1cm} \text{static homogeneous \((q = 0)\) conduction electron spin susceptibility in normal and SC state}

\( \mu, \mu_B \) \hspace{1cm} \text{magnetic moment, Bohr magneton} \hspace{1cm} \chi(q, \omega) \hspace{1cm} \text{magnetic susceptibilities for conduction electrons or localised moments}

\( m \) \hspace{1cm} \text{free electron mass} \hspace{1cm} \chi(q, \omega) \hspace{1cm} \text{magnetic susceptibilities for conduction electrons or localised moments}
1. Introduction

The Fermi liquid state in metals has two common instabilities. Superconductivity (SC) is due to pair formation of electrons and (spin-, charge-) density waves (CDW, SDW) are formed by pairing electrons and holes. Theoretically it has been suspected quite early that pair-wave functions other than s-wave as in conventional superconductors and density waves may exist. However it has taken surprisingly long to identify such ‘unconventional’ condensed pair states in real materials.

Now there is an abundance of non-s-wave superconductors which are frequently associated with anisotropic ‘nodal’ gap functions where the quasiparticle excitations of the SC state vanishes on points or lines on the Fermi surface. This leads to low temperature ‘power law’
behaviour in many physical quantities. Heavy fermion (HF) lanthanide and actinide superconducting compounds were the first that were supposed to have unconventional SC pair states mediated by low energy spin fluctuations. High $T_c$ cuprates which exhibit d-wave superconductivity are the most prominent and important examples. But nodal SC have also been found among the ruthenates and organic salts.

Conventional CDW and SDW states are ubiquitous in metals with the commensurate SDW or antiferromagnetic (AF) order being the most common. Compounds with confirmed unconventional density wavessofar are rather scarce and with certainty have only been found in organic metals and perhaps in uranium HF compounds and the ‘pseudogap’ phase of underdoped cuprates. This may in part be due to the difficulty detecting such ‘hidden’ order parameters which leave no signature in standard neutron or X-ray diffraction experiments.

In this review article we will summarize the knowledge on a class of unconventional superconductors and their competition and coexistence with magnetism and hidden order phases that has accumulated over the last decade. To be comprehensive we will restrict ourselves exclusively to intermetallic lanthanide (4f) and actinide (5f) systems. Except for occasional remarks we will leave out completely the cuprate, cobaltate and ruthenate superconductors which are beyond the scope of our work on intermetallic compounds. The oxide superconductors are mostly close to a Mott insulator transition and the underlying microscopic physics is very different from the intermetallic f-electron compounds. Although on the phenomenological level of SC and density wave order parameter classification and investigation strong similarities exist. Likewise we do not discuss unconventional organic superconductors. The focus here is on stoichiometric 4f- and 5f-compounds where the lanthanide or actinide atoms occupy regular sublattices. With the exception of rare earth borocarbides the compounds reviewed are heavy fermion metals to a varying degree. In our review we do not want to provide an exhausting compilation of the physical properties in this whole class of materials. We rather focus on a few important compounds where each displays an important aspect of unconventional SC and its relation to magnetism or hidden order that will be discussed in detail in its physical and theoretical implications.

The complex low temperature phase diagrams of the HF metals results from the partially filled f-shells of the lanthanide and actinide ions which preserve atomic-like character. Occupying the states according to Hund’s rules leads to magnetic moments. In a crystal their rotational degeneracy is partly lifted by the crystalline electric field (CEF) and the hybridization with broad conduction bands of the outer shell electrons. As a result a large number of low-energy excitations appears. In the ideal Fermi liquid case these excitations correspond to heavy quasiparticles whose effective mass $m^*$ is by orders of magnitude larger than the free electron mass $m$. The corresponding quasiparticle band width $T^*$ is of the order meV for true HF metals. The mass enhancement is reflected in large increase of the specific heat $\gamma$-value, the Pauli susceptibility and the $T^2$-coefficient of electrical resistivity. However close to a quantum critical point (QCP) which signifies the onset of density wave instabilities low temperature anomalies in these quantities appear which are characteristic signatures of a non-Fermi liquid state.
At this point, we emphasize that the Landau theory of Fermi liquids does not make assumptions concerning the microscopic nature of the ground state and the low-lying excitations, it merely gives a counting prescription. It does not address the question how these excitations emerge in an interacting electron system, this requires a microscopic treatment. Generally speaking heavy quasiparticles arise from the lifting of local degeneracies which result as a consequence of strong local correlations. Various mechanisms for heavy mass generation have been suggested. While in Ce compounds with well localised 4f\(^1\)-states the Kondo mechanism is appropriate, there is increasing evidence that the dual, i.e., partly localised and partly itinerant, nature of 5f-states is responsible for the mass renormalization in the U-compounds. In both scenarios it is assumed that the heavy quasiparticles predominantly have f-character.

Residual interactions among the quasiparticles lead to the pairing instabilities. Two candidates have been identified in HF compounds: Firstly pairing interactions via exchange of enhanced overdamped spin fluctuations of itinerant quasiparticles, presumably at an antiferromagnetic wave vector. This model is invoked for Ce-compounds, especially when SC appears near a quantum critical point. It may also contain some truth for the more itinerant U-HF compounds. However we now know for sure that in U-compounds with partly localised 5f electrons a different mechanism is at work: pairing mediated by the exchange of propagating internal excitations (‘magnetic excitons’) of the localised 5f subsystem. A variant of this mechanism may also be appropriate for the recently discovered Pr-skutterudite HF superconductor with quadrupolar instead of magnetic excitons involved.

A major topic of our review is the critical discussion of existing evidence for unconventional SC order parameters as witnessed frequently by the presence of nodal gap functions. Previously the identification of gap symmetries has been an elaborate guess work mostly built on indirect evidence from low temperature ‘power laws’. This situation has dramatically improved in recent years with the advent of genuine angular resolved magnetothermal and magnetotransport experiments in the vortex phase. They exploit the Doppler shift of SC quasiparticle energies which leads to field-angle oscillations in specific heat and thermal conductivity. Their analysis may lead to an unambiguous determination of nodal positions of the gap functions which facilitates a strong restriction and sometimes unique determination of the possible gap symmetry.

Superconductivity in HF-compounds frequently coexists with (spin-)density waves. In some cases they may be of the unconventional (hidden order) type. We discuss the competition and coexistence behaviour of these order parameters and related physical effects. Since both order parameters appear in the itinerant quasiparticle system this is a subtle interplay of Fermi surface geometry, pairing potentials and gap structures which can only be schematically understood in simplified toy models.

Coexistence behaviour is much simpler in the relatively new class of rare earth borocarbide superconductors which have comparatively high \(T_c\)’s. Although as electron–phonon superconductors they are set aside from our main focus on HF systems, they have been included in this review for various reasons: Firstly magnetism and SC superconductivity are carried by different species of electrons which only interact weakly through contact exchange interaction leading to a small effect of the local moment molecular field on the SC conduction electrons. This can be nicely treated by perturbation theory and allows a much better understanding of
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coexistence behaviour as compared to the HF systems. Also they provide the first example of homogeneous coexistence of SC and ferromagnetism for all temperatures below $T_c$. Secondly the nonmagnetic rare earth borocarbides have extremely large gap anisotropy ratios $\gtrsim 10^2$ which means that they are essentially nodal superconductors. In fact nonmagnetic borocarbides are the first which have been identified to have a SC gap with point nodes that is of fully symmetric $s+g$ wave type. Surely the standard electron–phonon mechanism has to be supplemented by something else, perhaps anisotropic Coulomb interactions to achieve this unconventional behaviour of borocarbides.

Considering the wide range of phenomena and questions involved we have to be quite selective in our review. We will concentrate on the exciting developments in 4f/5f intermetallic materials of the past decade referring to the literature for the earlier work. A general introduction to strongly correlated electron systems is given in the textbooks of Fulde (1995) and Fazekas (1999). Various aspects of heavy fermion physics are described in the previous review articles by Stewart et al. (1984), Ott (1987), Ott and Fisk (1987), Fulde et al. (1988), Grewe and Steglich (1991), Thalmeier and Lüthi (1991) and Zwicknagl (1992) as well as in the monographs by Kuramoto and Kitaoka (2000) and Hewson (1993). Reviews which focus on theories of unconventional HF superconductors are Sigrist and Ueda (1991), Sauls (1994) and Joynt and Taillefer (2002) and the monograph by Mineev and Samokhin (1999). Reviews discussing the rare earth borocarbide magnetic superconductors are given in Hilscher and Michor (1999) and Müller and Narozhnyi (2001). Earlier reviews of coexistence of superconductivity and localised magnetism can be found in Fulde and Keller (1982), Fulde and Zwicknagl (1990) and Fischer (1990).

Since we will not discuss oxide or organic unconventional superconductors we would like to refer to a few review articles where parallel developments, especially on pairing mechanism and gap function symmetries are discussed for these classes of materials: van Harlingen (1995), Scalapino (1995), Izyumov (1999), Moriya and Ueda (2000), Yanase et al. (2003) and Tsuei and Kirtley (2000) for cuprates, Mackenzie and Maeno (2003) for ruthenates and Maki and Won (1996) and Lang and Müller (2003) for unconventional organic superconductors.

The present review focuses on chosen topics of high current interest. We therefore select specific materials which allow us to discuss physical effects and theoretical concepts in detail.

This article is organized as follows: In sect. 2 we review the basic theoretical concepts and experimental techniques to identify pair condensate order parameters, especially we discuss the important new tool of angular resolved methods for SC gap investigation. Section 3 deals with the Ce-based heavy fermion compounds. The central question there is the competition between magnetism and Fermi liquid states which may become superconducting. While superconducting ground states seem to occur rather rarely in Ce-based heavy fermion systems at ambient pressure they are common in their U-based counterparts discussed in sect. 4. The major challenge in this context is the investigation of the unconventional and often exotic pairing mechanism and the identification of the symmetry of superconducting order parameters which may coexist with (hidden) long-range order. The low-temperature phases of rare earth borocarbides are reviewed in sect. 5. Of particular interest are the unusual magnetic ordering phenomena which may coexist with very anisotropic electron–phonon superconductivity. Section 6 is devoted to the newly discovered superconducting rare earth skutterudite cage
compounds which may exhibit a quadrupolar pairing mechanism. We conclude in sect. 7 by giving a summary and an outlook.

2. Theory and techniques

The theoretical understanding of superconductivity and magnetism in the heavy fermion systems is still in the state of rather schematic or illustrative models without real predictive power. The difficulty arises on two levels. Firstly the normal state quasiparticles themselves can so far be described only within effective one particle renormalised band pictures with empirical input parameters. For some compounds like UBe$_{13}$ and Ce-compounds close to the quantum critical point the SC transition even takes place in a state without well defined quasiparticles as witnessed by the observation of non-Fermi liquid (nFl) behaviour. Secondly the effective pairing interactions can only be described in an oversimplified way as in the spin fluctuation models and its variants. They commonly neglect the internal orbital structure of f-electron compounds due to the intra-atomic spin–orbit coupling and CEF potential. Attempts to include these terms have not been carried very far. Nevertheless it is important to understand these qualitative theories. We first discuss normal state quasiparticles, namely Kondo lattice models for Ce-compounds vs. dual 5f-electrons models for the U-compounds. Starting from this basis the renormalised band theory provides a way to describe the heavy quasiparticle bands within a Fermi liquid approach. From there approximate models for the effective pairing interactions may be obtained by standard many-body techniques. The symmetry classification of pairing order parameters is an important step to understand the nodal structure of the gap and solve the gap equations. Models for coexistence and competition of SC and density wave order parameters frequently observed in HF metals will also be discussed. Finally we make a survey of theoretical ideas and experimental methods to identify the nodal structure of SC and density type order parameters which is the most important ingredient to understand the physics of HF superconductors.

2.1. Heavy quasiparticles in Ce, U compounds and their interactions

A prerequisite for a microscopic theory of superconductivity in heavy-fermion compounds is a description of the normal state and of the low-energy excitations at low temperatures. We begin by reviewing the physical processes which lead to the high density of low-energy excitations reflected in the strongly enhanced specific heat. The corresponding microscopic many-body problems, i.e., the Kondo model and the dual model, can be solved for (effective) impurities but not for extended systems. The energy dispersion of the coherent quasiparticle states in a periodic lattice, however, can be calculated from an effective single-particle Hamiltonian where the effective and not necessarily local potential is devised to account for the relevant many-body effects. The residual interaction among the quasiparticles eventually leads to the instability of the normal Fermi liquid phase. We focus on Cooper pair formation induced by electron–electron interactions. Of particular importance are spin-fluctuation models which were adopted in the majority of papers during the past decade.
2.1.1. Kondo lattice model for Ce-compounds

The similarities in the behavior of Ce-based heavy-fermion systems to that of dilute magnetic alloys have led to the assumption that these systems are 'Kondo lattices' where the observed anomalous behavior can be explained in terms of periodically repeated resonant Kondo scattering. This ansatz provides a microscopic model for the formation of a singlet ground state and the existence of heavy quasiparticles. In addition, it explains why there is no magnetic pairbreaking associated with the presence of the f-electrons. An extensive review is given by Hewson (1993). The Kondo picture for the Ce-based heavy-fermion compounds is supported by the fact that both the thermodynamic properties at low temperatures (e.g., the specific heat, the magnetic susceptibility) as well as the temperature dependence of the spectroscopic data can be reproduced by an Anderson model with the same parameters (Gunnarsson and Schönhammer, 1983). The most direct evidence, however, comes from photoelectron spectroscopy. The characteristic features of a Kondo system can be summarized as follows (Allen, 1992; Malterre et al., 1996): At high temperatures, the combined PES/BIS spectra from photoemission and inverse photoemission exhibit two distinct peaks below and above the Fermi energy. These two features correspond to the valence transitions \( f^n \rightarrow f^{n\pm1} \), respectively. The changes in the occupation of the Ce 4f-shells are associated with energies of order eV. The high-temperature state can be modelled by weakly correlated conduction electrons which are weakly coupled to local f-moments. The f-derived low-energy excitations are those of a system of local moments. The direct manifestation of the low-energy scale is the appearance of a sharp peak in the f-spectral density near the Fermi energy when the temperature \( T \) is smaller than a characteristic ('Kondo') temperature \( T^* \). In Ce systems, this many-body feature, i.e., the 'Abrikosov–Suhl' or 'Kondo' resonance, is centered at \( E_F + kT^* \) slightly above the Fermi edge \( E_F \) with \( T^* \sim W \exp(\pi \varepsilon_f / N_f \Delta) \) up to a constant of order unity. Here \( W \) is the conduction band width, \( \varepsilon_f < 0 \) is the f-level position below the Fermi level \( E_F \equiv 0 \), \( N_f \) the f-level degeneracy and \( \Delta = \pi |V|^2 N(E_F) \) is the hybridization or charge fluctuation width which can be estimated from the width of the transition \( 4f^1 \rightarrow 4f^0 \) (\( V = \) hybridization matrix element, \( N(E_F) = \) conduction electron DOS). This energy scale characterises the dynamical screening of the impurity spin by conduction electron spin fluctuations within a “screening cloud” that extends to distances \( \sim \hbar v_F / kT^* \) away from the impurity. The evolution of the Kondo resonance with temperature was recently observed by high-resolution photoemission experiments. The spectra displayed in fig. 1 provide direct evidence for the presence of a Kondo resonance in the lattice. The resonance is a genuine many-body feature reflecting the small admixture of \( f^0 \)-configurations to the ground state and the low-lying excitations which are mainly built from \( f^1 \)-configurations. By hybridization with conduction states via transitions \( f^1 \leftrightarrow f^0 \), the local magnetic degeneracies of the singly occupied 4f-shells are lifted. The characteristic energy \( kT^* \sim 1–10 \) meV which can be surprisingly close to the value of the corresponding dilute system sets the scale for the anomalous low-temperature behavior. The width and the overall weight of the resonance are of the order \( \pi kT^* / N_f \) and \( \pi kT^* / N_f \Delta = 1 - n_f \ll 1 \), respectively. Here \( n_f \) denotes the occupation of the \( f^1 \) state which in the Kondo limit \( \Delta \ll |\varepsilon_f| \) is slightly less than one due to the finite hybridization matrix element \( V \) with conduction electrons. Impurity model calculations based on the NCA approximation for the Anderson model indicate that the Kondo resonance states exist as long as \( n_f \gtrsim 0.85 \). For smaller \( n_f \) due to
larger $V$ one enters the mixed valent regime. At sufficiently low temperatures $T \ll T^*$, the contribution of the narrow resonance peak to the thermodynamic and transport properties can be described in terms of a Landau theory with heavy fermionic quasiparticles as suggested by the renormalization group calculations for magnetic impurities immersed in a metallic host (Wilson, 1975). Based on the corresponding effective Hamiltonian Nozières (1974) introduced a narrow resonant phase shift to account for the impurity contribution to the low-energy properties.

The novel feature observed in stoichiometric Ce-compounds is the formation of narrow coherent bands of low-energy excitations (Garnier et al., 1997, 1998; Zwicknagl, 1999). Following this line of thought, heavy fermions arise from a decoherence-coherence crossover. The strong local correlations in Kondo lattices lead to an observable many-body effect, i.e., the change with temperature of the volume of the Fermi surface. At high temperatures, the $f$-degrees of freedom appear as localized magnetic moments, and the Fermi surface contains only the itinerant conduction electrons. At low temperatures, however, the $f$-degrees of freedom are now tied into itinerant fermionic quasiparticle excitations and accordingly, have to be included in the Fermi volume following Luttinger’s theorem. Consequently the Fermi surface is strongly modified. This scenario (Zwicknagl, 1993) was confirmed experimentally by
measurements of the de Haas–van Alphen (dHvA) effect (Lonzarich, 1988; Aoki et al., 1993; Tautz et al., 1995).

Competition between the formation of (local) Kondo singlets and the lifting of degeneracies by long-range magnetic order is clearly evident in many Ce-based heavy fermion compounds. In the high-temperature regime the moments of the Ce 4f-shells are coupled by the RKKY interaction which can favor parallel as well as antiparallel orientation of the moments at neighboring sites. In the majority of cases, there is a tendency towards antiferromagnetic alignment although also ferromagnetic HF systems are known. Model calculations for two Kondo impurities (Jayprakash et al., 1981, 1982; Jones and Varma, 1987) showed that antiferromagnetic correlations between the magnetic sites weaken the Kondo singlet formation reducing the characteristic energy scale $kT^*$ to rather small values. Since the latter plays the role of an effective Fermi energy for the heavy quasiparticles it is not surprising that the Fermi liquid description fails to be valid in systems with sufficiently strong antiferromagnetic correlations.

For Ce-based systems, the natural tuning parameter is the hybridization which measures the coupling between the strongly correlated f-electrons and the weakly correlated conduction states. This coupling, which can be increased by applying hydrostatic pressure or chemical pressure via proper element substitution, affects the f-electron system in two different ways. First, it is responsible for the indirect exchange interaction building up magnetic correlations between the moments at different sites. On the other hand, it leads to the formation of local singlets via the Kondo effect. The energy gain due to magnetic ordering follows a power law for weak hybridization whereas the Kondo temperature depends exponentially on the latter. Based on these considerations, Doniach (1977) suggested that for sufficiently weak hybridization the f-derived magnetic moments should order. With increasing hybridization, however, the magnetic ordering temperature should be suppressed and a Fermi liquid ground state characterized by Kondo-type correlations forms above a quantum critical point (QCP). At the QCP itself non-Fermi liquid behaviour is expected in the temperature dependence of physical quantities. The resulting schematic “Doniach phase diagram” has been widely used to understand qualitatively the variation with pressure of the anomalous low-temperature properties in heavy-fermion systems. It seems that in the majority of Ce-based heavy-fermion superconductors the superconducting phase develops in the vicinity of a quantum critical point.

2.1.2. Dual model for U-compounds

The Kondo picture, however, does not apply in the case of the actinide compounds. The difficulties with this model have been discussed in Cox and Zawadowski (1999). The difference between the Ce-based heavy-fermion compounds and their U-counterparts can be seen directly from the photoemission spectra (Allen, 1992). In U-based heavy-fermion compounds, the fingerprint character of the transitions $f^n \rightarrow f^{n\pm1}$ is lost. Instead of the well-defined f-derived peaks familiar for the Ce systems, we encounter a rather broad f-derived feature. This fact shows that the f-valence in the actinide heavy-fermion systems is not close to integer value as it is the case in Ce-based compounds. In fact, the f-valence of the U ions has been discussed rather controversially.
There is growing evidence that actinide ions may have localized as well as delocalized 5f electrons. This picture which was suggested by susceptibility measurements (Schoenes et al., 1996) is supported by a great variety of experiments including, e.g., photoemission and neutron inelastic scattering experiments on UPd$_2$Al$_3$ (Takahashi et al., 1995; Metoki et al., 1998; Bernhoeft et al., 1998) as well as muon spin relaxation measurement in UGe$_2$ (Yaouanc et al., 2002). The assumption is further supported by quantum chemical calculations on uranocene U(C$_8$H$_8$)$_2$ (Liu et al., 1998) which exhibit a number of low-lying excitations which are due to intra-shell rearrangements of 5f-electrons. There is clear evidence that the presence of localized 5f-states is even responsible for the attractive interactions leading to superconductivity (Sato et al., 2001). In addition the dual model should allow for a rather natural description of heavy fermion superconductivity coexisting with 5f-derived magnetism.

The above-mentioned observations form the basis of the dual model which assumes the coexistence of both itinerant and localized 5f-electrons. The former hybridize with the conduction states and form energy bands while the latter form multiplets to reduce the local Coulomb repulsion. The two subsystems interact which leads to the mass enhancement of the delocalized quasiparticles. The situation resembles that in Pr metal where a mass enhancement of the conduction electrons by a factor of 5 results from virtual crystal field (CEF) excitations of localized 4f$^2$-electrons (White and Fulde, 1981). The underlying hypothesis is supported by a number of experiments. Detailed Fermi surface studies for UGa$_3$ (Biasini, 2003) show that the experimental data cannot be explained by assuming all 5f-electrons to be itinerant nor by treating them as fully localized. The basic assumptions underlying the dual model were recently confirmed by measurements of the optical conductivity (Dressel et al., 2002) which show the evolution of the high effective mass at low temperatures. The formation of the heavy quasiparticles is also observed in ARPES (Denlinger et al., 2001).

The dual model provides a microscopic theory for the heavy quasiparticles in U-compounds. The method reproduces the dHvA data in UPt$_3$ (Zwicknagl et al., 2002) and UPd$_2$Al$_3$ (Zwicknagl et al., 2003). The calculation of the heavy bands proceeds in three steps schematically summarized in fig. 2. First, the band structure is determined by solving the Dirac equation for the self-consistent LDA potentials but excluding two U 5f ($j = 5/2$) states from forming bands. The choice of the itinerant and localized orbitals depends upon the symmetry of the crystal and the hybridization strengths. The localized 5f orbitals are accounted for in the self-consistent density and accordingly in the potential seen by the conduction electrons. The intrinsic bandwidth of the itinerant U 5f $j = 5/2$ electrons is taken from the LDA calculation while the position of the corresponding band center is chosen such that the density distribution of the conduction states as obtained within LDA remains unchanged. For UPt$_3$ and UPd$_2$Al$_3$, the f-occupancy per U atom for the delocalized 5f-electrons amounts to $n_f = 0.65$ indicating that we are dealing with a mixed valent situation. The calculations yield the dHvA frequencies which can be directly compared with experimental data. In the second step, the localized U 5f-states are calculated by diagonalizing the Coulomb matrix in the restricted subspace of the localized 5f-states. Assuming the $jj$-coupling scheme, the Coulomb matrix elements are calculated from the radial functions of the ab-initio band structure potentials. In a crystal, the degeneracies of the ground-state may be lifted by a CEF. This is in fact the case for UPd$_2$Al$_3$ where the resulting singlet ground state of the localized 5f$^2$ is given by
\[ |g \rangle = (1/\sqrt{2})(|J_z = 3\rangle + |J_z = -3\rangle) \] in the \( J = 4 \) subspace. The coupling between the localized and delocalized 5f-electrons is directly obtained from the expectation values of the Coulomb interaction \( H_C \) in the 5f\(^3\)-states. Finally, the renormalization of the effective masses which results from the coupling between the two 5f subsystems is determined. The enhancement factor is calculated from the self-consistent solution of the self-energy equation (White and Fulde, 1981) with the input taken from the ab-initio electronic structure calculations for the delocalized and the localized 5f-electrons.

The coexistence of itinerant and localized 5f-states is a consequence of the interplay between hybridization with the conduction electrons and local Coulomb correlations. This “partial localization” of the 5f-states is found in many actinide intermetallic compounds. The underlying microscopic mechanism is an area of active current research (Lundin et al., 2000; Söderlind et al., 2000). LDA calculations show that the hopping matrix elements for different 5f orbitals vary. But it is of interest to understand why only the largest one of them is important and why the other ones are suppressed.

Partial localization may arise from the competition between hopping and angular correlations. This can be seen by exact diagonalization of small clusters which model the U sites in heavy fermion compounds (Efremov et al., 2004). We keep only the degrees of freedom of the 5f shells, the conduction states being accounted for by (effective) anisotropic intersite hopping. The Hamiltonian reads

\[
H = -\sum_{(nm),J_z} t_{J_z}(c_{J_z}^\dagger(n) c_{J_z}(m) + \text{h.c.}) + H_C, \tag{1}
\]
where the first sum is over neighboring sites \( \langle nm \rangle \). Furthermore \( c_{j}^{\dagger}(n) \) (\( c_{j}(n) \)), creates (annihilates) an electron at site \( n \) in the 5f \( j = 5/2 \) state with \( j_{z} = -5/2, \ldots, 5/2 \). We will consider two and three sites models. Since the relevant correlations are local the results for these small clusters are qualitatively similar to those of four-site models. The effective hopping between sites results from the hybridization of the 5f-states with the orbitals of the ligands and depends generally on the crystal structure. Rather than trying to exhaust all possible different lattice symmetries, we shall concentrate here on the special case that hopping conserves \( j_{z} \). While this is certainly an idealization, it allows us to concentrate on our main interest, i.e., a study of the influence of atomic correlations on the renormalization of hybridization matrix elements. The parameters \( t_{j_{z}}(=t_{-j_{z}}) \) are chosen in accordance with density-functional calculations for bulk material which use \( jj_{z} \) basis states. The local Coulomb interactions can be written in the form

\[
H_{C} = \frac{1}{2} \sum_{n} \sum_{j_{z1}, \ldots, j_{z4}} U_{j_{z1}j_{z2}, j_{z3}j_{z4}} c_{j_{z1}}^{\dagger}(n) c_{j_{z2}}^{\dagger}(n) c_{j_{z3}}(n) c_{j_{z4}}(n) \tag{2}
\]

with Coulomb matrix elements

\[
U_{j_{z1}j_{z2}, j_{z3}j_{z4}} = \delta_{j_{z1}+j_{z2}= j_{z3}+ j_{z4}} \sum_{J} \langle \frac{5}{2} \frac{5}{2} j_{z1} j_{z2} | J J_{z} \rangle U_{J} \langle J J_{z} | \frac{5}{2} \frac{5}{2} j_{z3} j_{z4} \rangle. \tag{3}
\]

Here \( J \) denotes the total angular momentum of two electrons and \( J_{z} = j_{z1} + j_{z2} = j_{z3} + j_{z4} \). The sum is restricted by the antisymmetry of the Clebsch–Gordan coefficients \( \langle \frac{5}{2} \frac{5}{2} j_{z1} j_{z2} | J J_{z} \rangle \) to even values \( J = 0, 2, 4 \). We use in the actual calculations \( U_{J} \) values which are determined from LDA wavefunctions for UPt\(_{3}\) (Zwicknagl et al., 2002), i.e., \( U_{J=4} = 17.21 \) eV, \( U_{J=2} = 18.28 \) eV, and \( U_{J=0} = 21.00 \) eV. We expect \( U_{J=4} < U_{J=2} < U_{J=0} \) always to hold for Coulomb interactions, independently of the chemical environment. In contrast, the relative order of the hopping matrix elements will vary strongly from one compound to the next. The average Coulomb repulsion of about 20 eV is irrelevant for the low-energy physics of the model. It simply restricts the relevant configurations to states such that each site is occupied either by 2 or 3 f electrons. The low-energy sector is exclusively determined by the differences of the \( U_{J} \) values, which are of the order of 1 eV and thus slightly larger than typical bare f-band widths. The latter are obtained, e.g., from LDA calculations for metallic uranium compounds like UPt\(_{3}\). Restricting the model to \( f^{2} \) and \( f^{3} \) configurations is equivalent to let the various \( U_{J} \rightarrow \infty \) while their differences remain finite. To mimic the situation in the U-based heavy-fermion compounds we consider the intermediate valence regime. Note that in the absence of a magnetic field all states of the two-site model with five electrons will be at least doubly degenerate because of Kramers’ degeneracy.

The Hamiltonian equation (1) conserves \( J_{z} = \sum_{n} J_{z}(n) \) where \( J_{z} \) is the \( z \)-component of the total angular momentum of the system and the \( J_{z}(n) \) refer to angular momentum projections on individual sites. We shall therefore characterize the eigenstates by their \( J_{z} \) value. Since \( t_{j_{z}} = t = \text{const} \) the system is rotationally invariant. Then \( J^{2} \) provides an additional good quantum number. Strong on-site correlations result in a considerable enhancement of anisotropies in the bare hopping matrix elements. This can lead to a localization of electrons
in orbitals with relatively weak hybridization. The latter is effectively reduced to zero in those cases.

In order to quantify the degree of localization or, alternatively, of the reduction of hopping of a given \( j_z \) orbital by local correlations, we calculate the ratio of the \( j_z \)-projected kinetic energy \( T_{j_z} \) and the bare matrix element \( t_{j_z} \) and obtain

\[
\frac{T_{j_z}}{t_{j_z}} = \sum_{(nm),\pm} \langle \Psi_{gs} | (c_{\pm j_z}^\dagger (n) c_{\pm j_z}^\dagger (m) + h.c.) | \Psi_{gs} \rangle.
\]

The ground-state wavefunction \( |\Psi_{gs}\rangle \) contains the strong on-site correlations. A small ratio of \( T_{j_z}/t_{j_z} \) indicates partial suppression of hopping for electrons in the \( \pm j_z \) orbitals. Two kinds of correlations may contribute to that process. The first one is based on the reduction of charge fluctuations to atomic \( f^2 \) and \( f^3 \) configurations. This is a result for large values of \( U_j \) and can be studied by setting all \( U_j \) equal to a value much larger than the different \( t_{j_z} \). The second one is due to differences in the \( U_j \) values, i.e., \( U_{j=4} < U_{j=2} < U_{j=0} \). The differences in the \( U_j \) values are the basis of Hund’s rules. Hopping counteracts Hund’s rule correlations and vice versa. What we want to stress is the fact that those correlations can lead to a complete suppression of hopping channels except for the dominant one which shows only little change.

Results for the ratios \( T_{j_z}/t_{j_z} \) are shown in fig. 3 for a two-site model. As the relevant correlations are local the general results qualitatively agree with those found for a three-site cluster and four-site clusters (Pollmann and Zwicknagl, 2004). We can distinguish three different regimes with \( J_z = 15/2, 5/2 \) and \( 3/2 \), labeled I, II and III, respectively. One observes that in region I only the dominant hybridization of the \( j_z = 3/2 \) orbital survives while that

![Fig. 3. Values \( T_{j_z}/t_{j_z} \) for the two-site cluster along the line connecting linearly the points written below the figure. Regions with \( J_z = 15/2, 5/2 \) and \( 3/2 \), are labeled with I, II and III, respectively (Efremov et al., 2004).](image)
of the $j_z = 1/2$ and $j_z = 5/2$ orbitals is completely suppressed. On the other hand in regions II and III the correlation effects on different orbitals are not very different. These findings demonstrate that in particular Hund’s rule correlations strongly enhance anisotropies in the hopping. For a certain range of parameters this may result in a complete suppression of the effective hopping except for the largest one, which remains almost unaffected. This provides a microscopic justification of partial localization of 5f-electrons which is observed in a number of experiments on U compounds and which is the basis for further model calculations described later.

2.1.3. Fermi-liquid state and heavy quasiparticles: renormalized band theory

Neither the Kondo model for Ce-based heavy-fermion systems nor the dual model for their U-based counterparts can be solved for extended systems described by lattice models. The excitation spectra of Ce metal in the $\alpha$- and $\gamma$-phase were recently calculated applying the dynamical mean field theory (McMahan et al., 2003). The method yields a low-energy resonance but the experimentally observed change of the f-valence at the transition is not reproduced. A fully microscopic description is not available to describe experiments where inter-site effects become strongly manifest. Typical lattice effects are the formation of coherent heavy quasiparticle bands whose Fermi surfaces were observed experimentally.

The Landau–Fermi liquid (FL) theory assumes a one-to-one correspondence between the states of the complex interacting system and those of a gas of independent fermions which may move in an external potential (Landau, 1956, 1957, 1958; Abrikosov et al., 1975). The single-particle orbitals and energies are determined from an effective Hamiltonian. The characteristic properties of a system are reflected in an effective and not necessarily local potential $V_{\text{eff}}$ which describes the field of the nuclei and the modifications arising from the presence of the other electrons. The essential many-body aspects of the problem are then contained in the prescription for constructing the effective potentials which have to be determined specifically for the problem under consideration.

The quasiparticle energies reflect the interaction among the fermions and therefore may be altered when the overall configuration is changed. A characteristic feature of interacting Fermi liquids is that the energy dispersion $\epsilon_{\sigma}(k)$ depends on how many other quasiparticles are present,

$$\epsilon_{\sigma}(k) = E(k) + \sum_{k'\sigma'} f_{\sigma\sigma'}(k, k')\delta n_{\sigma'}(k').$$  \hspace{1cm} (5)

Here $E(k)$ denotes the energy dispersion of a dilute gas of quasiparticles. In systems with strong correlations it cannot be calculated from the overlap of single-electron wave functions. The interactions among the quasiparticles are characterized by the matrix $f_{\sigma\sigma'}(k, k')$. The deviations from the equilibrium distribution are given by $\delta n_{\sigma}(k)$.

Interaction effects must be accounted for in considering those situations where the quasiparticle distribution function deviates from that of the equilibrium case. In the phenomenological Landau–FL theory the characteristic properties of the quasiparticles, which can hardly be calculated microscopically, are expressed in terms of parameters which are determined from
experiment. Examples are the effective potentials, the interaction potential and the scattering amplitudes. An important property of the quasiparticles is that they can be considered as ‘rigid’ with respect to low-energy and long-wavelength perturbations. Only such processes can be described within this theoretical framework.

The energy dispersion \( E(k) \) of a dilute gas of noninteracting quasiparticles is parametrized by the Fermi wave vector \( k_F \) and the Fermi velocity \( v_F \)

\[
E(k) = v_F(\hat{k}) \cdot (k - k_F),
\]

where \( \hat{k} \) denotes the direction on the Fermi surface. The key idea of the renormalized band method is to determine the quasiparticle states by computing the band structure for a given effective potential. Coherence effects which result from the periodicity of the lattice are then automatically accounted for. The quantities to be parametrized are the effective potentials which include the many-body effects. The parametrization of the quasiparticles is supplemented by information from conventional band structure calculations as they are performed for “ordinary” metals with weakly correlated electrons. The periodic potential leads to multiple-scattering processes involving scattering off the individual centers as well as the propagation between the centers which mainly depends on the lattice structure and is therefore determined by geometry. The characteristic properties of a given material enter through the information about single center scattering which can be expressed in terms of a properly chosen set of phase shifts \( \{ \eta_i^\nu(E) \} \) specifying the change in phase of a wave incident on site \( i \) with energy \( E \) and symmetry \( \nu \) with respect to the scattering center. Within the scattering formulation of the band structure problem the values of the phase shifts at the Fermi energy \( \{ \eta_i^\nu(E_F) \} \) together with their derivatives \( \{ (d\eta_i^\nu/dE)_{E_F} \} \) determine the Fermi wave vectors \( k_F \) and the Fermi velocity \( v_F \).

The calculation of realistic quasiparticle bands proceeds in several steps as schematically summarized in fig. 4. The first step is a standard LDA band structure calculation by means of which the effective single-particle potentials are self-consistently generated. The calculation starts, like any other ab-initio calculation, from atomic potentials and structure information. In this step, no adjustable parameters are introduced. The effective potentials and hence the phase shifts of the conduction states are determined from first principles to the same level as in the case of ‘ordinary’ metals. The f-phase shifts at the lanthanide and actinide sites, on the other hand, are described by a resonance type expression

\[
\tilde{\eta}_f \simeq \arctan \frac{\tilde{\Delta}_f}{E - \tilde{\epsilon}_f}
\]

which renormalizes the effective quasiparticle mass. One of the two remaining free parameters \( \tilde{\epsilon}_f \) and \( \tilde{\Delta}_f \) is eliminated by imposing the condition that the charge distribution is not significantly altered as compared to the LDA calculation by introducing the renormalization. The renormalized band method devised to calculate the quasiparticles in heavy-fermion compounds thus is essentially a one-parameter theory. We mention that spin–orbit and CEF splittings can be accounted for in a straightforward manner (Zwicknagl, 1992).
2.1.4. Quasiparticle interactions and spin fluctuation theory

The low-energy excitations of heavy-fermion systems are described in terms of quasiparticle bands which yield the high density of states and specific heat $\gamma$-value. The preceding sections focussed on the case of a dilute gas of quasiparticles whose energy dispersion can be explicitly calculated by means of the renormalized band method. The many-body effects, however, lead to deviations from the picture of independent fermions which fill the rigid bands of the dilute gas of quasiparticles. An important consequence of the quasiparticle interactions is the instability of the normal Fermi liquid with respect to charge or spin density waves or superconductivity. In heavy-fermion compounds, quasiparticle interactions are strongly evident in the electronic compressibility whose values are comparable to those of a normal metal and do not reflect the enhancement of the specific heat. This experimental fact indicates that there must be a strong repulsion between two quasiparticles at the same lattice site.

The influence of the quasiparticle interactions on observable quantities is usually described in terms of a small set of interaction parameters. According to Landau, the compressibility $\kappa_e$ and susceptibility $\chi_s$ are given by

$$\frac{\kappa_e}{\kappa^0_e} = \frac{m^*/m}{1 + F_0^a} \quad \text{and} \quad \frac{\chi_s}{\chi^0_s} = \frac{m^*/m}{1 + F_0^a}.$$  (8)
where \( \kappa_0^e \) and \( \chi_0^s \) denote the compressibility and susceptibility of independent fermions of mass \( m \). From \( \kappa_0^e / \kappa_0^e \approx 1 \) follows the order of magnitude estimate \( F_0^s \approx m^* / m \approx 10^2 - 10^3 \) for the spin-independent isotropic part of the quasiparticle interactions. The spin-dependent isotropic part of the interaction, on the other hand, is reflected in the enhancement of the spin susceptibility \( \chi_s / \chi_0^s \). From the observed value \( \chi_s / \chi_0^s \approx m^* / m \) we infer that the enhanced spin susceptibility simply reflects the high density of quasiparticle states and that the corresponding Landau–FL parameter \( F_0^a \) plays only a minor role in heavy-fermion compounds. Although these results are not sufficient to specify the effective interaction in an anisotropic Fermi liquid they nevertheless impose important constraints on theoretical models.

In Landau–FL theory a two-particle scattering matrix \( T_{\alpha\beta;\gamma\rho}(k_3k_4; k_1k_2) \) is introduced which describes the scattering of two quasiparticles with momenta \( k_3k_4 \) and spins \( \gamma\rho \) into states with momenta \( k_1k_2 \) and spins \( \alpha\beta \) on the Fermi surface. The superconducting transition in a Fermi liquid is determined by a singularity in the scattering matrix of two quasiparticles with opposite momenta at the Fermi surface (see fig. 5). Assuming rotational invariance in spin space the general scattering matrix \( T_{\alpha\beta;\gamma\rho} \) can be expressed in terms of two scalar amplitudes. Conventional choices are either the singlet and triplet amplitudes in the particle–particle channel \( (\gamma\rho \to \alpha\beta) \), \( T_s \) and \( T_t \) or alternatively, the singlet and triplet amplitudes in one of the particle–hole channels, \( T^{(s)} \) and \( T^{(a)} \),

\[
T_{\alpha\beta;\gamma\rho} = -\frac{1}{2}(\sigma_2)_{\alpha\beta}(\sigma_2)_{\gamma\rho} T_s + \frac{1}{2}(\sigma_2)_{\alpha\beta} \cdot (\sigma_2)_{\gamma\rho} T_t
= \delta_{\alpha\beta}\delta_{\gamma\rho} T^{(s)} + \sigma_{\alpha\beta} \cdot \sigma_{\gamma\rho} T^{(a)},
\]

where \( \sigma_\mu; \mu = 1–3 \) denote the Pauli matrices. The two sets of scalar amplitudes are related by

\[
T_s = T^{(s)} - 3T^{(a)}; \quad T_t = T^{(s)} + T^{(a)};
T^{(s)} = \frac{1}{4}(T_s + 3T_t); \quad T^{(a)} = \frac{1}{4}(T_t - T_s).
\]

Fig. 5. Two-particle (e-e) scattering matrix \( T_{\alpha\beta;\gamma\rho}(k, -k; k', -k') \) for quasiparticle scattering with initial momenta \( k', -k' \) and spins \( \gamma\rho \) to final momenta \( k, -k \) and spins \( \alpha\beta \). SC transition corresponds to singular pair scattering with opposite momenta. It is approximated by ladder diagrams involving the irreducible (with respect to e-e scattering) four-point vertex for the effective interaction (wiggly line) which is the central quantity in microscopic models of SC, see also fig. 6 and eqs. (12), (13).
The symmetries can be obtained directly from the symmetries of the two-particle Green’s function. The Landau–FL parameters $F_0^s$ and $F_0^a$ are determined by the isotropic Fermi surface averages $A_0^{s,a} = \langle T^{s,a}(k_1 k_2; k_1 k_2) \rangle_{\text{FS}}$ of the two particle forward-scattering amplitude by $F_0^{s,a} = A_0^{s,a} [1 - A_0^{s,a}]^{-1}$. As mentioned above, the transition to the superconducting state is caused by a singularity in the scattering of a pair with opposite momenta $k_3 = -k_4 = k'$ into pair states with $k_1 = -k_2 = k$.

The Fermi liquid approach which attempts to construct phenomenological models for the scattering amplitude has been reviewed (Fulde et al., 1988; Zwicknagl, 1992). In the present review we shall rather concentrate on microscopic models for the effective pairing interaction.

Microscopic theories of superconductivity focus on the attractive interaction which results from the motion of the (heavy) quasiparticle through a polarizable medium. The theories can be divided into two major groups depending on whether the polarizable medium is distinct from the particles which are participating in the attraction or not. The BCS theory (Bardeen et al., 1957; Schrieffer, 1964) belongs to the former class. Here Cooper-pair formation is due to quasiparticle–phonon interactions which is appropriate for common metals. The spin fluctuation models assume that pairing is mediated by overdamped low lying magnetic excitations with a prefered wave vector in the itinerant electron system. The dual model for U-based heavy-fermion systems suggests a novel mechanism for the Cooper-pair formation, i.e., the exchange of weakly damped propagating magnetic excitons. In this case, the polarizable medium is provided by the CEF-split multiplets of the localized 5f-electrons. A model calculation for UPd$_2$Al$_3$ is presented in sect. 4.2, where we also compare theoretical predictions with experimental observations.

The majority of theoretical calculations for heavy-fermion systems adopts spin-fluctuation models for the quasiparticle attraction which belong to the second group. Early theoretical efforts (Anderson, 1984a; Valls and Tešanović, 1984) emphasized an analogy with superfluid $^3$He. Even though a quantitative microscopic theory of suprafluidity is still lacking for $^3$He it is generally accepted that spin fluctuations qualitatively explain observed features in $^3$He, in particular odd-parity spin-triplet pairing (Anderson, 1984b; Vollhardt and Wölfle, 1990).

The influence of spin fluctuations on the Cooper pair formation in metals was studied first by Berk and Schrieffer (1966) and Layzer and Fay (1971, 1974). It has found its most prominent applications in HF systems (Miyake et al., 1986) and especially in the theory of high-$T_c$ superconductors (Monthoux et al., 1991; Monthoux and Pines, 1994). The calculations proceed in close analogy to the strong-coupling theory of phonon-mediated superconductivity. At this point we have to add that the theory of spin fluctuation-mediated superconductivity does not have a natural small parameter which would allow for a systematic asymptotic expansion. In particular, there is no a priori justification for including only selected contributions in a diagrammatic expansion for the scattering matrix.

The pair scattering for two quasiparticles with opposite momenta at the Fermi surface is evaluated adopting the ladder approximation displayed in fig. 5. The problem is therefore reduced to finding the four-point vertex which is irreducible with respect to particle–particle scattering. The basic assumption is that the important structure in the scattering amplitudes comes from exchange of collective modes in the two particle–hole channels. The central quantity of these theories is the dynamic magnetic susceptibility $\chi(q, \omega)$ which can be determined
– in principle – by inelastic neutron scattering. Close to a magnetic instability, the susceptibility diverges for $\omega \to 0$ at some wave vector $Q$. This behaviour results from a singularity in the particle–hole scattering matrix which accounts for the dynamic effects of the induced spin polarizations. The strong particle–hole correlations affect the pair scattering amplitude in the particle–particle channel. In explicit calculations, the effective interaction associated with $\chi(q, \omega)$ originates in a residual on-site quasiparticle repulsion ($I$) described by the single band Hubbard model for the heavy quasiparticle bands $\epsilon_k$.

$$H_I = \sum_{k,s} \epsilon_k n_k + I \sum_i n_{i\uparrow} n_{i\downarrow}. \quad (11)$$

This is also the starting point in the theory of high-$T_c$ SC, albeit with bare Coulomb repulsion and 2D quasiparticle band. In the fluctuation exchange (FLEX) approach (Pao and Bickers, 1994; Dahm and Tewordt, 1995) the effective pair interactions and gap equations are obtained and solved self-consistently in the context of a strong coupling approach. For the complicated 3D Fermi surfaces of HF materials this is too difficult and one is restricted to nonretarded model calculations. In this context the effective interaction is obtained within the random phase approximation (RPA) for pairs of quasiparticles with opposite and equal spins from the diagrams displayed in fig. 6. To conserve rotational invariance in spin space the maximally crossed particle exchange contributions in fig. 6 have to be included. These terms were studied by Berk and Schrieffer (1966) who considered the influence of spin-fluctuations on singlet pairing. The case of triplet scattering was discussed by Nakajima (1973) for $^3$He. He showed that sufficiently close to a magnetic instability, the complete set of diagrams yields an effective spin-dependent interaction which is rotationally invariant in spin space

$$H_{\text{eff}} = \frac{1}{2} \sum_{k,k'} J(k-k') \sigma_{\alpha\gamma} \cdot \sigma_{\beta\rho} c_{k\alpha}^{\dagger} c_{-k\beta}^{\dagger} c_{-k'\rho} c_{k'\gamma}. \quad (12)$$

![Fig. 6. RPA diagrams for the effective interaction $J(k-k')$ between quasiparticles with opposite spins. The bare instantaneous interaction is denoted by $I$ (dashed line) as in eq. (11). Contribution from polarisation (bubble) diagrams contains odd number and from (maximally crossed) exchange diagrams contain any number of interaction lines ($I$). For equal quasiparticle spins the RPA yields only a sum of bubble diagrams with an even number of interaction lines.](image)
where

\[ J(q) = -\frac{1}{2} \frac{I}{1 - I\chi_0(q)} \simeq -\frac{1}{2} I^2 \chi(q) \quad \text{and} \quad \chi(q) = \frac{\chi_0(q)}{1 - I\chi_0(q)}. \tag{13} \]

Here \( \chi_0(q) \) is the static susceptibility of the non-interacting quasiparticles and \( \chi(q) \) the RPA susceptibility of the interacting system. The approximation is valid for the enhanced spin fluctuation regime \( I\chi_0(q) \leq 1 \). In this regime the effective pairing is therefore completely determined by the collective static magnetic susceptibility \( \chi(q) \).

Assuming a spherical FS the appropriate basis function for the SC order parameter are spherical harmonics of angular momentum \( l \). The interaction in the \( l \)-wave channel is given by \( (P_l = \text{Legendre polynomial})\)

\[ V_l = a_l 2 \int_0^1 dx x P_l(1 - 2x^2) [-J(2k_Fx)], \tag{14} \]

with \( x = 1 - (q/2k_F)^2 \) and \( a_l = 3 \) \((l \text{ even})\) or \( a_l = -1 \) \((l \text{ odd})\). If the effective interaction \(-J(q)\) strongly peaks for small \( q \) \((x \simeq 1)\), i.e., for FM spin fluctuations, the integrand is positive in this region and hence \( V_l < 0 \) for odd \( l \) and \( V_l > 0 \) for even \( l \).

Assuming that in HF systems the quasiparticle interaction is mediated by FM spin fluctuations one would expect odd-parity SC order parameters. For UPt\(_3\) however, the odd parity state was originally claimed to be inconsistent with the power law behavior observed in thermodynamic and transport properties at low temperatures. This statement was based on the assumption that the superconducting phase should be characterized by one of the symmetry-adapted order parameters which were derived in the limit of strongly coupled orbital and spin moment of the Cooper pair \( \text{(Volovik and Gorkov, 1985; Ueda and Rice, 1985; Sigrist and Ueda, 1991).} \) We shall comment on this subtle point in the next section.

The discovery of antiferromagnetic spin fluctuations in UPt\(_3\) \( \text{(Aeppli et al., 1987) prompted Miyake et al. (1986) to study the nature of pairing due to antiferromagnetic spin fluctuations.} \) The authors consider pairing in a single-band model with the effective electron interaction of eq. (12) where the nonretarded interaction \(-J(q)\) should have a maximum at an AF wave vector \( Q \), e.g., \( Q = (\frac{1}{4}, \frac{1}{4}, \frac{1}{4}) \) or \( Q = (\frac{1}{2}, \frac{1}{2}, 0) \). In the vicinity of its maximum at \( Q \), \(-J(q)\) is approximated by

\[ -J(q) = J_0 - J_1 \gamma_q, \tag{15} \]

where \( J_0, J_1 \) are positive constants and the function \( \gamma_q \) has a minimum at \( q = Q \). Examining this model for a parabolic single band Miyake et al. (1986) demonstrated that the resulting pairing interaction favors anisotropic even-parity SC order parameters in cubic symmetry \((V_l < 0 \text{ for even } l > 0)\). This turned out to be an influential result. Because of the general presence of AF spin fluctuations in HF metals it has lead to the attitude to expect generally singlet (even parity) SC in these compounds.

There are, however, counter-examples like UPt\(_3\) and UNi\(_2\)Al\(_3\) with AF spin fluctuations and experimentally identified triplet order parameter. The prediction of only even parity unconventional pair states therefore seems to be an artefact of the simplifications inherent in the model. One obvious deficiency is the assumption of cubic symmetry and basis functions.
adjusted for a spherical Fermi surface not appropriate for these compounds. More sophisticated versions of the model incorporating experimental data for the magnetic fluctuations via $\chi(q, \omega)$ and realistic band structures for the quasiparticles failed to reproduce the symmetry of the superconducting order parameter in UPt$_3$. In particular, the calculations do not produce a multicomponent $E_1$ or $E_2$ order parameter as stable solution. This failure is discussed in detail in Heffner and Norman (1996). Also the generalizations of standard spin-fluctuation theory which account for orbital effects (Norman, 1994) do not resolve the difficulties encountered in real materials.

Aspects of strong coupling effects within spin-fluctuation theories, especially the pair breaking role of low frequency bosons have been discussed by Millis et al. (1988). More recently magnetically mediated superconductivity in materials close to a magnetic instability was investigated in a series of papers by Monthoux and Lonzarich (1999, 2001, 2002) and McHale and Monthoux (2003). A phenomenological form for the retarded generalized magnetic susceptibility $\chi(q, \omega)$ defining the effective quasiparticle interaction is adopted

$$\chi(q, \omega) = \frac{\chi_0 \kappa_0^2}{\kappa^2 + \hat{q}^2 - i \omega \eta(\hat{q})},$$

where $\kappa$ and $\kappa_0$ are the inverse correlation lengths (in units of the inverse lattice constant) with and without strong magnetic correlations, respectively. The functions $\hat{q}^2$ and $\eta(\hat{q})$ are parametrized so as to directly compare ferromagnetic and antiferromagnetic spin fluctuations. The instability of the normal state is determined by solving the linearized Eliashberg equations which yield the transition temperatures $T_c$ and the mass renormalization. The questions addressed include the influence of dimensionality on the robustness of magnetic pairing and the relative stability of d-wave versus p-wave pairing.

Pairing instabilities may also appear in the electron–hole (Peierls-) rather than electron–electron (Cooper-) channel leading to CDW/SDW type instabilities described by $T^{(s)}$ and $T^{(a)}$ in eq. (10). They are strongly favored if the Fermi surface shows the nesting property $\epsilon_{k+Q} = -\epsilon_k$ characteristic for flat FS portions connected by the nesting vector $Q$ as shown later in fig. 7. In this case the on-site interaction term in the Hamiltonian equation (11) may be truncated in momentum space because the e-h scattering is dominated by processes with momentum transfer $Q$. In mf approximation the effective CDW/SDW pairing Hamiltonian is given in eq. (33). To obtain the full variety of electron–hole pair states one needs more general microscopic interactions, replacing the on-site Hubbard model of eq. (11) by an extended Hubbard model which include inter-site Coulomb repulsion and exchange (Gulacsi and Gulacsi, 1987; Schulz, 1989; Ozaki, 1992).

2.2. **Order parameters and their coexistence in strongly correlated electron systems**

Degenerate interacting Fermi systems are prone to instabilities due to pair condensation. The Pauli principle requires that states have to be filled up to the Fermi energy $E_F$ leading to a large kinetic energy. Rearranging the occupation of noninteracting states around the Fermi level may reduce the interaction energy considerably. The large on-site Coulomb interactions
will achieve this by reducing double occupancies of opposite spin states below the Hartree–Fock result. This leads to a strongly correlated electronic ground state which is ideally approached at low temperatures below a characteristic temperature scale $T^*$ without breaking of spatial or internal symmetries. Excitations from this state may be described within the Landau–Fermi liquid (FL) picture with quasiparticles that have a strongly enhanced effective electron mass $m^* \gg m$. Eventually however this state will become unstable against formation of electron (Cooper-)pairs or electron–hole (Peierls-)pairs due to the residual screened interactions between the quasiparticles. In the former case gauge symmetry is broken leading to a superconducting (SC) state, in the latter spatial symmetries and possibly spin rotational and time reversal symmetry are broken leading to charge-density wave (CDW) or spin-density wave (SDW) states. Which state is more favorable depends on the momentum and energy dependence of residual quasiparticle interactions and on the geometric properties of the Fermi surface and usually cannot be predicted with confidence for real materials. The effective Hamiltonian obtained previously describing the low energy pairing correlations of quasiparticles is most frequently used for studying possible SC states. The condensation into pairs is described by a gap function $\Delta(k)$ which characterises both the type of broken symmetry state and its new quasiparticle excitations. Its experimental determination is therefore of central importance. Unfortunately this is a difficult task except in the isotropic case where $\Delta(k)$ is a constant. In this section we outline the possible type of pair states and gap functions and their symmetry classification. We also discuss simple models of coexistence of SC and CDW/SDW type order parameters based on 2D FS models with nesting properties.

2.2.1. Order parameter classification

Many physical properties of superconducting materials are directly determined by the symmetry of the SC order parameter. The possible types of order parameters are restricted by crystal symmetry. This fact provides a classification scheme for different superconducting states and, in addition, allows one to construct the superconducting classes by means of group theory. Superfluids are characterized by off-diagonal long range order (ODLRO) which leads to nonvanishing correlations in the two-particle density matrix for large separations of particles at points $\mathbf{r}_1, \mathbf{r}_2$ and $\mathbf{r}_1', \mathbf{r}_2'$,

$$\langle \mathbf{r}_1 s_1; \mathbf{r}_2 s_2 | \rho(2) | \mathbf{r}_1' s_1'; \mathbf{r}_2' s_2' \rangle \rightarrow \psi_{s_1, s_2}^*(\mathbf{r}_1, \mathbf{r}_2) \psi_{s_1', s_2'}(\mathbf{r}_1', \mathbf{r}_2').$$

Here, $\mathbf{r}_i, s_i$ denote the fermion positions and spins, respectively. In systems with strong spin–orbit (s.o.) interactions, the indices $s_i$ refer to pseudo spins associated with Kramers degeneracy of conduction bands. The ordered phase is characterized by the order parameter $\psi$ which is a complex pseudo-wave function always connected with a spontaneous breaking of U(1) gauge symmetry. In general, it depends on the center-of-mass and relative coordinates, $\mathbf{R} = \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2), \mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ and the pseudo spins $s_1$ and $s_2$, respectively. We shall restrict ourselves to homogeneous systems neglecting the dependence on the center-of-mass variable $\mathbf{R}$. Then the order parameter is a function of the relative coordinate $\mathbf{r}$ only. Performing a Fourier transformation with respect to $\mathbf{r}$ with $\psi_{s_1 s_2}(\mathbf{r}) \rightarrow \psi_{s_1 s_2}(\mathbf{r}) \psi(\mathbf{k})$ and restricting the wave vector to the Fermi surface $\mathbf{k} = k_F(\hat{\mathbf{k}})\hat{\mathbf{k}}$ yields a pairing amplitude $\psi_{s_1 s_2}(\hat{\mathbf{k}})$ which depends upon the direction $\hat{\mathbf{k}}$ on the Fermi surface. In microscopic theories the gap function $\Delta_{s_1 s_2}$...
which also determines the excitation spectrum is commonly used as order parameter. It is given by
\[ \Delta_{s_1s_2} (\mathbf{k}) = - \sum_{\mathbf{k}' s_3s_4} V_{s_1s_2s_3s_4} (\mathbf{k}, \mathbf{k}') \psi_{s_3s_4} (\mathbf{k'}) , \]  
(18)
where \( V \) is the effective pairing interaction. For the spin fluctuation model we obtained \( V_{s_1s_2s_3s_4} (\mathbf{k}, \mathbf{k}') = - J (\mathbf{k} - \mathbf{k}') \sigma_{s_1s_2} \sigma_{s_3s_4} \) in eq. (12). Concerning the symmetry classification of the superconducting phases, one may use the gap function \( \Delta_{s_1s_2} \) or the pair amplitude \( \psi_{s_1s_2} \) as order parameter due to their identical transformation properties.

The fundamental property of \( \psi_{s_1s_2} \) or \( \Delta_{s_1s_2} \) is its behaviour as a two-fermion wave function in many respects. This expresses the fact that an ODLRO order parameter is not the thermal expectation value of a physical observable but rather a complex pseudo-wave function describing quantum phase correlations on the macroscopic scale of the SC coherence length. Its phase is a direct signature of the broken gauge invariance in the SC condensate. The Pauli principle then requires \( \Delta_{s_1s_2} \) to be antisymmetric under the interchange of particles
\[ \Delta_{s_1s_2} (\mathbf{k}) = - \Delta_{s_2s_1} (-\mathbf{k}). \]  
(19)
In addition, it transforms like a two-fermion wave function under rotations in position and spin space and under gauge transformations. The transformation properties yield a general classification scheme for the superconducting order parameter which is represented by a \( 2 \times 2 \)-matrix in (pseudo-)spin space. It can be decomposed into an antisymmetric \((s)\) and a symmetric \((t)\) contribution according to \( \Delta (\mathbf{k}) = \Delta_s (\mathbf{k}) + \Delta_t (\mathbf{k}) \) with
\[ \Delta_s (\mathbf{k}) = \phi (\mathbf{k}) i \sigma_2 \quad \text{and} \quad \Delta_t (\mathbf{k}) = \sum_{\mu=1}^{3} d_\mu (\mathbf{k}) \sigma_\mu i \sigma_2 , \]  
(20)
where \( \sigma_\mu \) denote the Pauli matrices. Antisymmetry \( \Delta (\mathbf{k}) = - \Delta^T (-\mathbf{k}) \) requires
\[ \phi (\mathbf{k}) = \phi (-\mathbf{k}) \quad \text{and} \quad d_\mu (\mathbf{k}) = - d_\mu (-\mathbf{k}) \]  
(21)
for the complex orbital functions \( \phi (\mathbf{k}) \) and \( d_\mu (\mathbf{k}) \) \( (\mu = 1-3) \). For brevity we will frequently write \( \Delta (\mathbf{k}) \) for \( \phi (\mathbf{k}) \) or \( |d (\mathbf{k})| \). The physical meaning of the order parameters \( \phi \) and \( d \) for singlet and triplet state, respectively, is evident from the identity
\[ \frac{1}{2} \text{Tr} |\Delta_s (\mathbf{k})|^2 = |\phi (\mathbf{k})|^2 \quad \text{and} \quad \frac{1}{2} \text{Tr} |\Delta_t (\mathbf{k})|^2 = |d (\mathbf{k})|^2 . \]  
(22)
Therefore the modulus of \( \phi \) and \( d \) is a measure for the total gap amplitude of the Cooper pairs at a given point \( \mathbf{k} \) on the Fermi surface. In addition, the direction of the vector \( d \) specifies the relative contributions of the three triplet pair states. The above relation holds for triplet states which satisfy \( d^* \times d = 0 \). They are called ‘unitary states’ because they are invariant under time reversal. In this case the vector \( d \) defines a unique direction in spin space for every point on the Fermi surface.
The order parameter can be chosen either as purely antisymmetric $\Delta_s$ or purely symmetric $\Delta_t$ when spin–orbit interaction can be neglected. Then the total spin is a good quantum number and may be used to classify the pair states. Accordingly the states $\Delta_s$ and $\Delta_t$ are called spin singlet and spin triplet states, respectively. In crystals which have an inversion center, pair states can also be classified with respect to their parity as even-parity ($\Delta_s$) and odd-parity states ($\Delta_t$).

In the 4f- and 5f-based heavy fermion superconductors the spin–orbit interaction is strong. As a consequence classification according to physical pair spins cannot be used. If their high-temperature crystal structures, however, have an inversion center then classification according to parity is still possible. We note here that recently the first example of a HF superconductor (CePt$_3$Si) which lacks inversion symmetry was discovered (Bauer et al., 2004) and its theoretical implications were discussed by Frigeri et al. (2004).

The simplest even-parity state is the isotropic state encountered in ordinary superconductors. This state is often referred to as "s-wave state". The isotropic order parameter does not depend on the direction $\khat$ and reduces to a complex constant $\phi = |\phi|e^{i\varphi}$. Its only degree of freedom is the Josephson phase $\varphi$. By far the most extensively studied examples of anisotropic pairing are the p-wave states realized in the superfluid phases of $^3$He, the d-wave pair state in high-$T_c$ superconductors and the f-wave states in UPt$_3$ and SrRu$_2$O$_4$. The odd parity (p, f) states among these examples are characterised by more than one order parameter component with internal phase degrees of freedom which appear in addition to the overall Josephson phase.

The general classification scheme for superconducting order parameters proceeds from the behavior under the transformations of the symmetry group $G$ of the Hamiltonian. It consists of the crystal point group $G$, the spin rotation group $SU(2)$, the time-reversal symmetry group $K$, and the gauge group $U(1)$. The latter two are, respectively, defined by

$$K \Delta(\khat) = \sigma_2 \Delta^*(\khat) \sigma_2 \quad \text{and} \quad \Phi \Delta(\khat) = e^{i\varphi} \Delta(\khat).$$

Concerning rotations in $\khat$- and spin space we distinguish two different cases: (1) If spin–orbit coupling is negligible spatial and spin rotations can be applied independently. For the elements $g \in G$ of the point group which act on $\khat$ one has

$$g \Delta(\khat) = \Delta(D_G^{(-)}(g) \khat),$$

where $D_G^{(-)}(g)$ is the three-dimensional representation of $G$ in $\khat$-space. For the elements $g \in SU(2)$ of the spin rotation group which act on the spin indices we obtain

$$g \Delta(\khat) = D_T(g) \Delta(\khat) D(g),$$

where $D(g)$ denotes the representation of $SU(2)$ for spin $\frac{1}{2}$. This transformation leaves even-parity states invariant. For odd-parity states where the order parameter can be represented by a vector $\mathbf{d}$ in spin space the transformation gives a conventional orthogonal rotation of the $\mathbf{d}$-vector according to

$$g \mathbf{d}(\khat) = D_G^{(+)}(g) \mathbf{d}(\khat).$$
In the presence of spin–orbit interaction the transformations of $\hat{k}$ and the spin rotations are no longer independent. The spins are frozen in the lattice and the operations of the point group amount to simultaneous rotations in $\hat{k}$ space and spin space:

$$g d(\hat{k}) = D_G^{(\pm)}(g) d(\hat{k}) D_G^{(-)}(g).$$

The appropriate choice of rotations corresponding to weak or strong spin–orbit coupling case is determined by microscopic considerations. Using the above transformation properties the singlet and triplet gap functions $\phi(\hat{k})$ and $d(\hat{k})$, respectively, may further be decomposed into basis functions $\phi_n^\Gamma(\hat{k})$ or $d_n^\Gamma(\hat{k})$ of the irreducible representations $\Gamma$ (degeneracy index $n$) of $G \times SU(2)$ (weak s.o. coupling) or $G$ (strong s.o. coupling).

The occurrence of long-range order at a phase transition described by an order parameter is most frequently associated with spontaneous symmetry breaking. The simplest superconductors where only gauge symmetry is broken are called conventional. In this case the SC order parameter has the same spatial symmetry as the underlying crystal, i.e., it transforms as a fully symmetric even parity singlet $A_{1g}$ representation of $G$. It should be noted, however, that conventional is not a synonym for isotropic, for any $G$ one can form $A_{1g}$ representations from angular momentum orbitals of higher order $l$, for example, $l \geq 2$ for tetragonal and hexagonal symmetry and $l \geq 4$ for cubic symmetry. On the other hand, a superconductor with additional broken symmetries besides gauge symmetry is called unconventional. It can have either parity. Any odd-parity SC state which has broken inversion symmetry is unconventional in this sense. From the effective attractive interactions as eq. (12) obtained from model Hamiltonians like eq. (11) containing ‘high energy’ interaction parameters the symmetry can be obtained by directly minimizing the free energy and solving the resulting gap equations. These calculations depend on the model parameters and approximation schemes and cannot make predictions for real compounds.

The usual procedure described in detail in the next section to determine the symmetry of SC order parameters is to select plausible candidate states corresponding to irreducible representations (or mixtures) of the symmetry group, calculate the expected (temperature, field) behavior of physical quantities and compare the predictions with experiment. The selection of candidate states exploits Michel’s theorem (Michel, 1980) according to which stable points of the free energy should correspond to states which are invariant under subgroups of the full symmetry group. However the theorem was only proven for free energy expressions which are polynomial in the order parameter. As long as we work sufficiently close to the transition temperature, where a Ginzburg–Landau expansion is valid the systematic construction of gap functions in terms of basis functions which are invariant under subgroups is a useful guideline. Exhaustive lists of the superconducting classes for the relevant crystal symmetries were given by Volovik and Gorkov (1985), Ueda and Rice (1985), Blount (1985), Gorkov (1987), Sigrist and Ueda (1991), Annett (1990) and Ozaki and Machida (1985), Machida and Ohmi (1998), Machida et al. (1999) and references cited therein. A recent summary is found in the textbook of Mineev and Samokhin (1999).

Quite generally the results as given above can be divided into two groups (1, 2) according to their treatment of spin–orbit interaction. Group (1) treats the orbital and (pseudo-)spin degrees of freedom separately, while group (2) assumes that the spin of the Cooper pair is frozen in.
the lattice. In mathematical terms: (1) assumes the symmetry group to be the direct product $G \times SU(2)$, while (2) considers the point group of the crystal to be the double group. The two schemes yield different predictions for the node structures. Treating the Cooper pair spin or $d$ as frozen in the lattice leads to odd-parity order parameters which vanish only in isolated points on the Fermi surface. This fact was first pointed out by Blount (1985) and is usually referred to as Blount’s theorem. Even-parity states, on the other hand, have gap functions which may vanish on lines and at isolated points on the Fermi surface.

Blount’s theorem seems to rule out odd-parity states in UPt$_3$ at first glance since there is strong evidence for node lines on the Fermi surface. As a consequence, the majority of early order parameter models for UPt$_3$ adopted multicomponent even-parity states. However the anisotropy of thermal conductivity, reversal of upper critical field anisotropy and Knight shift results in UPt$_3$ are better accounted for by an odd-parity order parameter. For an extensive discussion of this problem we refer to sect. 4.1. Another more recent case is UNi$_2$Al$_3$ where evidence for an odd parity state exists. It seems that Blount’s theorem is not respected in real HF superconductors.

Concerning this ambiguity we add the following remark. Doubtlessly spin–orbit interaction plays an important role in U compounds. However, the energy $\xi_{so} \approx 2$ eV associated with this relativistic effect is large compared to the HF quasiparticle band width $k_BT^* \approx 10$ meV. In fact it is even larger than the hybridization energies, leading to separate 5f LDA bands belonging to different total angular momenta $j = 5/2$ and $7/2$ where the latter is almost empty (Albers et al., 1986). With respect to pair formation spin–orbit coupling is therefore to be treated as a high-energy effect. This suggests that spin–orbit coupling should be included already in the properties of the normal state quasiparticle states. The latter can be classified according to the pseudo spins connected with Kramers degeneracy of band states.

The transformation properties of the odd-parity order parameters under spatial rotations is reduced to considering the behavior of the quasiparticle states. To leading order in the small ratio $kT^*/\xi_{so}$ we include the spin–orbit interaction in the calculation of the local atomic basis states, in a second step they are coherently superposed to form extended states. For bands derived from one doubly (Kramers-)degenerate orbital the elements of the point group should act only on the propagation vector. When the Cooper pairs, i.e., the two-particle states, are formed the orbital and spin degrees of freedom can be treated independently. With the spin–orbital interaction already included in the normal state quasiparticles one can use Machida’s states derived for vanishing spin–orbit interaction.

The two different methods in treating the spin–orbit interaction have their counterparts in atomic physics. The scheme put forward by Volovik and Gor’kov parallels Russell–Saunders coupling where the orbital momenta and spins of the individual electrons are coupled to the total orbital momentum $L$ and total spin $S$, respectively. These two quantities are coupled to form the total angular momentum $J = L + S$. Including the spin–orbit interaction in the quasiparticle states, on the other hand, closely parallels $jj$-coupling.

The symmetry classification of density wave order parameters may proceed in a similar way. Charge and spin density are physical observables, hence the CDW/SDW order parameter describe diagonal long range order (DLRO) meaning that they correspond to expectation
values of diagonal elements in the one-particle density matrix according to
\[
\langle r_{s1}| \rho^{(1)} | r_{s2} \rangle \rightarrow A_{s1s2} \exp(i Q r),
\] (28)

where \( Q \) is the wave vector of the density modulation and its amplitude is given by
\[
A_{s1s2} = \sum_k F^{Q}_{s1s2}(k) \quad \text{and} \quad F^{Q}_{s1s2}(k) = \langle c_{k+s1}^\dagger c_{k+Qs2} \rangle.
\] (29)

The electron–hole pair amplitude \( F_Q(k) \) may again be decomposed into singlet (CDW) and triplet (SDW) components \( \rho_Q(k) \) and \( s_Q(k) \), respectively, in analogy to SC pair amplitudes. Their corresponding gap functions are then given by the gap equations (cf. eq. (52))
\[
\Delta_C(k) = -i \langle \rho_Q(k) \rangle \quad \text{and} \quad \Delta_S(k) = -i \langle s_Q^2(k) \rangle.
\] (30)

for CDW and SDW (\( d \) vector along \( z \)), respectively, where \( I \) is the on-site Hubbard interaction. Likewise the \( k \)-dependence of gap functions may be classified according to the irreducible representations of the point group \( G \) with even and odd parity type when it contains the inversion symmetry. Several interesting aspects arise here: (1) Since gauge symmetry is obviously not broken its phase cannot be chosen arbitrarily, it rather is tied to the spatial transformation properties of the representation considered. Commonly the gap functions will either be real or imaginary for commensurate density waves. (2) On the other hand for electron–hole pairing there is no antisymmetry requirement hence the gap function for singlet (CDW) or triplet (SDW) can both be odd or even. (3) Conventional CDW and SDW states correspond to gap functions \( \Delta_C(k) \) and \( \Delta_S(k) \) which transform as fully symmetric (\( A_{1g} \)) representations, in the simplest case they are constants, independent of \( k \). In this case the amplitude of the CDW/SDW density modulation in real space obtained from the summation in eq. (29) is nonzero because the gap function does not change sign. If the gap functions belong to nontrivial representations, e.g., \( d_{x^2-y^2} \) in two dimensions they do change sign and the sum in eq. (29) is zero, i.e., there is no density modulation although there is a ‘hidden’ order parameter \( \Delta_C(k) \) or \( \Delta_S(k) \) which characterizes the electron–hole condensate. Such states have been given the misleading name ‘unconventional density waves’ in analogy to the usage for SC states. These hidden order parameters have recently been under intense discussion for both underdoped cuprates, organic materials and U-HF systems.

2.2.2. Pairing model for coexistence of SC and CDW/SDW
In U-HF compounds the coexistence of SC and SDW order parameters is frequently observed. In a purely itinerant picture this phenomenon is well studied on the basis of a mean field (mf) pairing Hamiltonian which allows for both type of instabilities. It has first been proposed for SC/CDW coexistence (Bilbro and McMillan, 1976) and then for conventional SC/SDW (Machida, 1981; Machida and Matsubara, 1981) coexistence and later generalized to unconventional anisotropic gap functions for both SC and CDW/SDW (Kato and Machida, 1988; Thalmeier, 1994).
The effective mf Hamiltonian for pair formation in both electron–hole (e-h or Peierls) and electron–electron (e-e or Cooper) channels is given by

\[ H = \sum_{k,s} \epsilon_k c_{k,s}^\dagger c_{k,s} + H_{e-h} + H_{e-e}. \]  

Introducing the charge and spin-densities (polarised along \( z \))

\[ \rho_k(Q) = c_{k,\uparrow}^\dagger c_{k,\uparrow} + c_{k,\downarrow}^\dagger c_{k,\downarrow}, \]
\[ s_k(Q) = c_{k,\uparrow}^\dagger c_{k,\downarrow} - c_{k,\downarrow}^\dagger c_{k,\uparrow}, \]

the electron–hole part can be separated in singlet (CDW) and triplet (SDW) part

\[ H_{e-h} = -\sum_k \left[ \Delta_C(k) \rho_k(Q) + \text{h.c.} \right] - \sum_k \left[ \Delta_S(k) s_k(Q) + \text{h.c.} \right]. \]

Likewise the electron–electron part which, for example, is the mf approximation of eq. (12) is given by

\[ H_{e-e} = -\sum_k \Delta_{SC}^{a'}(k) \left[ c_{k,s}^\dagger c_{-k,s'}^\dagger + \text{h.c.} \right]. \]

Here \( \Delta_C(k) \), \( \Delta_S(k) \) and \( \Delta_{SC}^{a'} \) are the CDW, SDW and SC gap functions which in general will depend on the momentum \( k \). The former two simply correspond to spin singlet (CDW) and triplet pairing (SDW) in the e-h channel. Analogously the superconducting gap matrix \( \Delta_{SC}^{a'}(k) \) may be decomposed into singlet and triplet parts described previously.

If only a single nonzero OP is present diagonalisation of \( H \) leads to quasiparticle excitations (\( A \equiv C, S \))

\[ E_k = \left[ \epsilon_k^2 + (\Delta_{SC}^k)^2 \right]^{1/2}; \quad \text{or} \quad E_k = \left[ \epsilon_k^2 + (\Delta_A^k)^2 \right]^{1/2}; \]

in the ordered phase. Here \( \Delta_{SC}^k \) represents \( \phi(k) \) in the singlet case and \( |d(k)| \) in the (unitary) triplet case, respectively. The general case of coexisting order parameters and coupled gap equations is discussed below.

In the spirit of the Landau theory of 2nd order phase transitions one has to assume that each of the gap functions transforms as an irreducible representation of the underlying symmetry group which consists of gauge transformations \( U(1) \), time reversal \( K \), spin rotations \( SU(2) \) and spatial transformations \( G \) as introduced before. For e-h pairing the former is preserved, for e-e pairing it is at least partly broken. If the gap functions belong to a fully symmetric or trivial representation (\( A_{1g} \) or \( T_{2g}^+ \)), i.e., if they are invariant under all spatial symmetry operations (including parity) of \( G \) then the condensed state describes a conventional CDW, SDW or superconductor, for all other representations the pair state is called unconventional as mentioned before. The most important difference in the two cases is due to their generally different nodal structure. Whereas for \( A_{1g} \) in general \( |\Delta_i(k)| > 0 \) (\( i = A, SC \)) and hence quasiparticle excitations are gapped, for the unconventional states frequently zeroes of the gaps \( \Delta_i(k) = 0 \) or \( d(k) = 0 \) on points or lines on the Fermi surface are possible but not necessary. In the SC case one basically has two types of nodes: (i) Symmetry enforced gap nodes which always...
Fig. 7. Left panel: Cooper pairs \((-\mathbf{k}, \mathbf{k})\) and electron–hole (Peierls) pairs \((-\mathbf{k}, -\mathbf{k} + \mathbf{Q})\) for the n.n. tight binding Fermi surface (thick line) with perfect nesting vector \(\mathbf{Q}\). Saddle points \((S)\) of \(\varepsilon(\mathbf{k})\) at \((0, \pm \pi)\) and \((\pm \pi, 0)\) lead to DOS peak at the Fermi energy. Therefore unconventional pair states can only have nodes away from S, i.e., at the ‘Dirac’ points \(D\) \((\frac{\pm \pi}{2}, \frac{\pm \pi}{2})\) where the quasiparticle spectrum takes the form of eq. (54). This is the case for a \(d_{x^2-y^2}\)-type gap function \(\Delta(\mathbf{k})\) which is indicated schematically. Right panel: Corresponding quasiparticle DOS \(N(E)\) \((W =\) tight binding band width) for normal state (dotted) and with \(d_{x^2-y^2}\)-gap with amplitude \(\Delta_0\). The Fermi level is at \(E_F = 0\).

appear when the gap function transforms as an irreducible representation whose remaining symmetry group in the SC phase contains elements composed of spatial transformation and discrete gauge transformations and/or time reversal. (ii) Accidental nodes which depend on the specific choice of basis functions within a single representation or which appear as a result of superposition of different representations (‘hybrid gap functions’). Then gap nodes exist only for special superposition parameters (‘fine tuning’). Such nodes (or at least very large gap anisotropies \(\Delta_{\text{max}}(k_{\text{max}})/\Delta_{\text{min}}(k_{\text{min}})\)) may also be present in conventional superconductors where the hybrid gap function is constructed from fully symmetric representations \(A_1g\) of various degree and only \(U(1)\) symmetry is broken. Both types of nodes have been found and examples will be discussed in later sections. The precise conditions for the symmetry enforced gap nodes are discussed in Mineev and Samokhin (1999). On the other hand in some compounds, for example, borocarbides (sect. 5) and possibly skutterudites (sect. 6) the second type of SC gap nodes is observed. The quasiparticle spectrum of eq. (35) around the nodes defined by \(\Delta_{\text{SC}}^k = 0\) or \(\Delta_{\text{A}}^k = 0\) determines completely the low temperature physics of the ordered state. Knowledge about their position, multiplicity and dimension (points or lines) is therefore essential.

2.2.3. Coupled gap equations and results for coexistence
The interplay between superconductivity and antiferromagnetism has been in the focus of interest for a long time. The coexistence of the two ordering phenomena is well understood in those materials where they occur in different electronic subsystems which, in addition, are
not coherently coupled. Well-known examples are the Chevrel phases RMo$_6$S$_8$, the rhodium borides RRh$_4$B$_4$ or the borocarbides RNi$_2$B$_2$C where the influence of local moments of the lanthanide elements (R) on SC can be modelled by a molecular magnetic field acting on the SC conduction electrons (sect. 5). This results in a weaker pair interaction as compared to the paramagnetic phase and therefore leads to anomalies in the upper critical field $H_{c_2}$ below the onset of magnetic order.

The interplay of superconductivity with itinerant-electron magnetism where both SC and CDW/SDW are carried by conduction electrons continues to be a theoretical challenge. Qualitative phase diagrams reflecting the dominant order parameters and their mutual coexistence or expulsion have been derived within a mf approximation in order to interpret experiments. The concepts developed for transition metal alloys or organic superconductors have been extended to the case of heavy-fermion systems (Kato and Machida, 1988; Konno and Ueda, 1989). The basic assumption is that the long-range order, i.e., superconductivity or spin-density wave results from an instability of the Fermi surface of the strongly renormalized heavy quasiparticles. This assumption implies that we have at least an approximate separation of energy scales with $T_c(T_N) \ll T^*$ where $T_c$, and $T_N$ denote the superconducting and magnetic transition temperatures while $T^*$ is the HF quasiparticle band width.

Kato and Machida (1988) adopt a two-dimensional $(t, t')$-tight binding model for the quasiparticle bands resulting in a Fermi surface that exhibits nesting features with the commensurate wave vector $Q = (\frac{1}{2}, \frac{1}{2}, 0)$. The effective interaction which consists of an on-site repulsion and an attractive pairing leads to both SDW and SC instabilities, the former supported by the nesting feature. The formation of a spin-density wave changes the symmetry of the system. This requires a modified classification scheme for the superconducting order parameters which can appear in a second order phase transition within the SDW state. The presence of both types of order parameters $\Delta_A (A = C, S)$ and $\Delta_{SC}$ may lead to induced SC pair amplitudes with pair momentum $Q$ which strongly influence the coexistence behaviour. The competing order parameters for various symmetry types are determined from coupled gap equations for $\Delta_A(k)$ and $\Delta_{SC}(k)$. In Kato and Machida (1988) the former was taken as constant (conventional SDW) while the latter allowed to belong to any nonconventional representation. In Thalmeier (1994) the coexistence study was generalised to include also unconventional density waves $\Delta_A(k)$. In the general case coexistence, competition as well as expulsion of the two order parameters may be observed.

For the tight binding model described above there are two types of coupled gap equations which are due to the different even/odd transformation properties of $\Delta_{SC}(k)$, $\Delta_A(k)$ under the two discrete transformations $k \rightarrow -k$ (inversion) and $k \rightarrow k + Q$ (translation).

Case (I): there is no induced SC pairing with nonzero wave vector.

Case (II): a finite induced pairing amplitude $\langle c_{k+Q_1} c_k \rangle$ for SC pairs with nonzero pair momentum exists.

Competition of SC and density wave pairs is much stronger in the second case because the induced pairing inevitably leads to a large loss in condensation energy.
In case I for the perfect nesting model \((t' = 0)\) the gap equations are given simply by

\[
1 = V \sum_k \frac{f_{\text{SC}}(k)^2}{2E_k} \tanh \frac{E_k}{2T} \quad \text{and} \quad 1 = I \sum_k \frac{f_A(k)^2}{2E_k} \tanh \frac{E_k}{2T},
\]

where \(E_k = [\epsilon_k^2 + |\Delta_{\text{SC}}|^2 + |\Delta_A|^2]^{1/2}\) is the quasiparticle energy and the form factors \(f_{\text{SC}}(k), f_A(k)\) which correspond to irreducible basis functions of \(D_{4h}\) point group are defined via \(\Delta_{\text{SC}}(k) = \Delta_{\text{SC}}^0 f_{\text{SC}}(k)\) and \(\Delta_A(k) = \Delta_A^0 f_A(k)\). Furthermore interaction constants \(I\) and \(V\) determine whether SDW or SC transition happens first. In HF compounds frequently we have \(T_N > T_c\). In this weakly competitive case coupled gap equations are formally the same as for each of the gap functions individually. They influence each other only through the quasiparticle energies where both gap functions appear. Numerical solution shows that for this case coexistence is always possible and (assuming \(T_N > T_c\)) the transition from SDW to SDW + SC coexistence phase happens always in a second order phase transition, irrespective how small \(T_c\) is.

In case II the gap equations are formally different from individual gap equations:

\[
1 = V \sum_k \frac{f_{\text{SC}}(k)^2}{4|\Delta_{\text{SC}}|^2} \left( \frac{|\Delta_{\text{SC}}^+| + |\Delta_A^+|}{E_k^+} \tanh \frac{E_k^+}{2T} + \frac{|\Delta_{\text{SC}}^-| - |\Delta_A^-|}{E_k^-} \tanh \frac{E_k^-}{2T} \right),
\]

\[
1 = I \sum_k \frac{f_A(k)^2}{4|\Delta_A|^2} \left( \frac{|\Delta_A^+| + |\Delta_{\text{SC}}^+|}{E_k^+} \tanh \frac{E_k^+}{2T} + \frac{|\Delta_A^-| - |\Delta_{\text{SC}}^-|}{E_k^-} \tanh \frac{E_k^-}{2T} \right),
\]

where the two quasiparticle bands \(E_k^\pm = [\epsilon_k^2 + (|\Delta_{\text{SC}}^\pm| \pm |\Delta_A^\pm|)^2]^{1/2}\) are now split due to the presence of two order parameters. Numerical solutions for case II show a much more competitive behaviour of gap functions \(\Delta_A\) and \(\Delta_{\text{SC}}\). If \(T_c\) is comparable in size to \(T_N\) the SDW state may be destroyed at \(T_c\) and replaced by the SC phase in a first order phase transition. For smaller \(T_c\) again a coexistence state SDW + SC appears in a second order phase transition but generally the order parameters in this regime show strong competition resulting in nonmonotonic temperature behaviour of gap functions.

The coexistence/competition behaviour of \(\Delta_A(k)\) and \(\Delta_S(k)\) is therefore quite different in cases I and II and it also depends strongly on the relative nodal structure of \(\Delta_A(k)\) and \(\Delta_S(k)\). Generally speaking if their nodes are on the same positions on the FS (or if at least one of them is a conventional gap function without nodes) this enhances competition, nonmonotonic temperature behaviour and tendency to expulsion of one OP. On the other hand an ‘orthogonal’ nodal structure for both (e.g., \(d_{xy}\) symmetry for SDW and \(d_{x^2-y^2}\) for SC) supports coexistence because the two gaps become maximal at different parts of the FS and this also favors monotonic temperature dependence of the gap amplitudes.

To illustrate this behaviour we briefly discuss an example which may be relevant to CeCu_2Si_2. Qualitatively the complex phase diagram of CeCu_2Si_2 can be described by a simple mean-field model for a conventional spin density wave ground state (competing with \(d\)-wave superconductivity within a two band model proposed by Dahm (2001)). The calculated phase diagram displayed in fig. 8 shows that a state with \(d_{xy}\) symmetry can become stable within the SDW phase. This is in contrast to the pairing states with \(d_{x^2-y^2}\) symmetry considered by Kato.
Fig. 8. Left panel: Experimental results for coexistence and competition between superconductivity and SDW in CeCu$_2$(Si$_{1-x}$Ge$_x$)$_2$ (Gegenwart et al., 1998). Ge substitution acts like a (negative) chemical pressure. Here $A =$ SDW and SC is the superconducting state. $T_A$ depends linearly on chemical pressure, hence $g \sim 1 - x$. Right panel: Two band model for $A$ and d-SC order in mf theory from Steglich et al. (2001). Here $A$ denotes a conventional SDW state with constant $\Delta_A$ and d-SC has $\Delta_{SC}(k) = \Delta_{SC0} \sin k_x \sin k_y$: $A + SC$ denotes the coexistence region. Solid and dashed lines correspond to second and first order transitions, respectively. The effective tight-binding hopping integral is $t_{eff}$, it should also scale linearly with $1 - x$. Inset: FS topology with two bands. Dash-dotted line indicates location of SDW gapping.

and Machida (1988) in a one band model. The experimentally observed unusual stability of the $(A + SC)$ phase might follow from the peculiar Fermi surface of the heavy quasiparticles displayed in fig. 18. This is suggested by model calculations which start from a more complex Fermi surface model. Assuming two well-separated sheets centered at $\Gamma$ and $M$, respectively, allows for a solution where the SDW gaps exist only on part (centered around $\Gamma$) of the Fermi surface while the quasiparticles on the remaining parts (centered around $M$) condense into a superconducting state. The calculated temperature–‘pressure’ phase diagram (assuming that the electron band width $\sim t_{eff}$ scales linearly with pressure) is qualitatively in agreement with experimental results (Gegenwart et al., 1998) for CeCu$_2$Si$_2$. Indeed the $A$-phase has now been identified as conventional SDW state with an incommensurate propagation vector $Q = (0.22, 0.22, 0.55)$ and a moment of $0.1 \mu_B$ per Ce-atom by neutron diffraction experiments (Stockert et al., 2004). The experimentally determined magnetic structure therefore requires a more refined treatment of the SDW/SC coexistence behaviour.

2.3. Methods to investigate the symmetry of order parameters

Phase transitions with the appearance of a spontaneous symmetry breaking are characterised by order parameters. In the spirit of Landau theory they belong to representations of the high temperature symmetry group and also characterise the class of remaining symmetries in the low-temperature phase. Furthermore they partly determine the type of possible excitations in the ordered phase which in turn influence the low-temperature thermodynamic and transport properties. The investigation of the order parameter symmetry is therefore of singular
importance. In the case of order parameters corresponding to expectation values of physical observables like spin- and charge densities this is in principle straightforward as they reveal their existence and symmetry in X-ray and neutron diffraction experiments. However for ‘hidden’ order, e.g., quadrupole ordering or ‘unconventional’ density waves (orbital antiferromagnetism) there is no large obvious signature in diffraction experiments and evidence for its presence has to be obtained indirectly by other means. The superconducting order parameter does not correspond to the expectation value of a classical observable but rather to the appearance of quantum mechanical phase rigidity of Cooper pairs on a macroscopic scale (the coherence length). For unconventional superconductors determination of the gap function dependence on the momentum of paired states is of fundamental importance. It is also extraordinarily difficult. The classical experimental techniques which we discuss in this section rely on the interpretation of temperature dependences of physical quantities obtained by averaging over the gap function momentum dependence and, therefore, the conclusion about their proper symmetry is always ambiguous. Recently a much more powerful method relying on the field angle resolved measurement of specific heat and thermal conductivity in the vortex phase for $T \ll T_c$ has been developed. It leads directly to the determination of the relative position of node lines and/or points of the SC gap function with respect to crystal axes. This leads to a strong reduction of the number of possible SC order parameters and possibly allows a unique determination. Therefore we devote an extra subsection to this new technique. On the other hand we shall not discuss phase sensitive methods based on Josephson tunneling which have been unsuccessful in HF compounds and point contact spectroscopy or $\mu$SR methods due to their difficult interpretation. For unconventional density waves which also correspond to gap functions of nontrivial momentum dependence in the particle–hole channel, the field angle dependent investigation of magnetoresistance has similar potential but is less developed.

2.3.1. Detection of superconducting order parameter symmetry

In conventional electron–phonon superconductors with an almost isotropic gap numerous thermodynamic, static and dynamic transport measurements and also resonance methods can give information about the size of the superconducting gap. The SC transition affects these quantities in two ways: (i) the quasiparticle DOS exhibits a gap $\Delta$ with a square root singularity of the DOS: $N_s(E) = N_0 E / (E^2 - \Delta^2)^{1/2} \ (E \geq \Delta)$, (ii) coupling of external probe fields to the quasiparticles involves a coherence factor $1/2 \left( 1 \mp \Delta^2 / EE' \right)$ where $\mp$ corresponds to perturbations even or odd under time reversal, respectively. In the former case (I) the DOS singularity and the vanishing coherence factor for $E \simeq \Delta$ compensate leading to a steep drop of the corresponding physical quantity, e.g., of ultrasonic attenuation below $T_c$. In the latter case (II) the coherence factor is unity and therefore a Hebel–Slichter type anomaly as, e.g., in the NMR-relaxation rate develops below $T_c$. At low temperatures ($T \ll T_c$) invariably experimental quantities determined by electron–hole excitations tend to zero exponentially due to the finite superconducting gap. Therefore the most obvious and seemingly easiest method to look for evidence of unconventional superconductivity is the search for deviations from the exponential low-temperature dependence. This strategy relies on the fact that nontrivial (unconventional) $k$-dependent gap functions $\Delta(k)$ in many cases (though not always) exhibit node points or lines on the Fermi surface leading to a nonvanishing $N_s(E)$ for all
energies \( E > 0 \) even in the superconducting state. The asymptotic behaviour of \( N_s(E) \) for \( E/\Delta \ll 1 \) is \( \sim E^2 \) for (first order) point nodes and \( \sim E \) for line nodes. Then this should ideally lead to ‘power law’ behaviour \( \sim T^n \) for physical quantities like specific heat, penetration depth, NMR relaxation rate and many others for \( T \ll T_c \). The experimentally observed exponent \( n \) would then allow to determine whether \( \Delta(\mathbf{k}) \) has point nodes \((n = 3)\), line nodes \((n = 2)\) or both if the exponent \( n \) is in between, or whether gapless regions of the Fermi surface \((n = 1)\) exist. For nondirectional quantities like \( C(T) \) this would however say nothing about the position of nodes relative to the crystal axes. Unfortunately this picture is so oversimplified as to make it useless. Firstly, experimental determination of the exponent \( n \) very often includes temperatures where \( T \ll T_c \) is not valid and more importantly impurity scattering in anisotropic superconductors has dramatic effects (Sigrist and Ueda, 1991; Fulde et al., 1988). In conventional superconductors Anderson’s theorem ensures that the low temperature thermodynamic properties are not affected by impurity scattering. On the other hand normal impurities for unconventional pairs act strongly pairbreaking. This is especially important in HF compounds where the normal state electrons are already strongly correlated and impurities act as unitary scattering centers. Their effect for anisotropic superconductors has originally been studied by Buchholtz and Zwicknagl (1981) for p-wave states and later extended and applied (Hirschfeld et al., 1986; Scharnberg et al., 1986; Schmitt-Rink et al., 1986; Hotta, 1993; Sun and Maki, 1995) for other cases like d-wave. In the unitary scattering limit a resonant residual quasiparticle DOS at low energies develops (Hirschfeld et al., 1986) invalidating the abovementioned power laws. On the other hand hybrid nodal superconductors like \( s + g \) wave type in borocarbides exhibit the opposite behaviour: Impurity scattering immediately leads to the opening of a gap (Yuan et al., 2003) with resulting low temperature exponential behaviour. It is therefore more reasonable to investigate the \( T \)-dependence of physical quantities depending on the gap anisotropy for the whole temperature range below \( T_c \) and compare with experiments, rather than looking at the often ill-defined and contradictory low-\( T \) power laws. This will be discussed now in a few cases.

### Specific heat:

The specific heat in the anisotropic SC state is given by an expression formally identical to the s-wave case:

\[
C_s = 2k\beta \sum_{\mathbf{k}} \left( -\frac{\partial f_{\mathbf{k}}}{\partial E_{\mathbf{k}}} \right) \left[ E_{\mathbf{k}}^2 + \frac{1}{2} \beta \frac{\partial \Delta_{\mathbf{k}}^2}{\partial \beta} \right] \quad \text{and} \quad \frac{\Delta C_s}{C_n} = \frac{3}{2} \frac{8 \zeta(3)}{\zeta(4)} \frac{\langle f_{\mathbf{k}}^2 \rangle_{FS}}{\langle f_{\mathbf{k}}^4 \rangle_{FS}}, \tag{38}
\]

where \( \beta = 1/kT \), \( E_{\mathbf{k}} = (\epsilon_{\mathbf{k}}^2 + \Delta_{\mathbf{k}}^2)^{1/2} \), \( f_{\mathbf{k}} = f(E_{\mathbf{k}}) \) is the Fermi function and \( C_n = \gamma T \). The specific heat jump given by the second formula has the BCS value \( \Delta C_s/C_n = 1.43 \) for the isotropic s-wave gap. In fig. 9 model calculations for \( C_s/\gamma T \) by Hasselbach et al. (1993) for all possible gap symmetries in tetragonal D_{4h} group are shown and compared with experimental results for URu_2Si_2. The latter show remarkably linear behaviour over the whole experimental temperature range. It is obvious that a perfect fit is not possible with any gap representation and at low \( T \) a tendency for saturation due to an impurity induced residual DOS is visible. A comparison of theoretical and experimental \( \Delta C(T_c) \) jumps however favors either a \( E_u(1,1) \) state \( \Delta(\mathbf{k}) = \Delta(k_x + k_y)^2 \) or \( B_{1g} \) state \( \Delta(\mathbf{k}) = \Delta(k_x^2 - k_y^2)^2 \) for which one has \( 2\Delta(0)/kT_c = \ldots \)
Fig. 9. Calculated specific heat for various superconducting order parameters in comparison to experimental results for URu$_2$Si$_2$ (Hasselbach et al., 1993). The specific heat jump $\Delta C_s(T_c)$ agrees best for $E_{u}(1,1)$ or $B_{1g}$ state.

Table 1

<table>
<thead>
<tr>
<th>Order Parameter</th>
<th>Isotropic</th>
<th>Axial</th>
<th>Polar</th>
<th>$E_{1g}$ ($D_{6h}$)</th>
<th>$E_{2u}$ ($D_{6h}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(\theta, \phi)$</td>
<td>1</td>
<td>sin $\theta$</td>
<td>cos $\theta$</td>
<td>2 sin $\theta$ cos $\theta$</td>
<td>$a$ sin$^2$ $\theta$ cos $\theta$</td>
</tr>
<tr>
<td>$2\Delta(0)/kT_c$</td>
<td>3.52</td>
<td>4.06</td>
<td>4.92</td>
<td>4.22</td>
<td>4.26</td>
</tr>
<tr>
<td>$\Delta C_s/C_n$</td>
<td>1.43</td>
<td>1.19</td>
<td>0.79</td>
<td>1.00</td>
<td>0.97</td>
</tr>
</tbody>
</table>

The presence of nodes reduces $T_c$ compared to s-wave case for the same gap amplitude the BCS ratio (table 1) is always larger for anisotropic SC. However for real compounds this increase may also be partly due to strong coupling effects. Likewise the ratio $\Delta C_s/C_n$ is always maximal for isotropic SC.

Thermal conductivity: The thermal conductivity tensor is a directional quantity depending on the orientation of temperature gradient and heat current. At low temperatures its anisotropy may contain important information on the relative position of nodal points or lines with respect to crystal axes. For example, this has proved decisive in the identification of the $E_{2u}$ SC order parameter of UPt$_3$ (sect. 4.1). The uniaxial thermal conductivity tensor in the unitary scattering limit for a spherical Fermi surface is explicitly given (Norman and Hirschfeld, 1996; Kübert and Hirschfeld, 1998; Machida et al., 1999; Graf et al., 2000; Wu and Joynt, 2002) as

$$\frac{k_i(T)}{k_n} = \frac{9}{2\pi^2 T_c} \int_0^\infty d\omega \left(\frac{\omega}{T}\right)^2 \frac{\text{sech}^2 \frac{\omega}{2T} K_i(\omega, T)}{K_1(\omega, T)},$$

(39)

$$K_i(\omega, T) = \frac{\langle \hat{k}^2_i \text{Re}[\omega^2 - |\Delta_k(T)|^2]^{1/2} \rangle}{\langle \text{Re}[\omega^2 - |\Delta_k(T)|^2]^{1/2} \rangle},$$

where $\hat{k}_i = (\hat{i} \cdot \hat{k}) = \cos \theta_i$ and $\hat{i}$ are the unit vectors along $x, y, z$ axes.

At low temperatures the $\mathbf{k}$-averaging in $k_i(T)$ is determined only by the quasiparticles close to the nodal region. Because of the direction cosines in the averages of eq. (39) coming from the quasiparticle velocities, $k_i$ depend on the position of the nodal points or lines with respect
to the axis $\hat{\mathbf{i}}$. When the nodal direction in $\mathbf{k}$-space is parallel to the direction $\hat{\mathbf{i}}$ of the heat current a large contribution to $\kappa_i(T)$ is obtained and a small one if it is perpendicular. This may lead to a uniaxial thermal conductivity anisotropy characterised by the ratio (Norman and Hirschfeld, 1996)

$$\frac{\kappa_c(0)}{\kappa_a(0)} = \lim_{\omega \to 0} \frac{\text{Re}(\hat{k}_i^2[\omega^2 - |\Delta_\mathbf{k}(T)|^2]^{1/2})}{\text{Re}(\hat{k}_l^2[\omega^2 - |\Delta_\mathbf{k}(T)|^2]^{1/2})}. \tag{40}$$

As discussed in sect. 4.1 this anisotropy ratio of thermal conduction has proved quite useful in the identification of the order parameter symmetry in $\text{UPt}_3$ (see fig. 31).

**Ultrasonic attenuation:** Sound attenuation has in addition to the propagation direction ($\hat{\mathbf{q}}$) another polarization ($\hat{\mathbf{e}}$) degree of freedom and may therefore give even more information on the gap nodes than the thermal conductivity. In HF metals sound attenuation can generally be considered in the hydrodynamic limit where $\omega \tau, q l \ll 1$ with $q, \omega$ denoting wave number and frequency ($\sim 100$ MHz) of the sound wave respectively and $l = v_F \tau$ is the mean free path in the normal state. In this case conduction electrons act like a viscous medium to the sound waves and the attenuation coefficient may be expressed as a correlation function of the electronic stress tensor (Tsuneto, 1961; Kadanoff and Falko, 1964). Its evaluation for unconventional SC states using a quasiclassical approximation leads to (Machida et al., 1999; Graf et al., 2000)

$$\frac{\alpha_{ij}(T)}{\alpha_n} = \frac{1}{2T} \int_0^\infty d\omega \text{sech}^2 \frac{\omega}{2T} A_{ij}(\omega, T),$$

$$A_{ij}(\omega, T) = \frac{1}{\Pi_{ij}} \left\{ \frac{1}{\omega} \text{Re} \left[ \omega^2 - |\Delta_\mathbf{k}(T)|^2 \right]^{1/2} \right\}, \tag{41}$$

$$\Pi_{ij} = \hat{k}_i \hat{k}_j - \frac{1}{3} \delta_{ij}.$$

The projection factor $\Pi_{ij}$ ($i = \text{polarisation}, \ j = \text{propagation direction}$) determines which of the quasiparticles having a $\mathbf{k}$-vector described by polar and azimuthal angles $\vartheta, \varphi$ contribute strongly to the attenuation. Consider two cases with $ij = ab$ where $\Pi_{ij}^2 \sim \sin^2(2\varphi)$ and $ij = ac$ with $\Pi_{ij}^2 \sim \sin^2(2\vartheta)$. The quasiparticle contribution to $\alpha_{ij}(T)$ therefore vanishes for nodal directions $\varphi = n(\pi/2)$ and $\vartheta = n(\pi/2)$, respectively. For example, if the gap has a node line in the basal plane ($\vartheta = 0, \pi$) it gives maximal contribution to the attenuation for $ij = ab$ but minimal contribution for $ij = ac$. The same argument holds for point nodes on the $c$-axis where $\vartheta = 0, \pi$. This situation is indeed realized in the $E_{2u}$ type $B$-phase of $\text{UPt}_3$ (sect. 4.1) and one can see in fig. 10 that the attenuation for $ac$ configuration is much smaller than for $ab$. The anomalies around the split $T_c$ are complicated and analyzed in Graf et al. (2000). Therefore polarization and propagation dependence of ultrasonic attenuation is a powerful method to distinguish between gap representations with different nodal structure. Finally it should be mentioned that collective modes in unconventional superconductors hardly contribute to the attenuation since one has an off-resonance situation due to $v_s \ll v_F$ (Kee et al., 2000).
NMR relaxation and Knight shift: In an external field nuclear spins exhibit a Larmor precession with a frequency $\omega_0$. In metals this is modified by the hyperfine coupling to conduction electrons which leads to spin-flip processes as witnessed by the NMR-relaxation rate $T_1^{-1}$ and the Knight shift of the resonance frequency $\delta\omega_0$. A review of these important effects for HF superconductors is given by Tou et al. (2003). The relaxation rate $T_1^{-1}$ is determined by the availability of resonant electron–hole excitations. In the normal state this leads to the Korringa law $T_1^{-1} \sim T$. In the superconducting state the presence of $\Delta(k)$ should lead to a faster decrease with temperature depending on the type of node structure, i.e., the low energy behaviour of the quasiparticle DOS. According to Sigrist and Ueda (1991) for singlet pairs one has

$$\frac{T_{1n}}{T_1} = 2 \int_0^\infty dE \left( -\frac{\partial f}{\partial E} \right) N_s(E) N_s(E + \omega_0) \left[ 1 + \frac{|\langle \Delta_k \rangle_{FS}|^2}{E(E + \omega_0)} \right].$$

For s-wave pairs the type II-coherence factor together with the singular DOS leads to the appearance of the Hebel–Slichter peak. Because for unconventional pair states the Fermi surface average in eq. (42) vanishes, there is no difference between type I and II coherence factors and the relaxation rate is determined by the DOS alone. Because in the presence of nodes there is no divergence in $N_s(E)$ no Hebel–Slichter peak will appear below $T_c$. Its absence may therefore be taken as a sign of an unconventional pair state. Furthermore the low temperature behaviour of $T_{1n}^{-1}$ should be $\sim T^3$ for line nodes and $\sim T^5$ for point nodes. Invariably the former is observed in HF systems, often in conflict with ‘power laws’ of other quantities. A puzzling feature is the absence of any crossover for $T \ll T_c$ to Korringa behaviour in the impurity dominated gapless regime.

In addition to the relaxation the hyperfine interaction leads to a Knight shift of the resonance frequency given by

$$\delta\omega_0(T) \sim |\psi_0|^2 \chi_s(T) H,$$

where $\psi_0$ is the conduction electron wave function at the nucleus and $\chi_s(T)$ is the spin susceptibility of conduction electrons. Without spin–orbit coupling present in an s-wave singlet state...
this drops to zero exponentially below $T_c$, leading to a pronounced $T$-dependence of $\delta\omega_0$. For a triplet SC state the condensate spin or the $\mathbf{d}$-vector of the pair can be freely rotated and $\chi_s(T)$ should not change leading to $T$-independent $\delta\omega_0$. Therefore the existence or non-existence of a $T$-dependent Knight shift below $T_c$ is usually interpreted as direct evidence for singlet or triplet superconductivity, respectively. In the former case the Knight shift should be proportional to the Yoshida function given by the FS average

$$Y(T) = \left( \frac{1}{2T} \int_0^\infty \frac{d\epsilon}{\cosh^2 \frac{\epsilon}{2T}} \right)_{\text{FS}}$$

with a low $T$ behaviour $Y(T) \sim T^2$ for point nodes and $\sim T$ for line nodes. Unfortunately the situation may be completely changed under the presence of strong spin–orbit (s.o.) coupling, for both the conduction electrons themselves and for the conduction electron-impurity scattering. If the mean free path due to conduction electron-impurity s.o. scattering is much smaller than the coherence length, the Knight shift variation with temperature approaches zero also for singlet pairing (Abrikosov, 1988). On the other hand in a triplet pair state a conduction electron s.o. coupling may lead to the pinning of the triplet pair $\mathbf{d}$-vector along one of the crystal axis. Then the $T$-dependence of $\delta\omega_0$ should be very anisotropic, vanishing for $\mathbf{H} \perp \mathbf{d}$ and large for $\mathbf{H} \parallel \mathbf{d}$. A moderately anisotropic Knight shift is indeed found in UPt$_3$ (sect. 4.1). These complications make a quantitative analysis as for the previous quantities difficult, in fact it is rarely performed for HF compounds. The arguments for or against singlet or triplet pairing using Knight shift results should therefore be taken with caution.

**Upper critical field:** The temperature dependence and anisotropy of the upper critical field $H_{c2}$ may contain important information both about the question of singlet vs. triplet pairing and the anisotropy of the gap function $\Delta(\mathbf{k})$. A general discussion of the problem within the semiclassical approach has been given in Rieck et al. (1991). Theoretical analysis in the context of a generalized Ginzburg–Landau theory depends very much on the crystal symmetry and order parameter model. It will be briefly discussed in sect. 4.1 for UPt$_3$. In this case the importance of uniaxial anisotropy to decide between singlet and triplet pairing has been realized (Choi and Sauls, 1991; Sauls, 1994; Yang and Maki, 1999). In a pure isotropic superconductor the upper critical field $H_{c2}(0)$ is limited by orbital effects leading to

$$\mu_B H_{c2}(0) = 0.693 T_c |dH_{c2}/dT|_{T_c}.$$  

The actual critical field may be much lower caused by the effect of paramagnetic ‘Pauli-limiting’ due to the Zeeman energy of the Cooper pair. If the field is larger than $\mu_B H_P = \Delta_0/\sqrt{2} = 1.25 T_c$ the upper critical field is limited by the breaking up of singlet Cooper pairs. According to eq. (45) a large critical field slope should also lead to a larger $H_{c2}(0)$. In UPt$_3$ the opposite is true leading to a ‘crossing’ of $H_{c2}(T)$ curves for $a$- and $c$-directions. This led to the proposal that $H_{c2}(0)$ is strongly reduced by Pauli-limiting effects for field along $c$ but not along $a$. This can only be explained by assuming triplet pairing with strongly pinned $\mathbf{d}$-vector parallel to $c$. As will be discussed in sect. 4.1 this interpretation is not undisputed. In addition to the question of the spin state a nontrivial $\mathbf{k}$-dependence of the gap should also lead to anisotropies in the upper critical field (slope) beyond those due to effective mass anisotropies.
This proved difficult or inconclusive for HF superconductors investigated so far (Keller et al., 1995).

2.3.2. Specific heat and magnetotransport in the vortex phase: a genuine angular resolved method

The classical methods to determine $\Delta(k)$ discussed above all suffer under the same major deficiency: the angular dependence, or at least nodal positions in k-space are never determined directly. Instead the physical quantities investigated are obtained by averaging over the quasiparticle and hence gap energies in k-space. And then it is hoped that the averaged quantities still show clear signatures depending on the gap symmetry. Naturally this leads to ambiguous results for any one method and only a comparison of all gap models for a number of methods may lead to a conclusion on the gap symmetry.

However, recently a new and powerful method has been established that is able to locate directly the nodal positions of the SC gap in k-space (Izawa et al., 2001, 2001a, 2002). In this method the field-angle dependence of specific heat or thermal conductivity is investigated. Typical oscillations are observed whose periodicity, phase and shape give direct information on the type of nodes (point- or line-like) and their direction in k-space. Therefore possibilities for allowed gap functions can be much more restricted. Notice however that still the full k-dependence of $\Delta(k)$ cannot be determined.

At the heart of this method is the ‘Volovik effect’ in the vortex state of the superconductor, it was first proposed by Volovik (1993) for d-wave superconductors. There it was shown that for nodal gap functions a continuum of quasiparticle states exists outside the vortex cores which dominate the specific heat and can easily carry a thermal current even perpendicular to the vortex lines. These states are due to quasiparticles channeling out of the vortex core region through the nodal points or lines. This is an important new aspect as compared to conventional nodeless gap functions where one has only bound states in the vortex core. For the quasiclassical limit $\xi_0 k_F \gg 1$ they also form a quasi-continuum but they can carry a heat current only parallel to the vortex direction. The theory of magnetothermal transport has subsequently been developed by many authors (Barash et al., 1997; Kübert and Hirschfeld, 1998; Vekhter et al., 2001; Won and Maki, 2000; Dahm et al., 2000; Won and Maki, 2001a, 2001b). The Volovik effect can nicely be explained within the quasiclassical picture where momentum (k) and position (r) coordinates commute and the quasiparticle energy $E(k, r)$ and occupation $f(k, r)$ depend on both variables. The dependence on r comes from the fact that the energy of quasiparticles channeling into the inter-vortex region gets Doppler shifted due to the r-dependent superfluid velocity field $v_s(r)$. This leads to $E(k, r) = E(k) - v_s(r) \cdot k$. Here $E(k)$ is the zero-field quasiparticle energy and the second term is the Doppler shift energy. This position dependent shift leads to a finite residual DOS which will depend both on field magnitude and direction:

$$\frac{N_s(E, H)}{N_n} \simeq \frac{1}{\Delta} \left\langle \frac{1}{|E - v_s(r) \cdot k|} \right\rangle.$$  \hspace{1cm} (46)

The double average is performed both over the velocity field coordinate r and the quasiparticle momentum k. It will depend on the direction of the magnetic field $H(\theta, \phi)$ with respect to
the nodal directions ($\theta$ and $\phi$ are polar and azimuthal angles of $\mathbf{H}$ with respect to the $c$-axis). An illustration of this direction dependence is given in fig. 11. The variation of the field angles ($\theta, \phi$) is therefore expected to lead to a periodic variation in $N(E, \theta, \phi)$ and hence in the specific heat and thermal conductivity components. Oscillations will be strongest (i) in the low temperature limit $T \ll \tilde{v}\sqrt{eH} \ll \Delta(0)$ when only quasiparticles from the nodal regions contribute to the residual DOS and (ii) if the node structure is not smeared to much by impurity scattering, i.e., one is in the ‘superclean limit’ with $(\Gamma\Delta)^{1/2} \ll \tilde{v}\sqrt{eH}$ where $\Gamma$ is the impurity scattering rate and $\tilde{v} = \sqrt{v_a v_c}$ with $v_a$ and $v_c$ giving the Fermi velocities along $a$- and $c$-axis in uniaxial symmetry. This limit can also be expressed as $\Gamma/\Delta \ll H/H_c \ll 1$.

For hybrid nodal gaps such as $s + g$ wave gap in the borocarbides the superclean condition should be replaced by $\Gamma \ll T \ll \tilde{v}\sqrt{eH}$ due to the completely different effect of impurities in this case (sect. 5). Including the effect of impurity scattering one obtains for the residual DOS at the Fermi level (Won and Maki, 2000):

$$\frac{N_s(0, \mathbf{H})}{N_n} = \text{Re}[g(0, \mathbf{H})],$$

$$g(0, \mathbf{H}) = \left(\frac{C_0 - ix}{\sqrt{(C_0 - ix)^2 + f^2}}\right).$$

$$C_0 = \frac{\Gamma}{\Delta g(0, \mathbf{H})}.$$

Here $f(k) = \Delta(k)/\Delta$ and $x = |v_s \cdot \mathbf{k}|/\Delta$ are the normalized gap function and Doppler shift energy, respectively, while $C_0$ is a normalized scattering rate. The above condition for the superclean limit can be expressed as $C_0 \ll x$.

The averaging is performed by restricting the $\mathbf{k}$-integration to the node region (since $T \ll \Delta$) and approximating the $\mathbf{r}$-integration over the superfluid velocity field of the vortex lattice by that of a single vortex (assuming $H \ll H_c$) with cutoff $d = 1/\sqrt{eH}$ given by the half-distance between two vortices (Won and Maki, 2001a). It is important to note that the averaging of the Doppler shift energy has to be done simultaneously in $\mathbf{r}$ and $\mathbf{k}$. The field (angle-)dependent specific heat $C_s$, spin susceptibility $\chi_s$ and superfluid density are then given by

$$\frac{C_s(\mathbf{H})}{\gamma T} = \frac{\chi_s(\mathbf{H})}{\chi_n} = g(0, \mathbf{H}) \quad \text{and} \quad \frac{\rho_s(\mathbf{H})}{\rho_s(0)} = 1 - g(0, \mathbf{H}).$$

Fig. 11. Illustration of the Volovik effect (Vekhter et al., 2001). $\mathbf{k}_i$ ($i = 1$–4) denote the nodal directions of the gap. Left: Field $\mathbf{H}$ along nodal direction where $v_s$ is orthogonal to $\mathbf{k}_2$, $\mathbf{k}_4$; these nodal directions do not contribute to the Doppler shift, the residual DOS will be at minimum. Right: Field along antinodal direction, all $\mathbf{k}_i$ ($i = 1$–4) directions contribute to the Doppler shift, the residual DOS is maximal.
The explicit expression for \( C_s(H, \theta, \phi) \) depends on the node structure of \( \Delta(k) \) and is obtained as

\[
C_s(H, \theta, \phi)/\gamma T = \frac{\tilde{v} \sqrt{eH}}{\Delta} I(\theta, \phi).
\]  

(49)

Where for example for the d-wave order parameter with \( f(k) = \cos(2\phi) \) and two orthogonal line nodes parallel to \( c \) one has for planar magnetic field (Won and Maki, 2000)

\[
I\left(\frac{\pi}{2}, \phi\right) \approx 0.95 + 0.028 \cos(4\phi).
\]  

(50)

Thus the specific heat should exhibit a four-fold oscillation with field rotation in the \( ab \)-plane with a minimum in the nodal and the maximum in the anti-nodal direction. The amplitude of the oscillation is not large because of the \( k \)-averaging in eq. (47) along the node line. It may be larger for a gap function with point nodes as in the case of borocarbides which is discussed in detail in sect. 5. This has been experimentally investigated by Park et al. (2003) as shown in fig. 12. It also shows the typical cusp-like minima of point nodes as opposed to smooth minima observed for gap functions with line nodes. The smoothing is also due to the additional averaging along the line node. A most significant result is the observed \( \sqrt{H} \) dependence of the specific heat magnitude which is also a fingerprint of nodal superconductivity (fig. 55). However one must keep in mind that this should be observed for \( H \ll H_c^2 \), a global approximate behaviour over the whole field range as sometimes found in conventional superconductors has no relation to the above effect.

Similar field-angle dependence of thermal conductivity can be calculated starting from the Ambegaokar–Griffin formula generalized to anisotropic gap functions. In the d-wave case one obtains in the superclean limit (Won and Maki, 2000).

\[
\frac{\kappa_{xx}}{\kappa_n} = \frac{2}{\pi} \left\langle x \right\rangle^2 = \frac{2 \tilde{v}^2 eH}{\Delta^2} I\left(\frac{\pi}{2}, \phi\right)^2.
\]  

(51)

Due to the magnetic field there is also an off-diagonal ‘thermal Hall conductivity’ \( \kappa_{xy} \) which has been identified in high-\( T_c \) compounds (Ocana and Esquinazi, 2002). The four-fold oscil-
lations in $\kappa_{xx}(\phi)$ have been found in the HF superconductor CeCoIn$_5$ with node directions $\phi = 0, \pi/2$ corresponding to maxima in $\kappa_{xx}(\phi)$ in the anti-nodal directions $\phi = \pm \pi/4$ as shown in fig. 13. This lead to the proposal of a $d_{x^2} - y^2$ gap functions as in high-$T_c$ compounds. For this geometry, where both field and heat current are in the $ab$-plane, there is also a term that shows a two-fold rotation $\sim \cos(2\phi)$ which is possibly due to the heat current contribution of vortex core states. This contribution is zero or finite for heat current perpendicular or parallel to vortex lines, respectively. Therefore a more favorable geometry is to rotate the field around the heat current direction, e.g., measuring $\kappa_{zz}$ as function of the azimuthal angle for a given polar angle $\theta$. In this case only oscillations due to the nodes should appear. Various examples will be discussed later (sects. 4.1, 5 and 6).

Finally we note that the field angle-oscillations in specific heat and thermal conductivity cannot be explained by normal state Fermi surface effects. Calculations based on the Kubo for-

2.3.3. Detection of density wave type order parameters

Density wave order parameters are of ‘diagonal long range order’ (DLRO) type and corre-

Fig. 13. Thermal conductivity $\kappa_{xx}(\phi)$ for field in the $ab$-plane as function of the azimuthal angle $\phi$ for CeCoIn$_5$ (Izawa et al., 2001a). A four-fold oscillation compatible with $d_{x^2} - y^2$ SC state is clearly seen.
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erator in eq. (32). Their gap functions for e-h pair states are determined by self-consistent equations formally identical to the BCS gap equations as obvious from eq. (36). The physical meaning of the gap function in the e-h pairing case becomes clear by calculating the expectation values of charge ($C$) and spin ($S$) densities in eq. (32):

$$
\langle \rho_Q \rangle = \sum_k \Delta_C(k) F(k) \quad \text{and} \quad \langle M^z_Q \rangle = \sum_k \Delta_S(k) F(k)
$$

with $F(k) = \tanh(E_k/2T)/2$ and $E_k = \sqrt{\epsilon(k)^2 + \Delta_i(k)^2}$ ($i = C, S$) denoting the quasi-particle energy and the gap functions $\Delta_i(k)$ are given by eq. (30).

Conventional density waves: In the conventional ‘s-wave’ CDW and SDW case with $k$-independent gap functions $\Delta_{C,S}(k) = \Delta$ the sum in eq. (52) leads to a finite charge or spin density Fourier component for the modulation vector $Q$. This also holds for a fully symmetric ($A_{1g}$) gap function which is $k$-dependent but satisfies $\Delta(gk) = \Delta(k)$ for any group element $g \in G$. Due to nonvanishing $\langle \rho_Q \rangle$ and $\langle M^z_Q \rangle$, conventional CDW and SDW states are straightforward to identify because macroscopic densities either couple to the electric and magnetic fields of external probes or, as in the case of CDW, lead to observable distortions of the underlying crystal lattice. In neutron and (magnetic) X-ray diffraction experiments one can therefore observe additional lattice or magnetic superstructure Bragg reflections that originate in the scattering from density modulations $\langle \rho_Q \rangle \cos(Q \cdot r)$ and $\langle M^z_Q \rangle \cos(Q \cdot r)$. This is the standard method to observe broken spatial and time reversal symmetries, i.e., structural and magnetic phase transitions. There are also numerous other physical quantities, both static and dynamic which are affected by the appearance of conventional CDW and SDW modulations for which we refer to the book by Grüner (1994) and references cited therein.

Unconventional density waves: These order parameters also correspond to DLRO of density operators but the gap functions $\Delta_k$ now belong to a nontrivial representation of $G$ sometimes called ‘unconventional density waves’. In this case there will be elements $g \in G$ with $\Delta(gk) = -\Delta(k)$ signifying a sign change of $\Delta_i(k)$ between different sectors of $k$-space. Under these circumstances cancellation in eq. (52) occurs and macroscopic charge and spin-densities $\langle \rho_Q \rangle$ and $\langle M^z_Q \rangle$ vanish even though the order parameter corresponds to a physical observable (therefore the name convention for these states is misleading). A more formal proof of this cancellation effect was given in Thalmeier (1994). Now one has a perplexing situation of ‘hidden order’. Although there is a condensation of e-h pairs associated with quasiparticle energies and thermodynamic signatures similar to conventional (s-wave) CDW and SDW, the order parameter cannot be identified by the usual diffraction type experiments. This makes the search for such ‘hidden order’ a difficult task, and in fact it has not been unambiguously successful in any specific material. Some of the most prominent candidates for unconventional density waves are: the HF compound URu$_2$Si$_2$ (Ikeda and Ohashi, 1998) to be discussed in sect. 4.4; the underdoped pseudo-gap phase of high-$T_c$ materials (Benfatto et al., 2000; Chakravarty et al., 2001); and the organic conductors (Dóra and Virosztek, 2001; Basletic et al., 2002; Korin-Hamzic et al., 2002), especially Bechgaard-salts (Maki et al., 2003a, 2003b). To discuss the properties of unconventional CDW and SDW e-h pair states and
their physical signature we restrict ourselves in the following to the 2D tight binding model with nearest neighbor (n.n.) hopping which in this respect is the only model so far investigated in any detail. In this case the tight-binding band $\epsilon_k = -2t(\cos k_x + \cos k_y)$ has the important perfect nesting property $\epsilon_{k \pm Q} = -\epsilon_k$ leading to parallel Fermi surface sections at half filling ($E_F = 0$) as shown in fig. 7. This also leads to a singular behaviour of the DOS according to $N(0) \sim \ln(t/|\epsilon|)$ due to the saddle points (S) of $\epsilon_k$ at $k = (0, \pm \pi)$ and $(\pm \pi, 0)$. The various possibilities of density wave and superconducting pair states assuming three types of interactions ($U$ = on-site Coulomb, $V$ = n.n. Coulomb, $J$ = n.n. exchange) were studied and classified (Gulacsi and Gulacsi, 1987; Schulz, 1989; Ozaki, 1992). From the many possible phases the most important ones are conventional s-SC, s-CDW, SDW, and unconventional d-SC, d-CDW, SDW states. Historically the latter two states have also been called orbital antiferromagnet (OAF) and spin-nematic (SN) states, respectively (Nersesyan and Vachnadze, 1989; Nersesyans et al., 1991; Gorkov and Sokol, 1992) which points to their physical origin in staggered charge or spin currents on the 2D square lattice, respectively. Their pairing amplitudes or gap functions have a d-wave like $k$-dependence given by

$$\Delta_{d,C,S}^d(k) = i\Delta_0(\cos k_x - \cos k_y)$$

and illustrated in fig. 7. The d-wave gaps have nodes which are located at the four ‘Dirac’ points (D) $k = (\pm \frac{\pi}{2}, \pm \frac{\pi}{2})$. The node points are in ‘orthogonal’ (rotated by $\frac{\pi}{4}$) configuration to the saddle points (S) because at the latter with their singular DOS the gap value of a density wave state (and also SC state) has to achieve maximum value to be stable. In other words the ordered state consists mostly of e-h pairs $(k, k + Q)$ with $k$ close to the saddle point regions. This allows one to study the problem in a simplified continuum theory (Schulz, 1989; Thalmeier, 1996). As indicated the phase of gap functions for d-density waves with commensurate nesting $Q$ is not arbitrary. Because of the symmetry $\Delta_{C,S}^d(k)^* = -\Delta_{C,S}^d(k)$ the gap function has to be purely imaginary. Due to this property the ordered states are connected with staggered persistent charge (C) or spin (S) currents around the lattice plaquettes. In real space the pattern of currents at the lattice links is indicated in fig. 14.

As a means of identification of d-density wave states one then has to look for signatures of persistent currents in the ground state and for evidence of nodes in the quasiparticle excitation spectrum in thermodynamic and (magneto-)transport properties.

**Neutron Diffraction:** For d-CDW the charge currents break time reversal symmetry while the spin-currents in d-SDW do not (under time reversal both current and spin direction are reversed leaving the spin current invariant), they do, however, break spin rotational symmetry. Therefore d-CDW states may lead to the appearance of staggered orbital moments. Because the currents are spread out over a whole plaquette, the generated magnetic fields are of the order $\sim 10$ G associated with orbital moments $\sim 10^{-2}\mu_B$ per plaquette. The contribution to elastic and inelastic neutron cross section has been discussed (Chakravarty et al., 2001) in context with the d-CDW scenario for the spin gap phase of underdoped cuprates (Chakravarty et al., 2001a), but it is generally valid for the tight binding model used here. While the scattering from spin moments is determined by the form factor $g(q)$ of atomic electron densities which falls off moderately fast with momentum transfer $q$, the scattering cross section from
Fig. 14. Left: Illustration of staggered charge or spin current pattern in the square lattice for d-CDW or d-SDW. Right: Magnetoresistance \( j \perp ac\)-plane oscillations in the low temperature d-CDW state of the organic quasi-1D conductor \( \alpha \)-(BEDT-TTF)\(_2\)KHg(SCN)\(_4\). The oscillations result from the Landau quantisation of energy levels eq. (55) around the nodal Dirac points in fig. 7. The circles are from experiment \( (T = 1.4 \, \text{K}, B = 157 \, \text{T}, \phi = 45^\circ) \), and the full line from d-CDW calculations (Maki et al., 2003a).

Orbital moments is determined by the orbital current distribution \( j(q) \), namely proportional to \( |\langle j(q) \rangle|/q^2 \sim 1/q^4 \) which falls off much more quickly with momentum transfer due to the spread-out orbital moment density. Although this method would be rather straightforward it is quantitatively difficult due to the small orbital moment size and only applicable for d-CDW.

A more indirect but more versatile method consists in looking for signatures of the gap nodes in the d-density wave states. For low temperatures \( T \ll \Delta \) the nodal regions dominate thermodynamics and transport properties. Specific heat and thermal conductivity is equivalent to that of d-SC which have been discussed before. Susceptibility and frequency dependent electrical conductivity however are characteristic for the d-density wave states. Of course there is no vortex phase and the Doppler shift method cannot be applied.

**Giant diamagnetism:** The susceptibility has been analysed in detail (Nersesyan and Vachnadze, 1989; Nersesyan et al., 1991). Strong anomalies in the diamagnetic susceptibility for both d-CDW and -SDW are predicted at low fields. This is due to the peculiar conical or ‘relativistic’ quasiparticle spectrum around the nodal Dirac points (D) in fig. 7. For \( T \ll \Delta_0 \) the spectrum can be linearized and consists of two bands

\[
E_{\pm}(k) = \pm \sqrt{v_F^2 k_x^2 + v_\Delta^2 k_y^2}
\]

with \( v_F = 2\sqrt{2} \, ta \) and \( v_\Delta = \sqrt{2} \, \Delta_0 a \) giving the group velocity perpendicular and parallel to the original Fermi surface in fig. 7, respectively, which are related to the effective mass via \( m^* = \Delta_0/2v_Fv_\Delta \). Here the lower band \((-\)\) is completely filled and the upper band \((+\)\) empty, thus the Fermi surface in the ordered state \( (\Delta_0 > 0) \) consists of two inequivalent points \( k_{1,2} = (\pi, \pm \pi) \), the other Dirac points are equivalent since \( Q \) is now a reciprocal lattice vector. The
linearization corresponds to restricting to a \((2 + 1)\)-dimensional continuum field theory of chiral massless relativistic fermions with ‘anisotropic’ velocities \(v_F\) and \(v_\Delta\) (Nersesyan and Vachnadze, 1989). The quasiparticle DOS close to the Fermi level \(E_F = 0\) is given by \(N(E) = |E|/(\pi v_F v_\Delta)\) therefore one has low-\(T\) power laws \(C \sim T^2/(\xi \Delta_0)\) and \(\chi_\parallel^p \sim \mu_B^2 T/\Delta_0\) for the specific heat and the in-plane spin susceptibility. The most striking effect however should appear in the diamagnetic susceptibility anomaly caused by the persistent plaquette currents. This is obtained through the relativistic Landau quantisation of the spectrum which leads to Landau levels

\[
E_{n}^\pm = \pm \sqrt{|\Omega_\perp|} n \quad \text{and} \quad \Omega_\perp = 2eH_\perp v_F v_\Delta/c
\]

which have a degeneracy \(v(H) = |eH|/2\pi c\) \((c = \text{speed of light})\) per unit area. Furthermore, \(H_\perp = H \cos \theta\) where \(\theta = \text{polar field angle with respect to the plane normal vector } \hat{n}\). This equation holds for fields with \(\sqrt{|\Omega_\perp|} \ll \Delta_0\). From eq. (55) the \(H\)-dependent total energy is obtained leading to a \((2D)\) diamagnetic susceptibility

\[
\chi_{2D} \sim \sqrt{\Delta_0 \left|\frac{H}{h}\right|^{1/2} \cos^{3/2} \theta}
\]

which predicts an enhancement factor of \((\Delta_0/\Omega_c) \gg 1\) compared to the normal state diamagnetic susceptibility. The divergence in eq. (56) will be arrested below a tiny crossover field where \(3D\) behaviour due to interlayer hopping sets in and perfect diamagnetism with \(\chi_{3D}(H \to 0) = -1/4\pi\) will be approached. Thus for temperatures \(T \ll \Delta_0\) and almost zero field the staggered currents of d-CDW or SDW should achieve a similar perfect screening effect as supercurrents in the d-SC state.

The conical quasiparticle spectrum and associated ‘relativistic’ Landau quantisation should also leave its mark on the (magneto-)resistance which we only briefly mention since sofar it has only been applied to d-CDW candidates among quasi-1D organic conductors. In this case a finite carrier concentration \((|E_F| > 0)\) should be present. This is achieved by including a finite next-nearest-neighbor (n.n.n.) hopping element \(t'\) in the model. It does not appear in the linearized spectrum eq. (55) however. Analogous to the susceptibility, the magnetoresistance is determined by the Landau levels of the d-CDW state leading to typical oscillations (fig. 14) in its angular dependence which have been identified in two examples (Dóra et al., 2002; Basletic et al., 2002; Maki et al., 2003a).

**Finite frequency probes:** Finally we discuss finite frequency probes for d-DW states like optical conductivity (Yang and Nayak, 2002). It exhibits non-Drude like behaviour at low frequencies because of arbitrary low excitation energies for \(q = 0\) interband \((E_- \leftrightarrow E_+)\) transitions at the nodal (Dirac) points. For perfect nesting \((E_F = 0)\) at low temperatures one obtains (for \(\omega \tau \gg 1, \tau = \text{quasiparticle lifetime}\))

\[
\sigma(\omega, T) = (\ln 2)e^2\alpha T \delta(\omega) + \frac{1}{8} e^2\alpha \left| f\left(\frac{-\omega}{2}\right) - f\left(\frac{+\omega}{2}\right)\right|
\]

where \(\alpha \simeq (v_F/v_\Delta)\) and \(f\) denotes the Fermi function. The first part is the Drude term and the second is the interband term which should extend to arbitrary low frequencies and contribute
to the d.c. conductivity because of the vanishing gap at the node points. The weight of the Drude peak vanishes for $T \to 0$ while the interband contribution stays finite at zero frequency. Since the thermal conductivity has only a Drude contribution, the Wiedemann–Franz law $\kappa \sim \sigma T$ should be strongly violated in the d-DW state and one would rather expect $\kappa \sim \sigma T^2$. This result holds as long as $E_F < kT$ and also for finite quasiparticle scattering rate $\Gamma = 1/\tau$ if $\Gamma < E_F$. In addition the d-DW state may lead to anomalous field dependence of electrical and thermal Hall conductivity.

The peculiar excitation spectrum eq. (55) of a d-DW should also leave its imprint in the inelastic neutron scattering cross section (Chakravarty et al., 2001) which is proportional to the imaginary part of the dynamical spin susceptibility.

3. Ce-based heavy-fermion superconductors

The discovery of superconductivity in CeCu$_2$Si$_2$ by Steglich et al. (1979) initiated the rapid development of heavy fermion physics. For nearly two decades, this material was the only Ce-based heavy-fermion superconductor at ambient pressure. Only recently, superconductivity at ambient pressure was found in the new class of heavy-fermion materials Ce$_n$M$_m$In$_3n+2m$ where M stands for the transition metal ion Co, Ir or Rh and $m = 1$ while $n = 1$ or $n = 2$ (Thompson et al., 2001). Typical examples are CeCoIn$_5$ (Petrovic et al., 2001b) and CeIrIn$_5$ (Petrovic et al., 2001a).

The SC phases are characterized by BCS-type pair correlations among the heavy quasiparticles. In addition the strong on-site correlations introduced by the partially filled Ce-4f shells are reflected in several aspects. Firstly, the SC gap functions are anisotropic due to the local quasiparticle repulsion. Secondly, the competition between the non-magnetic Fermi liquid state and magnetically ordered phases leads to pronounced strong-coupling effects. They highlight the complex low-frequency dynamics in these systems which result from the competition between Kondo effect and long-range magnetic order.

The main emphasis of the current experimental and theoretical studies is on the fundamental question which factors actually determine the low-temperature phases, i.e., when does a Ce compound become a heavy-fermion superconductor or why does it order magnetically. The subtle interplay between Kondo effect and long-range magnetic order can be monitored experimentally in pressure studies where isostructural relatives of the heavy-fermion superconductors are tuned from magnetic phases at ambient pressure to superconducting states. Similar behavior is found in doping experiments where constituents of the metallic host are successively replaced. Examples for pressure-induced superconductors are CeCu$_2$Ge$_2$ (Jacard et al., 1992), CePd$_2$Si$_2$ (Grosche et al., 1996; Mathur et al., 1998), CeNi$_2$Ge$_2$ (Grosche et al., 2000) and CeRh$_2$Si$_2$ (Movshovich et al., 1996), CeSn$_3$ (Walker et al., 1997) and CeIn$_3$ (Grosche et al., 1996).

Finally, a different type of competition between superconductivity and magnetic order may be realized in CeCu$_2$Si$_2$. In this compound, the strongly renormalized Fermi liquid seems to be unstable with respect to the formation of both superconducting pairs and an (incommensurate)
spin-density wave. The actual ground state depends sensitively on the composition of the sample.

In the present section, we review the properties of the stoichiometric Ce compounds exhibiting competition between heavy-fermion superconductivity and long-range magnetic order. We shall not discuss the highly complex phase diagrams obtained in alloy systems. In addition, we summarize recent attempts at a theoretical description of the complex low-frequency dynamics resulting from the competition between Kondo effect and RKKY interaction.

3.1. \textit{CeM}_2\textit{X}_2

The archetype heavy-fermion superconductor \textit{CeCu}_2\textit{Si}_2 as well as the pressure-induced superconductors \textit{CeCu}_2\textit{Ge}_2, \textit{CePd}_2\textit{Si}_2, \textit{CeNi}_2\textit{Ge}_2 and \textit{CeRh}_2\textit{Si}_2 crystallize in the tetragonal \textit{ThCr}_2\textit{Si}_2 structure. The unit cell is shown in fig. 15. The isostructural compounds \textit{CeM}_2\textit{X}_2 with \textit{M} = \textit{Ru}, \textit{Ni}, \textit{Pd}, \textit{Cu}, \textit{Ag}, \textit{Au} and \textit{X} = \textit{Si}, \textit{Ge} exhibit a great variety of possible ground states and have been extensively studied to clarify the interplay between the formation of magnetic order and heavy fermion behavior.

3.1.1. \textit{Electronic properties, Fermi surfaces and heavy quasiparticles}

To study the electronic structure, we compare the results for two different models treating the Ce 4f degrees of freedom as localized (atomic-like) states and as delocalized yet strongly renormalized electrons. The first procedure provides a good quantitative description of the properties at elevated temperatures, high excitation energies, and above the metamagnetic transition. The latter ansatz yields a model for the Fermi liquid state.
With respect to the electronic properties, the compounds CeM$_2$X$_2$ fall into two distinct categories, the key feature being the presence or absence of transition metal d-states at the Fermi level. We apply this criterion to classify the CeM$_2$X$_2$ compounds exhibiting superconductivity. The first group is formed by CePd$_2$Si$_2$, CeNi$_2$Ge$_2$ and CeRh$_2$Si$_2$ which have a large conduction electron density of states at the Fermi level. A well-studied model for the normal state of these systems is provided by CeRu$_2$Si$_2$ which we shall consider in detail below. The macroscopic properties of this compound closely resemble those of CeNi$_2$Ge$_2$ and CePd$_2$Si$_2$. The general conclusions derived for CeRu$_2$Si$_2$ should therefore apply also to the pressure-induced superconductors. The second group is given by CeCu$_2$Si$_2$ and CeCu$_2$Ge$_2$ where the metal d-bands are filled.

The low-temperature behavior of CeRu$_2$Si$_2$ is well described by a paramagnetic Fermi liquid with weak residual interactions. The relevant low-energy excitations are heavy quasiparticles as inferred from the linear specific heat coefficient $\gamma \simeq 350$ mJ/mol K$^2$ (Steglich et al., 1985). The electronic structure was calculated both for the local moment regime at high temperatures and for the Fermi liquid state. Both types of calculations start from ab-initio crystal potentials which are determined self-consistently within the LDA. Details of the calculation can be found in Runge et al. (1995). In the local moment regime, the Fermi surface is determined exclusively by the conduction states. The strongly renormalized Fermi liquid state, on the other hand, is described by the renormalized band method using $\tilde{\Delta}_f \simeq 10$ K in eq. (7) for the intrinsic width of the quasiparticle band. The value is consistent with inelastic neutron data (Regnault et al., 1988) as well as thermopower and specific heat data (Steglich, 1985). CEF effects are accounted for by adopting a $I^7$ ground state. The details of the calculation are described in Zwicknagl (1992).

The renormalized band scheme gives the correct Fermi surface topology for CeRu$_2$Si$_2$ and thus consistently explains the measured dHvA data (Zwicknagl et al., 1990; Zwicknagl, 1993). The Fermi surface consists of five separate sheets among which are four closed hole surfaces centered around the $Z$-point. The remaining one is a complex multiply-connected surface where extremal orbits of rather different character exist. In particular, one can find typical particle-like orbits as well as others for which a hole picture would be more appropriate. The agreement between calculated and experimental results with respect to general topology is rather good (for further details see (Zwicknagl, 1992, 1993)).

The character of quasiparticles in CeRu$_2$Si$_2$ varies quite strongly over the Fermi surface. There are three $Z$-centered hole ellipsoids with rather light quasiparticles. The states on the pillow-shaped $Z$-centered hole surface displayed in fig. 16, however, have predominantly f-character and therefore yield the dominant contribution to the specific heat. Experimentally, one finds heavy and light quasiparticles coexisting on the multiply-connected sheet.

The validity of the Fermi liquid picture is concluded from a comparison of the effective masses on the fourth pillow-shaped sheet as given in table 2. From the large linear specific heat the renormalized band scheme deduces a characteristic energy $kT^* \simeq 10$ K which in turn implies heavy masses of the order of $m^*/m \simeq 100$. This value was confirmed by experiments (Albessard et al., 1993; Aoki et al., 1993; Tautz et al., 1995) where the $\psi$ orbit with $m^*/m \simeq 120$ was observed. The corresponding Fermi surface cross section is in agreement with estimates from the renormalized band theory. This proves that the heavy quasiparticles
CeRu$_2$Si$_2$: Fermi surface sheets for quasiparticles with f-character. The labels $\psi$, $\alpha$, $\delta$, $\epsilon$ refer to the branches observed in dHvA experiments (Lonzarich, 1988; Albessard et al., 1993). Left: Hole surface centered around the Z-point of the Brillouin zone with effective mass $m^* \sim 100m$ which dominates the specific heat $\gamma$-value. For localized f-electrons at elevated temperatures, the hole surface expands extending to the boundaries of the Brillouin zone while the multiply connected electron-like surface shrinks. The expansion of $\psi$ is confirmed by photoemission experiments. Right: Multiply-connected electron-like sheet.

Table 2

<table>
<thead>
<tr>
<th>$\psi$</th>
<th>Experiment</th>
<th>Theory (approx.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>area [MG]</td>
<td>53.6</td>
<td>62</td>
</tr>
<tr>
<td>$m^*/m$</td>
<td>120</td>
<td>100</td>
</tr>
</tbody>
</table>

exhaust the low-energy excitations associated with the f-states in HF systems. The shape of the Fermi surface of heavy quasiparticles implies the existence of nesting vectors. This fact suggests that the Fermi liquid state might become unstable with respect to particle–hole pairing, leading for example to the formation of a SDW state.

The change in the volume of the Fermi surface when going from $T \ll T^*$ to $T \gg T^*$ is observed by comparing the Fermi surface of CeRu$_2$Si$_2$ to that of its ferromagnetic isostructural counterpart CeRu$_2$Ge$_2$ where the f-states are clearly localized. In a series of beautiful experiments (King and Lonzarich, 1991) it was demonstrated that the Fermi surfaces of these two compounds are rather similar. However, the enclosed Fermi volume is smaller in the case
Fig. 17. Photoemission results for LaRu$_2$Si$_2$ (a) in comparison to CeRu$_2$Si$_2$ (b) at $T = 25$ K above the Kondo temperature $T^* = 15$ K of CeRu$_2$Si$_2$. Band structures are similar for both compounds (Denlinger et al., 2001).

of CeRu$_2$Ge$_2$, the difference being roughly one electron per unit cell. More direct evidence is provided by recent photoemission experiments (see fig. 17). Denlinger et al. (2001) have shown that at temperatures around 25 K, the Fermi surface of CeRu$_2$Si$_2$, is that of its counterpart LaRu$_2$Si$_2$ which has no f-electrons.

The similarity of the Fermi surface topologies for localized and itinerant Ce $4f$ states is a characteristic feature of the CeM$_2$X$_2$ systems with partially filled M d-bands. In addition, we mention that the topology of the Fermi surface for the heavy quasiparticles is qualitatively reproduced by standard band structure calculations based on the LDA (Zwicknagl, 1988).

This, however, is not the case for CeCu$_2$Si$_2$. As the Cu d-bands are filled there are only two major two-fold degenerate bands crossing the Fermi energy, and the effective hybridization becomes rather anisotropic. To calculate the quasiparticle bands in CeCu$_2$Si$_2$ by means of the renormalized band method, we adopt the doublet-quartet CEF scheme suggested by Goremychkin and Osborn (1993). The ground state is separated from the excited quartet by $\delta \simeq 340$ K.

The results for the Fermi surface (Zwicknagl and Pulst, 1993; Pulst, 1993) can be summarized as follows: two separate sheets of the Fermi surface for heavy and light quasiparticles are found. The light quasiparticles have effective masses of the order of $m^*/m \simeq 5$. They can be considered as weakly renormalized conduction electrons and the corresponding Fermi surface is rather similar to the LDA prediction (Harima and Yanase, 1991). The observed Fermi surface cross sections (Hunt et al., 1990) can be explained by both the renormalized band structure as well as by the LDA bands. There is, however, a characteristic difference between the Fermi surfaces for the light quasiparticles as derived from the two schemes: The LDA calculation predicts a small closed surface centered around the $\Gamma$-point which is absent in the dHvA-experiment although the corresponding cross sections are rather small and the masses are expected to be light.

Of particular interest are heavy quasiparticles of effective masses $m^*/m \simeq 500$ which are found on a separate sheet. The topology of this surface is rather different from the corresponding LDA result as can be seen from fig. 18. It mainly consists of columns parallel to the tetragonal axis and small pockets. The topology of the Fermi surface suggests that the strongly correlated Fermi liquid state should become unstable at sufficiently low temperatures. Firstly, it exhibits pronounced nesting features which may eventually lead to the formation of a ground
state with a spin density modulation. This will be discussed in detail below. Secondly, the topology of this surface depends rather sensitively on the position of the Fermi energy. The band filling and hence the $f$-valence are critical quantities. Reducing the $f$-occupancy from the initial value of $n_f \simeq 0.95$ by $\sim 2\%$ leads to changes in the topology as shown by Pulst (1993) and Zwicknagl and Pulst (1993). As a result, the quasiparticle density of states (DOS) should exhibit rather pronounced structures in the immediate vicinity of the Fermi energy which, in turn, can induce instabilities (Kaganov and Lifshits, 1979).

An external magnetic field couples to the $f$-degrees of freedom. It leads to a Zeeman splitting of order $\mu_{\text{eff}} B$ of the heavy quasiparticle bands and hence can move the structures in the DOS relative to the Fermi energy. The effective moment $\mu_{\text{eff}}$ of the $f$-states which contains the CEF effects depends on the direction of the external magnetic field. From the renormalized band calculation we determine the values of the critical magnetic fields when the pronounced structures are moved to the Fermi energy. Due to CEF effects on the $f$-states this critical value of the external magnetic field will depend upon its orientation. The explicit values of the critical fields $B_{\text{crit}}^c$ and $B_{\text{crit}}^a$ for field directions parallel to the $c$- and $a$-axes as listed in table 3 agree rather well with the observed critical magnetic fields for the transition into the ‘$B$-phase’ (see fig. 19).

Having discussed the basic electronic structure of the CeM$_2$X$_2$ heavy fermion compounds we now turn to their low-temperature properties.

Fig. 18. CeCu$_2$Si$_2$: Left: FS of heavy quasiparticles ($m^*/m \simeq 500$) calculated by using the renormalized band method. It consists of ellipsoids and modulated columns which are oriented parallel to the tetragonal axis. The calculations adopt the CEF scheme suggested by Goremychkin and Osborn (1993) consisting of a singlet ground state separated from an excited quartet by a CEF splitting $\delta \sim 340$ K. Therefore $\delta \gg T^* \simeq 10$ K (obtained from the $\gamma$-value). Consequently quasiparticle properties are strongly anisotropic. The topology of the renormalized band FS differs qualitatively from LDA results. Right: Enlarged FS columns exhibiting nesting in a column with a nesting vector $Q = (0.23, 0.23, 0.52)$ (Zwicknagl and Pulst, 1993).
3.1.2. Superconductivity and itinerant antiferromagnetism in \( \text{CeCu}_2\text{Si}_2 \)

\( \text{CeCu}_2\text{Si}_2 \) exhibits highly complex phase diagrams at low temperatures. This fact partially results from an extreme sensitivity of the physical properties with respect to variations of the stoichiometry. Thorough investigations of the ternary chemical phase diagram (for a review see Steglich et al. (2001)) showed that the actual ground state is mainly determined by the average occupation of the non-f sites surrounding the Ce-ion.

Apart from the composition, the phases are strongly affected by magnetic fields. The \( B-T \) phase diagram (Bruls et al., 1994) for a high-quality single-crystal is displayed in fig. 19. The phase transitions were deduced from anomalies in thermodynamic quantities including the elastic constants, thermal expansion and magnetostriction coefficients. In the low-temperature regime, there is a SC phase surrounded by a phase ‘A’ which becomes unstable in high magnetic fields. Similar phase diagrams are obtained from resistivity measurements, e.g. (Steglich et al., 2000a). In addition to these ‘A-SC-type’ crystals, there exist also ‘A-type’ systems which do not superconduct as well as ‘SC-type’ compounds where the A-phase is suppressed (fig. 8). The high-field phase \( B \), on the other hand, is present in all three cases. It is interesting to note that the transition into the \( B \)-phase occurs close to the calculated critical fields \( B_{\text{crit}}^a \) and \( B_{\text{crit}}^c \) listed in table 3. At these values, the Fermi surface of the heavy quasiparticles is expected to change drastically.
Fig. 20. Left panel: Neutron diffraction intensity in CeCu$_2$Si$_2$ at temperatures above and below the $A$-phase transition temperature $T_A$. Incommensurate peak corresponds to $\mathbf{Q} = (0.215, 0.215, 0.53)$ (Stockert et al., 2004). Right panel: Nesting of heavy FS columns (fig. 18) leads to a peak in the static susceptibility $\chi(\mathbf{q})$ at $\mathbf{q} = \mathbf{Q}$. The intensity map of $\chi(\mathbf{q})$ (value increasing from dark to bright) in the reciprocal $(h, h, l)$-plane as calculated for the renormalized bands at $T = 100$ mK. The experimental $\mathbf{Q}$ at 50 mK from the left panel shows perfect agreement with the calculated maximum position of $\chi(\mathbf{q})$.

Much effort has been devoted to the characterization of the $A$-phase which originally had the appearance of another ‘hidden order’ phase. However later the spin density wave character first inferred from resistivity results (Gegenwart et al., 1998) was supported by specific heat and high-resolution magnetization measurements (Steglich et al., 2000). The transition temperature $T_A$ is suppressed by increasing the 4f-conduction electron hybridization and eventually vanishes. This can be achieved by applying hydrostatic pressure or choosing a few percent excess of Cu. The ordered moments are expected to be rather small. The propagation vector $\mathbf{Q}$ can be estimated by extrapolation from neutron diffraction studies in single-crystalline CeCu$_2$(Si$_{1-x}$Ge$_x$)$_2$. The central result is that the wave vector of the magnetic order changes only slightly with the Ge concentration while the ordered moment per Ce site, however, decreases strongly from $\mu \simeq 0.5 \mu_B$ for $x = 0.5$ to $\mu \simeq 0.1 \mu_B$ for $x = 0$.

Recent neutron scattering experiments (Stockert et al., 2004) (fig. 20) for the stoichiometric compound ($x = 0$) show a spin density wave which forms below $T_N \simeq 0.7$ K. The experimental propagation vector $\mathbf{Q}$ is close to $(0.215, 0.215, 0.53)$ and the ordered moment amounts to $\mu \simeq 0.1 \mu_B$. The instability of the Fermi liquid state with respect to the SDW state follows from the nesting properties of the heavy quasiparticle surface shown in fig. 18 which leads to a peak at $\mathbf{q} = \mathbf{Q}$ in the static susceptibility $\chi(\mathbf{q})$ with $\mathbf{Q}$ close to the above experimental value (fig. 20).

3.1.3. Pressure-induced superconductivity in CePd$_2$Si$_2$ and CeNi$_2$Ge$_2$

The compounds CeNi$_2$Ge$_2$ and CePd$_2$Si$_2$ offer the possibility for studies of the antiferromagnetic instability in a heavy-fermion system. In the vicinity of the quantum critical point, non-
Fermi liquid behaviour and superconductivity are observed. At ambient pressure, CePd$_2$Si$_2$ orders in an antiferromagnetic structure below a Néel temperature $T_N \simeq 8.5$ K. The magnetic structure is characterized by a propagation vector $\mathbf{Q} = (1/2, 1/2, 0)$. The ordered moment lying in the basal plane is reduced to $\sim 0.66 \mu_B$ by CEF effects and possibly Kondo screening (van Dijk et al., 2000). The coefficient of the linear specific heat $\gamma \simeq 250$ mJ/mol K$^2$ and the quasielastic line width suggest a Kondo temperature $T^* \simeq 10$ K which is close to the Néel temperature $T_N$. The interplay between the Kondo effect and the RKKY interaction is reflected in the depression of $T_N$ with increasing pressure. The antiferromagnetic order is suppressed by a critical pressure $p_c$ between 28 and 29.5 kbar. There is a superconducting phase between 23 and 33 kbar with a maximum $T_c \simeq 290$ K close to $p_c$. This fact indicates that the appearance of superconductivity should be related to the magnetic instability (Demuer et al., 2001).

CeNi$_2$Ge$_2$ has a slightly smaller lattice constant than CePd$_2$Si$_2$ and exhibits non-Fermi liquid behavior at ambient pressure (Gegenwart et al., 1999). No phase transition is found down to lowest temperatures. However magnetic correlations appear gradually with decreasing temperature (Fak et al., 2000). They have a characteristic energy of 4 meV and an incommensurate wave vector $\mathbf{Q} = (0.23, 0.23, 0.5)$ which is rather similar to the ordering vector in CeCu$_2$Ge$_2$. They also exhibit quasi-two-dimensional character with strong correlations between moments in the [110] planes. The electronic structure was recently studied by angle-resolved photoemission (Ehm et al., 2001). The low-temperature data ($T \simeq 20$ K) exhibit strongly dispersing bands in agreement with LDA band structure calculations. Close to the Fermi level two features with high spectral intensity can be distinguished one of which has predominantly Ni-3d character. The other one can be assigned to a non-dispersive Kondo resonance. These findings indicate that CeNi$_2$Ge$_2$ is in the intermediate temperature regime below the single-site Kondo temperature $T^*$ and above the coherence temperature where extended heavy band states form. The magnetic moments of the Ce ions are already screened by the Kondo effect as indicated by the $f$-spectral weight at the Fermi level. The coherence between the sites in the periodic lattice, on the other hand, is not yet fully developed.

Traces of possible superconductivity at ambient pressure have been found in resistivity measurements (Steglich et al., 1997; Gegenwart et al., 1999; Grosche et al., 2000). The preliminary phase diagram (Braithwaite et al., 2000) shows that the transition temperature $T_c$ goes through a maximum around 30 kbar and vanishes above 65 kbar. The upper critical field data $H_{c2}(T)$ of the system under pressure are analyzed by comparing them with calculated curves assuming weak coupling in the clean limit. The analysis reveals two interesting points. First, the values of $H_{c2}$ at low temperatures considerably exceed the Pauli limit which indicates the possibility of triplet pairing. Second, the initial slope $-(dH_{c2}/dT)_{T=T_c}$ exhibits only a weak decrease in the range between 0 kbar and 23 kbar which is much weaker than the change expected from the variation of the specific heat.

### 3.2. CeMIn$_5$

The compounds CeMIn$_5$ (M = Co, Ir, Rh) are a newly reported family of heavy fermion systems (Hegger et al., 2000; Petrovic et al., 2001a) which exhibit a superconducting transition
at comparatively high temperatures. The values of the critical temperatures reach $T_c = 2.3$ K in CeCoIn$_5$ (Petrovic et al., 2001b). We mention that the isostructural compound PuCoGa$_5$ is the first Pu-based superconductor with $T_c = 18.5$ K (Sarrao et al., 2003). The discovery of this new class of heavy-fermion compounds has opened a way to systematically investigate the evolution from the antiferromagnetic to superconducting state as a function of pressure.

The structurally layered compounds CeMIn$_5$ crystallize in the tetragonal HoCoGa$_5$-structure (Grin et al., 1979) which is displayed in fig. 21. They belong to the broader family of materials Ce$_n$M$_m$In$_{3n+2m}$ which can be considered as built from ‘CeIn$_3$’ and ‘MIn$_2$’ subunits stacked along the tetragonal c-axis. The infinite-layer parent CeIn$_3$ crystallizes in the cubic AuCu$_3$ structure. The layering introduces pronounced anisotropies in the electronic and structural properties which, in turn, influence the magnetic and superconducting properties as well as the behavior in the quantum critical regime. Varying the stacking sequence, i.e., the parameters $n$ and $m$, allows for a study of the general role on dimensionality in the competition between the various ground states.

The subsequent section is mainly devoted to the properties of the $n = m = 1$ materials commonly referred to as single-layer systems. We begin, however, by briefly summarizing the properties of the infinite-layer parent CeIn$_3$. At ambient pressure it orders antiferromagnetically at $T_N \approx 10$ K with an ordering vector $\mathbf{Q} = (0.5, 0.5, 0.5)$ (Morin et al., 1988) and an ordered moment of $0.4\mu_B$. The magnetic ordering temperature decreases monotonically with pressure. In a narrow range around a critical pressure $p_c = 2.6$ GPa resistivity vanishes below the onset temperature at 0.2 K. Recent measurements of $T_1^{-1}$ and the ac-susceptibilities (Kawasaki et al., 2002) confirm the bulk character of superconductivity. The results, however, yield a much lower value $T_c = 95$ mK than the onset temperature. The absence of the coherence peak is taken as evidence of an unconventional pair state.
The single-layer \((n = 1)\) members with \(M = \text{Rh}, \text{Ir}, \text{Co}\) and the bilayer \((n = 2)\) member with \(M = \text{Rh}\) can be considered as pressure variants of the parent \(\text{CeIn}_3\). This is suggested by the lattice constants of \(\text{CeIn}_3\) \((a = 4.689\ \text{Å}, c = 7.542\ \text{Å})\), \(\text{CeRhIn}_5\) \((a = 4.668\ \text{Å}, c = 7.515\ \text{Å})\), \(\text{CeCoIn}_5\) \((a = 4.62\ \text{Å}, c = 7.56\ \text{Å})\) and \(\text{Ce}_2\text{RhIn}_8\) \((a = 4.665\ \text{Å}, c = 12.244\ \text{Å})\). Like the parent, the compounds \(\text{CeRhIn}_5\) and \(\text{Ce}_2\text{RhIn}_8\) order antiferromagnetically at ambient pressure and become pressure-induced superconductors. The magnetic ordering temperatures \(T_N\), however, are reduced while the superconducting transition temperatures \(T_c\) are an order of magnitude higher than the maximum value found in \(\text{CeIn}_3\). \(\text{CeIrIn}_5\) \((T_c = 0.4\ \text{K})\) and \(\text{CeCoIn}_5\) \((T_c = 2.3\ \text{K})\) become superconducting at ambient pressure.

Structure data as well as pressure studies indicate that superconductivity in \(\text{CeMIn}_5\) occurs close to a quantum critical point (QCP). The lattice parameters suggest that the antiferromagnetic compound \(\text{CeRhIn}_5\) and the superconductor \(\text{CeCoIn}_5\) fall on different sides of the QCP in the Doniach phase diagram.

Finally we want to add two comments. The layered crystal structure of the materials seems to suggest that the compounds may be approximately described as 2-dimensional systems. However, this is not justified in view of magnetic properties. Neutron scattering data on \(\text{CeRhIn}_5\) and \(\text{Ce}_2\text{RhIn}_8\) have shown that these materials are magnetically 3-dimensional despite their layered structure. The Fermi surface topologies of the alloys \(\text{Ce}_x\text{La}_{1-x}\text{RhIn}_5\) (Alver et al., 2001) are found to be nearly independent of the composition parameter \(x\).

### 3.2.1. Electronic properties and Fermi surfaces

The electronic properties of the \(\text{CeMIn}_5\) compounds have been studied theoretically by band structure calculations based on the LDA. Of particular interest are the Fermi surfaces which can be directly compared with experimental dHvA data. The central result is the confirmation of the qualitative picture that Ce 4f-degrees of freedom should be considered as localized moments in \(\text{CeRhIn}_5\) (at ambient pressure) while a description in terms of itinerant, strongly renormalized fermions is more appropriate in \(\text{CeIrIn}_5\) and \(\text{CeCoIn}_5\). The conclusions are based on the following findings by Shishido et al. (2002) and references therein: For the reference compound \(\text{LaRhIn}_5\) without f-electrons the Fermi surface as well as the effective masses of the quasiparticles are well described by the LDA band structure. The quasi-twodimensional character reflects the tetragonal lattice structure. The topology of the main Fermi surfaces is also in good agreement with that of the Ce-counterpart \(\text{CeRhIn}_5\) indicating the localized nature of the Ce-4f electron in this system. A detailed quantitative comparison between calculated and measured cross-sections, however, is difficult due to the complicated antiferromagnetic structure. Angle-resolved photoemission experiments (Fujimori et al., 2003) performed at 15 K showed rather small 4f weight close to \(E_F\). The presence of the localized 4f-states in \(\text{CeRhIn}_5\) is reflected in the rather large quasiparticle masses \(m^*\) which are enhanced by a factor 7–9 over those of \(\text{LaRhIn}_5\). The Fermi surfaces of \(\text{LaRhIn}_5\) and the localized 4f-compound \(\text{CeRhIn}_5\) are highly different from those of the HF systems \(\text{CeIrIn}_5\) and \(\text{CeCoIn}_5\) which are reasonably well explained by the itinerant 4f-band model.

Let us now turn to the properties of the 4f-electrons in the local moment regime. In the tetragonal CEF the degeneracy of the 4f \(j = 5/2\) state is lifted. The level scheme has been determined using susceptibility and thermal expansion coefficient data for \(\text{CeRhIn}_5\), \(\text{CeIrIn}_5\)
and CeCoIn$_5$ (Takeuchi et al., 2001; Shishido et al., 2002). In all three compounds, the CEF ground state has predominantly $j_z = \pm \frac{5}{2}$-character while the first and second excited states are mainly derived from $j_z = \pm \frac{3}{2}$- and $j_z = \pm \frac{1}{2}$-states. However the CEF level schemes differ with respect to their excitation energies. In the Rh and Ir systems, the $j = \frac{5}{2}$-manifold is split into three Kramers doublets with the excitation energies $E(\pm \frac{3}{2}) \sim 60$ K and $E(\pm \frac{1}{2}) \sim 300$ K whereas these excited states are (almost) degenerate in the Co-compound separated by $\sim 150$ K from the ground state.

Finally, we mention angle-resolved photoemission studies of CeRhIn$_5$ and CeIrIn$_5$ (Fujimori et al., 2002a, 2003). The spectral contributions from the ligand states, i.e., from the Rh, Ir and In states, agree rather well with the results of LDA band structure calculations. However, no feature originating from the Ce 4f-states is observed near the Fermi level. This fact may have an explanation within the Kondo scenario: The experiments were carried out at $T = 14$ K which is already equal to the estimated Kondo temperature of CeIrIn$_5$ given by $T^* \sim 15$ K.

### 3.2.2. Unconventional superconductivity in CeCoIn$_5$

CeCoIn$_5$ appears to be a typical heavy-fermion material. The effective magnetic moment of $\sim 2.59 \mu_B$ as determined at high temperatures ($T > 200$ K) from a polycrystalline average is consistent with the free-ion value for Ce$^{3+}$ $(2.54 \mu_B)$. The $\rho(T)$ dependence is rather weak above a characteristic temperature $T^* \sim 30$ K. The crossover to a coherent state is reflected in a rapid decrease at lower temperatures. The electronic contribution to the low-temperature specific heat in the normal state ($T > 2.3$ K) varies linearly $C \sim \gamma T$ with a large coefficient $\gamma \sim 350$ mJ/mol K$^2$ indicating substantial mass renormalization (Petrovic et al., 2001a). The normal Fermi liquid becomes unstable at $T_c = 2.3$ K where the heavy quasiparticles form Cooper pairs. This can be seen from the discontinuity in the specific heat with $\Delta C(T_c)/\gamma T_c \simeq 5$ as shown in fig. 22. The enhancement over the universal BCS values for isotropic SC points to pronounced strong coupling effects. The BCS value of 1.43 for isotropic superconductors should be considered as an upper bound on weak-coupling theory which is reduced for anisotropic pair states as obvious from table 1.

The superconducting state seems to be of unconventional d-wave singlet type. The presence of line nodes is inferred from the power-law behavior exhibited by various thermodynamic and transport properties at low temperatures. The latter include the specific heat (Petrovic et al., 2002; Hegger et al., 2000) and NMR relaxation rate (Kohori et al., 2001; Zheng et al., 2001). The most direct evidence, however, is provided by thermal conductivity measurements in a magnetic field rotating within the $a$–$b$ plane (Izawa et al., 2001; Matsuda and Izawa, 2003) (fig. 13). In addition, the observation of a zero-bias conductance anomaly in point-contact spectroscopy supports the hypothesis of an unconventional pair state (Goll et al., 2003). This is also suggested by the Knight shift measurements of Kohori et al. (2001) which give support for an even singlet SC order parameter with line nodes such as the $d_{x^2-y^2}$ gap function deduced from the above thermal conductivity results or the $d_{xy}$ gap function according to specific heat results (Aoki et al., 2004).

The properties of this unconventional superconductor are of great interest. A first issue is the sensitivity to nonmagnetic impurities which are expected to act as strong pairbreakers. Substitution of Ce-ions with La-ions results in a depression of the transition tempera-
tured $T_c$ (Petrovic et al., 2002). However the measured $T_c$-reduction is weaker than the value expected for isotropic ($s$-wave) scatterers in a $d_{x^2-y^2}$-state. This suggests that La-substitution introduces anisotropic scattering. Anisotropies in the scattering matrix element may strongly reduce pairbreaking as compared with pure $s$-wave scattering (Fulde and Zwicknagl, 1990; Fulde et al., 1988). The upper critical field $H_{c2}$ is simply reduced by the impurities while its anisotropy remains unaltered.

A magnetic field can suppress superconductivity by acting on the orbits of the charged quasiparticles and by acting on their spins. Both mechanisms determine the upper critical magnetic field $H_{c2}(T)$ which separates the normal state for $H > H_{c2}(T)$ from the superconducting vortex phase at $H < H_{c2}(T)$. The relative importance of the two depairing effects determines the order of the phase transition in a type-II superconductor. While the omnipresent and usually dominating orbital pairbreaking is associated with a second order phase transition, a first order transition is anticipated for dominant spin depairing. In superconductors with sufficiently weak orbital pairbreaking a change from second order at $T_0 < T < T_c$ to first order at $T < T_0$ should be observed (for a review see Saint-James et al. (1969)). The crossover, however, was not observed in conventional superconductors due to the presence of strong spin–orbit scattering. CeCoIn$_5$ appears to be an ideal candidate for the observation of the first order phase transition in a magnetic field. Firstly, it is a very clean type-II superconductor where the orbital pairbreaking is rather weak due to the high effective mass of the quasiparticles. Secondly, the magnetic susceptibility is enhanced which further lowers the Chandrasekhar–Clogston field. The first order phase transition at $T_0$ shown in the phase diagram of fig. 22 was originally inferred from a step in thermal conductivity by Izawa et al. (2001). It is reflected in a pronounced sharpening of both the specific heat and thermal expansion, a large magnetoelastic effect and a steplike change in the sample volume (Bianchi et al., 2002).

![Fig. 22. Left panel: Linear specific heat coefficient of CeCoIn$_5$ vs temperature, for zero field anomalous nFI behaviour is observed. $\Delta C(T_c)/\gamma T_c \simeq 5$ is strongly enhanced over the isotropic BCS value 1.43. Inset shows corresponding entropies. Right panel: $H_{c2}$ curve from specific heat data. At $T_0 \simeq 1.1 \text{ K}$ the transition changes from second to first order. Lower inset shows entropy gain as function of $T$ starting from 0.13 K, for increasing field (right to left: 8.6 T–11.4 T) a step is evolving. Upper inset shows specific heat jump at the transition line $T_{FFLO}$ to the Fulde–Ferrell–Larkin–Ovchinnikov state (Bianchi et al., 2003).](image)
The change from the second-order nature of the transition observed at zero and low magnetic field to first order at high fields occurs at \( T_0 \approx 1.1 \text{ K} = 0.48T_c \). This value should be compared with the estimate of \( 0.33T_c \) deduced from the Chandrasekhar–Clogston field of a d-wave order parameter and the orbital critical field obtained from extrapolating the behavior close to the zero-field transition to low temperatures.

The structure of the flux line lattice which forms at low temperatures in the presence of an external magnetic field is of particular interest. A comprehensive review of this topic for s-wave superconductors has been given by Brandt (1995). An ideal isotropic type-II superconductor favors the hexagonal flux lattice. However, as the energy gain with respect to the square lattice is only 2%, relatively weak perturbations like FS or gap anisotropies may lead to distorted hexagonal or even square vortex lattices. CeCoIn\(_5\) is a clean type-II superconductor for which one anticipates a rather complex variation with magnetic field of the flux line lattice. Recent small-angle neutron scattering experiments (Eskildsen et al., 2003) have imaged the vortex lattice of CeCoIn\(_5\). At low fields, a hexagonal lattice is observed. The latter undergoes a “phase transition” to a square lattice when the field is increased. The orientation of the square flux line lattice relative to the crystal lattice is consistent with the expectations for the suggested d-wave order parameter. The square lattice for d-wave superconductors has been predicted theoretically by Won and Maki (1995) and Berlinsky et al. (1995). Finally at low temperatures and close to \( H_{c2} \) specific heat measurements (Bianchi et al., 2003) seem to indicate the presence of another inhomogeneous phase for both a- and c-field directions which was interpreted as the long-sought Fulde–Ferrell–Larkin–Ovchinnikov (FFLO) phase which is characterised by Cooper pairs with finite net momentum. The FFLO phase for d-wave SC order parameter of CeCoIn\(_5\) was studied theoretically in Won et al. (2004).

3.3. Superconductivity close to a quantum critical point

The symmetry and mechanism of SC in the CeMIn\(_5\) compounds have not yet been identified in any comprehensive way. However there are strong indications of unconventional order parameters. The interest in the SC of these compounds results from the observation that under pressure and chemical substitution the SC pair correlations develop in the vicinity of a QCP. Recently a few U-compounds have been found which fall also in this class. Theories to describe SC at the QCP are at present highly controversial and will be discussed there to some extent (sect. 4.3).

Here we shall briefly focus on the genuine QCP phenomenon in Ce-systems and its theoretical description. The compounds under consideration have a tendency to AF order. Two different scenarios are discussed for the transition from the HF state to a phase with long-range magnetic order.

The first approach views the magnetic order as itinerant magnetism of the heavy quasiparticle system. The theory starts on the Fermi liquid side which is characterized by a large Fermi surface including the f-degrees of freedom. The low-energy excitations are fermionic quasiparticles and their collective excitations. Close to the QCP, the static susceptibility is assumed to diverge at a specific wave vector \( \mathbf{Q} \) which signifies the transition to a spin density wave state. The quasiparticles are strongly scattered by spin fluctuations along “hot lines”
which are connected by $Q$. This scattering modifies the low-temperature thermodynamic and transport properties which exhibit anomalous scaling relations close to QCP. They differ from those of the familiar Fermi liquid and therefore are at the focus of experimental interest. The theoretical derivation of the QCP scaling relations proceeds in close analogy to the spin-fluctuation theory of ferromagnetism (Moriya, 1985) starting from the self consistent RPA expression for the magnetic susceptibility of the heavy Fermi liquid (Moriya and Takimoto, 1995). The theory was recast in the renormalization group language (Hertz, 1976; Millis, 1993). The ansatz yields scaling relations for the specific heat, the susceptibility, and the Wilson ratio which differ from the well-known Fermi liquid results (Continentino, 2001; Sachdev, 1999). The scaling behavior obtained for various different models and many non-Fermi liquid compounds are summarized in Stewart (2001). The anomalous transport properties exhibited by metals near antiferromagnetic quantum phase transitions are of particular interest in the experimental study of Ce-based heavy fermion systems. The theory (Rosch, 1999) exploits the fact that in the vicinity of the magnetic instability the magnetic susceptibility reduces to the simple form

$$\chi^{-1}(q, \omega) \simeq \frac{1}{(q_0 \xi)^2} + \frac{\omega_q - i\omega}{\Gamma}$$

with

$$\frac{1}{(q_0 \xi)^2} \simeq r + c \left( \frac{T}{\Gamma} \right)^{3/2} \quad \text{and} \quad \omega_q \simeq \frac{1}{q_0^2} (q - Q)^2.$$  

Here $r$ measures the (almost $T$-independent) distance from the quantum critical point, $\Gamma$ denotes the characteristic energy scale of the fluctuations and $c$, $q_0$ are constants. The frequency magnetic $\omega_q$ vanishes at the ordering vector $Q$ of the magnetic structure. In the dirty limit the variation with temperature of the resistivity is given by

$$\rho(T) = \rho(0) + \Delta \rho(T) \quad \text{with} \quad \Delta \rho(T) \sim T^{3/2}. \quad (60)$$

In the clean limit, on the other hand, the transport is dominated by the carriers with the longest life times, i.e., by the quasiparticles far away from the ‘hot’ regions on the Fermi surface yielding $\rho(T) \sim T^2$. Equation (60) is valid only at very low temperatures $T/\Gamma < 1/\hat{\rho}_0$ where $\hat{\rho}_0$ denotes the residual resistivity ratio $\hat{\rho}_0 = \rho(0)/\rho(273 \text{ K})$. At intermediate temperatures $1/\hat{\rho}_0 < T/\Gamma < 1/\sqrt{\hat{\rho}_0}$ the resistivity rises linearly with temperature

$$\Delta \rho(T) \sim T \sqrt{\hat{\rho}_0}. \quad (61)$$

The important feature of the weak-coupling picture as described above is that $\Gamma$, which is related to the characteristic energy of the Fermi liquid phase still plays the role of the scaling energy. This fact is not surprising since only parts of the Fermi surface contribute to the anomalous properties. One should note however that this theory is not fully self-consistent because the AF fluctuation wave vector is not connected with any FS feature since the latter is assumed as spherical.

The loss of the energy scale $\Gamma$ is a characteristic feature of an alternative scenario which emphasizes the role of local correlations in quantum critical behavior (Si et al., 1999, 2001;
Coleman, 1999). The dynamical competition between the Kondo and RKKY interactions is analyzed by an extension (Smith and Si, 2000; Chitra and Kotliar, 2000) of the dynamical mean-field theory (see (Georges et al., 1996) and references therein). The resulting momentum-dependent susceptibility has the general form

\[ \chi^{-1}(q, \omega) = M(\omega) + J(q), \]

where the spin fluctuation self energy \( M(\omega) \) is calculated from the effective impurity problem. A zero-temperature phase transition will occur when \( \chi(q, \omega = 0) \) diverges. Anomalous behavior is expected whenever the (effective) local dynamics has an anomalous frequency dependence. Different scenarios are discussed in Si (2001). We finally mention that the compound YbRh\(_2\)Si\(_2\) (where Yb has one f-hole instead of one f-electron as in Ce) has been identified as one of the most promising and cleanest QCP systems where these theoretical models can be tested (Custers et al., 2003).

4. U-based heavy-fermion superconductors

Heavy Fermion superconductivity is found more frequently in intermetallic U-compounds than in Ce-compounds. This may be related to the different nature of heavy quasiparticles in U-compounds where the 5f-electrons have a considerable, though orbitally dependent, degree of delocalisation. The genuine Kondo mechanism is not appropriate for heavy quasiparticle formation as is the case in Ce-compounds. This may lead to more pronounced delocalised spin fluctuations in U-compounds which mediate unconventional Cooper pair formation as discussed in sect. 2. The AF quantum critical point scenario invoked for Ce compounds previously also does not seem to be so important for U-compounds with the possible exception of UGe\(_2\). On the other hand AF order, mostly with small moments of the order \( 10^{-2} \mu_B \) is frequently found to envelop and coexist with the SC phase in the \( B-T \) plane.

A common trait of U-compounds is the varying degree of localisation of 5f-electrons. It has recently become clear that Fermi surface properties of, e.g., UPt\(_3\) (Zwicknagl et al., 2002) and UPd\(_2\)Al\(_3\) (Zwicknagl et al., 2003) can be well described by treating two of the three 5f-electrons as localised in orbitals of specific symmetry and small CEF splitting of the order of meV. This approach also leads to a natural mechanism for the formation of heavy quasiparticles via the scattering of delocalised 5f-electrons by the internal CEF excitations of the localised 5f-electrons. This type of ‘dual model’ seems much closer to the truth than either a purely delocalised LDA type description which fails to explain the large masses or Kondo lattice models with fully localised 5f-states whose associated Kondo type anomalies above the Kondo temperature in transport and thermodynamics are not present in U-compounds.

In the following we will discuss to some length a number of prominent examples of U-compounds which have extensively been investigated experimentally and partly theoretically, mostly in the context of phenomenological Ginzburg–Landau (GL) theories or BCS type theories with magnetic effective pairing interactions. One is however far from being able to predict the spin state (parity) or even symmetry type of the gap function. Simple models
of AF spin fluctuations would predict singlet even parity gap functions but well characterised examples of triplet odd parity U-superconductors exist.

Indeed the hexagonal compound UPt$_3$ (Stewart et al., 1984) exhibits triplet pairing and it sticks out as the most interesting case of unconventional SC with a multicomponent order parameter whose degeneracy is lifted by a symmetry breaking field due to the small moment AF order. On the other hand in UPd$_2$Al$_3$ (Geibel et al., 1991a) SC coexists with large moment AF and probably spin singlet pairing is realised, it also exhibits a new kind of magnetic pairing mechanism mediated by propagating magnetic exciton modes. The sister compound UNi$_2$Al$_3$ (Geibel et al., 1991) is an example of coexistence of large moment AF with a SC triplet order parameter. In URu$_2$Si$_2$ (Palstra et al., 1985) the SC order parameter symmetry is still undetermined. The interest in this compound is focused more on the enveloping phase with a 'hidden' order parameter presumably of quadrupolar type or an 'unconventional' SDW. The oldest cubic U-HF superconductor UBe$_{13}$ (Ott et al., 1983) and its thorium alloy U$_{1-x}$Th$_x$Be$_{13}$ is also the most mysterious one. While for the pure system there is a single SC phase of yet unknown symmetry, in the small Th concentration range two distinct phases exist which may either correspond to two different SC order parameters or may be related to a coexistence of SC with a SDW phase. In addition in UBe$_{13}$ SC order appears in a state with clear non-Fermi liquid type anomalies. More recently the coexistence of ferromagnetism and SC in UGe$_2$ (Saxena et al., 2000) has been found. This is the only case of U-compounds where quantum critical fluctuations might be involved in the SC pair formation. Due to the FM polarisation the triplet gap function contains only equal spin pairing.

4.1. Multicomponent heavy fermion superconductor UPt$_3$

The intermetallic compound UPt$_3$ which crystallizes in the hexagonal Ni$_3$Sn structure is regarded as the archetype of a strongly correlated Fermi liquid. The existence of heavy quasiparticles is inferred from enhanced thermodynamic and transport coefficients, for example $\gamma \sim 420$ mJ/(mol U K$^2$) (Frings et al., 1983) and anisotropic Pauli-like susceptibility $\chi_c \sim 50 \cdot 10^{-9}$ m$^3$/(mol U) and $\chi_{a,b} \sim 100 \cdot 10^{-9}$ m$^3$/(mol U) (Frings et al., 1983; Stewart et al., 1984). In addition, the resistivity rises quadratically with temperature with a $T^2$-coefficient $A = 0.49 \mu\Omega/(\text{cm K}^2)$ (Taillefer and Lonzarich, 1988). The formation of the coherent heavy Fermi liquid state is clearly seen in the optical conductivity (Marabelli et al., 1987; Degiorgi, 1999; Dressel et al., 2002) and in the dynamical magnetic susceptibility as measured by neutron scattering (Bernhoeft and Lonzarich, 1995). Direct evidence for the existence of heavy quasiparticles comes from the observation of quantum oscillations (Tailfefer et al., 1987; Taillefer and Lonzarich, 1988; McMullen et al., 2001; Kimura et al., 1998) where Fermi surfaces and effective masses up to $m^* = 135 m$ were determined. The average dHvA mass enhancement $m^*/m_b \simeq 20$ compared to the band mass $m_b$ is found to be consistent with the specific heat enhancement $\gamma_{\text{exp}}/\gamma_{\text{calc}} = 17$ (Norman et al., 1988). These results prove convincingly that UPt$_3$ in its normal state is a heavy Fermi liquid.

The normal Fermi liquid state becomes unstable at low temperatures. At $T_N \simeq 5.8$ K an antiferromagnetic phase with extremely small ordered moments $\mu \simeq 0.035 \mu_B/U$ develops (Heffner et al., 1989; Hayden et al., 1992; Lussier et al., 1996). The most exciting phenomena are associated with the superconducting state. The existence of two clearly
distinct transition temperatures $T_{c1} = 530$ mK and $T_{c2} = 480$ mK (Fisher et al., 1989; Brison et al., 1994) implies that the superconducting phases are characterized by a multi-component order parameter. The discontinuities in the specific heat at the superconducting transitions show that the Cooper pairs are formed by the heavy quasiparticles of the normal heavy Fermi liquid state.

From this we conclude that a quantitative theory for the origin of the heavy quasiparticles in UPt$_3$ is a prerequisite for a detailed understanding of the fascinating low-temperature properties of this compound.

4.1.1. Dual model and heavy quasiparticles

Although a comprehensive picture of the low temperature ordered phases of UPt$_3$ has emerged, a complete theoretical understanding of the origin of 5f-derived heavy quasiparticles is still missing. The number of itinerant U 5f-electrons as well as the microscopic mechanism yielding the high effective masses are still controversial. It has been considered a success of the LDA that de Haas–van Alphen (dHvA) frequencies can be related to the areas of extremal orbits on the Fermi surface obtained by band-structure calculation which treats the U 5f-states as itinerant (Albers et al., 1986; Norman et al., 1988). From these findings, however, one should not conclude that the U 5f-states are ordinary weakly correlated band states. The calculated energy bands are much too broad for explaining the effective masses while on the other hand, they are too narrow to fit the photoemission data (Allen, 1992).

The key feature is likely the dual nature of the U 5f-states, i.e., the presence of both localized and delocalized U 5f-electrons. The theoretical investigation proceeds in three steps. Firstly, band-structure calculations have been performed starting from the self-consistent LDA potentials but excluding the U 5f $j = \frac{5}{2}, j_z = \pm\frac{5}{2}$ and $j_z = \pm\frac{1}{2}$ states from forming bands. The localized 5f-orbitals are accounted for in the self-consistent density and likewise in the potential seen by the conduction electrons. The 5f-bands are calculated by solving the Dirac equation (Albers et al., 1986). The intrinsic bandwidth of the itinerant U 5f $j = \frac{5}{2}, j_z = \pm\frac{3}{2}$ is taken from the LDA calculation while the position of the corresponding band center is chosen such that the density distribution of the conduction states as obtained within LDA remains unchanged. The resulting position of the f-band relative to the calculated Pt d-states is consistent with photoemission data. It was found that the U 5f-bands with $j_z = \pm\frac{3}{2}$ hybridize strongly near the Fermi level so that the f occupancy per U atom for the delocalized 5f-electrons amounts to $n_f = 0.65$ indicating that we are dealing with a mixed valent situation. The theoretical Fermi surface shown in fig. 23 is formed by two bands which are doubly degenerate and which are derived from the 5f $|j = \frac{5}{2}, j_z = \pm\frac{3}{2}\rangle$ states. The thermodynamically most important orbit is assigned to the $\Gamma$-centered strongly anisotropic electron surface shown in the left part of fig. 23. The A-centered part of the Fermi surface shown in the right part of fig. 23 has open orbits spanning the entire Brillouin zone for magnetic fields oriented along the $a$-direction in the basal plane. This feature is consistent with magnetoresistance measurements (McMullen et al., 2001). Now we turn to the discussion of the localized U 5f-states. The multiplet structure of the localized $f^2$-states is calculated by diagonalizing the Coulomb matrix. The spin–orbit splitting is rather large and therefore a $jj$-coupling scheme is used. This simplification gives six 2-particle states built from $|j = \frac{5}{2}, j_z = \pm\frac{5}{2}\rangle$ and
Fig. 23. UPt₃: FS of heavy quasiparticles as calculated within the dual model (Zwicknagl et al., 2002). One of the U 5f-electrons \((j = 5/2, j_z = 3/2)\) is treated as itinerant and is included in the FS while the remaining two are localized. Then two bands contribute to the FS. Left: \(\Gamma\)-centered heavy electron sheet which dominates \(\gamma\). Right: A-centered open orbit sheet consistent with magnetoresistance results (McMullen et al., 2001).

\(|j = 5/2, j_z = \pm 1/2\rangle\). The resulting eigenstates are generally no longer eigenstates of the total angular momentum \(J^2\), but remain eigenstates of \(J_z\). The Coulomb matrix elements are calculated following Condon and Shortley (1957). Inputs are the Slater–Condon parameters \(F^K\) (Coulomb integrals) and \(G^K\) (exchange integrals) which are evaluated with the radial function \(R_{j,5/2}(r)\) for U which is determined from a self-consistent band structure potential. Diagonalization of the matrix yields a doubly degenerate ground-state \(J_z = \pm 3\) which must be an eigenstate of \(J = 4\). Note that the Pauli principle permits even values of \(J\) only, i.e., \(J = 0, 2, 4\) in our case. The states \(|j = 5/2, j_z = 5/2; J = 4, J_z = \pm 3\rangle\) have an overlap of 0.865 with the Hund’s rule ground-state \(^3\text{H}_4\) derived from the LS-coupling scheme. Therefore the choice of \(jj\) vs. LS coupling should only weakly affect the results obtained for the ground-state multiplet. The two-fold degeneracy of the ground-state is lifted by a CEF yielding the two singlet states

\[
|\Gamma_3 \rangle = \frac{1}{\sqrt{2}}(|J = 4; J_z = 3\rangle + |J = 4; J_z = -3\rangle),
\]

\[
|\Gamma_4 \rangle = \frac{1}{\sqrt{2}}(|J = 4; J_z = 3\rangle - |J = 4; J_z = -3\rangle).
\]  

(63)

Note that \(|\Gamma_4\rangle\) has also been suggested as ground state of UPd₂Al₃ (Grauel et al., 1992; Böhm et al., 1992). The excited eigenstates of the Coulomb matrix are neglected, they are assumed to be separated by a rather high excitation energy from the ground state. The coupling between the localized and delocalized f-electrons is directly obtained from the expectation values of the Coulomb interaction \(H_C\) (eq. (2)) in the \(5f^3\)-states yielding the transition matrix element

\[
|M| = \left|\langle f^1; \frac{5}{2}, \frac{3}{2} | \otimes \langle \Gamma_4 | H_C | \Gamma_3 \rangle \otimes | f^1; \frac{5}{2}, \frac{3}{2} \rangle\right| = 0.19 \text{ eV}.
\]  

(64)

Finally, we determine the mass enhancement which arises from the scattering of the itinerant U 5f-electrons due to virtual excitations of the localized \(f^2\) subsystem. The latter are
characterized by a CEF splitting energy $\delta$ while the relevant energy scale for the ‘itinerant’ 5f-states is set by the effective band width $W^*$. To leading order in the ratio $\delta/W^*$ the mass enhancement $m^*/m_b$ is obtained from the fluctuation exchange (FLEX) contribution to the band self energy as shown in fig. 24 in close analogy to the case of Pr metals (White and Fulde, 1981). This leads to an expression

$$\frac{m^*}{m_b} = 1 - \left(\frac{\partial \Sigma}{\partial \omega}\right)_{\omega=0},$$

where $\omega_n$ are Matsubara frequencies, $m_b$ is the LDA band mass and $a_{5f}$ denotes the 5f weight of conduction electron states close to $E_F$. Furthermore the local 5f susceptibility and conduction electron propagator are given, respectively, by

$$\chi(i\omega_n - i\omega_{n'}) = \tanh\left(\frac{\delta}{2T} \frac{2\delta}{\delta^2 - (i\omega_n - i\omega_{n'})^2}\right),$$

$$g(i\omega_n) = \int dE \frac{N(E)}{i\omega_n - E - \Sigma(i\omega_n)}.$$

Here $2N(E)$ is the total electronic DOS obtained from the LDA bandstructure with two of the 5f-electrons kept localised.

This calculation procedure leads to a good agreement of theoretical and observed quasi-particle masses (Zwicknagl et al., 2002) for the main Fermi surface sheets. The results are summarized in fig. 25 as well as in table 4. If the dependence upon the wave vector $k$ is neglected one obtains an isotropic enhancement of the effective mass $m^*$ over the band mass $m_b$. The explicit evaluation requires as input parameters the density of states and the f-weight per spin and U-atom at the Fermi energy, $N(0)$ and $4a_{5f}^2$, respectively, as well as the average energy $\delta$ of CEF excitations. Solving for the selfconsistent conduction electron propagator with the input determined from the ab-initio calculations of the electronic structure ($N(0) \approx 15.5$ states/(eV cell), $|M|^2 = 0.036$ eV$^2$, $4a_{5f}^2 = 0.13$) and the estimate $\delta \approx 20$ meV we finally obtain the effective masses listed above.

The dual model and its partial 5f localisation scenario therefore provides for a satisfactory explanation for both the FS structure and mass renormalisation of the heavy quasiparticles observed in UPt$_3$. The physical reason for the strongly orbital dependent effect of local electron...
correlations on the different LDA hybridization matrix elements has been investigated separately (sect. 2). Finally we want to point out that the prescribed way of treating the 5f-electrons in UPt₃ is also applicable to other uranium compounds, notably UPd₂Al₃ (sect. 4.2).

4.1.2. Pairing and the spin–orbit coupling problem

There is little doubt that UPt₃ is an unconventional superconductor with a multicomponent superconducting order parameter. Theoretical models which have been proposed are reviewed by Sauls (1994) and Joynt and Taillefer (2002) and will be discussed in the next section. Here we shall briefly comment on the issue of the importance of spin–orbit coupling in the pairing scheme. In all theoretical treatments the superconducting phases of UPt₃ are characterized by BCS-type gap functions $\Delta_{s_1,s_2}(\mathbf{k})$ which satisfies the antisymmetry relation eq. (19). The wave vector $\mathbf{k} = k_F(\hat{\mathbf{k}})\hat{\mathbf{k}}$ is restricted to the Fermi surface since the coherence length $\xi_0$ is much larger than the interatomic distance or, equivalently, the Fermi wave length $\lambda_F = 2\pi/k_F$ satisfies

$$\xi_0 = \frac{\hbar v_F}{kT_c} \gg \lambda_F. \quad (67)$$

As a consequence, the gap function $\Delta(\mathbf{k})$ is a matrix function of $\mathbf{k}$ only or its corresponding polar and azimuthal angles $\vartheta$ and $\varphi$. We have neglected indices for interband pairing because the available phase space is much smaller than in the case of intraband pairs.
The indices \( s_1, s_2 \) refer to the (pseudo-)spin labels of the quasiparticles. Due to the large spin–orbit interaction the quasiparticle states are no longer eigenstates of the spin. In the absence of an external magnetic field the states \(|ks\rangle\) and \(|-k-s\rangle\) show Kramers degeneracy. In UPt\(_3\), the crystal structure has inversion symmetry. This fact implies that all four quasiparticle states \(|ks\rangle, |-ks\rangle, |k-s\rangle\) and \(|-k-s\rangle\) are degenerate. As a result, the superconducting order parameter can be decomposed into the contributions

\[
\begin{align*}
\text{even-parity (pseudo-spin singlet, } S = 0): & \quad \Delta_{s_1s_2}(k_F) = \Delta(k_F)(i\sigma_y)_{s_1s_2}, \\
\text{odd-parity (pseudo-spin triplet, } S = 1): & \quad \Delta_{s_1s_2}(k_F) = \Delta(k_F) \cdot (i\sigma_y)_{s_1s_2}.
\end{align*}
\]

(68)

Because the energy splitting of \( j = 5/2 \) and \( j = 7/2 \) states due to spin–orbit coupling is larger than the bandwidth due to hybridization and overlap (Albers et al., 1986) their mixing can be neglected. Consequently the pseudospin indices \( s_1, s_2 \) are good quantum numbers and effectively the classification of the gap function can proceed as for the weak spin–orbit coupling case (sect. 2).

4.1.3. Multicomponent superconducting order parameter

The hexagonal heavy Fermion compound UPt\(_3\) can justly be called the flagship of unconventional superconductivity, despite having a critical temperature less than one Kelvin. It is set aside from all other unconventional superconductors so far because it exhibits two superconducting phase transitions which have to be interpreted as a direct signature of the fact that the SC order parameter is a complex vector with more than one component. In all other unconventional superconductors, e.g., in high-\( T_c \) compounds the order parameter is still a complex scalar as in ordinary superconductors, albeit belonging to a nontrivial (but one-dimensional) representation of the symmetry group as witnessed by nodes in the anisotropic gap function. Only recently another example of a multicomponent superconductor, the Pr-skutterudite compound PrOs\(_4\)Sb\(_{12}\) (sect. 6), may have been found. The exciting discovery of the split SC transitions in UPt\(_3\) at \( T_{c1} = 530 \) mK and \( T_{c2} = 480 \) mK in specific heat measurements by Fisher et al. (1989) has lead to an enormous amount of experimental and theoretical work on UPt\(_3\) whose historical evolution has been described in detail in a recent review by Joynt and Taillefer (2002). Therefore in the following discussion we shall restrict to the essential aspects of SC in UPt\(_3\). The additional small moment AF observed in UPt\(_3\) \( (T_N = 5.8 \) K, \( \mu = 0.035 \mu_B) \) plays a key role in the identification of the SC order parameter since the in-plane staggered magnetization acts as a symmetry breaking field (SBF) to the SC multicomponent order parameter. The SBF is believed to be responsible for the appearance of two SC transitions which otherwise would merge into one, in fact this has later been proven by specific heat pressure experiments (Trappmann et al., 1991) on the SC \( T_{c1,2}(p) \) and by complementary neutron diffraction under pressure (Hayden et al., 1992). Naturally there are also two critical-field curves. They intersect at a tetracritical point in the \( B-T \) plane that is present for all field directions. Therefore one can identify three distinct SC phases \( A, B, C \) in the \( B-T \) plane (fig. 29) corresponding to different choices of the orientation of the complex vector SC-order parameter. These phase boundaries have first been identified by ultrasound velocity measurements (Bruls et al., 1990) and have been confirmed using the
same method (Adenwalla et al., 1990). Subsequently many different methods like thermal expansivity (van Dijk et al., 1993; van Dijk, 1994), field dependent specific heat (von Lönnneysen, 1994), etc. have yielded equivalent \( B-T \) phase diagrams. An important observation is a crossover of the anisotropy ratio of upper critical fields as function of temperature indicating that \( H_{c2} \) shows Pauli limiting for \( H \parallel c \) but not for \( H \perp c \) (Shivaram et al., 1987; Choi and Sauls, 1991). Again the asymptotic \( (T \ll T_c) \) low temperature behaviour of thermodynamic and transport coefficients, most notably thermal conductivity (Lussier et al., 1996; Graf et al., 1999) with power law behaviour points to a gap function with nodes. The precise node structure is different for the three phases due to the internal degrees of freedom for a multicomponent order parameter. Despite the wealth of experimental results on \( \text{UPt}_3 \) there is no unequivocal consensus on the symmetry and node structure of the superconducting gap. The many different proposals that have been put forward at various times are summarized in Joynt and Taillefer (2002). Here we restrict to the most commonly accepted \( E_{2u} \) (f-wave) model for the order parameter which was originally proposed by Choi and Sauls (1991), Sauls (1994) and in the form used here by Norman and Hirschfeld (1996). This odd parity spin triplet order parameter is consistent with \(^{195}\text{Pt}\) Knight shift measurements and the observed linear in \( T \)-dependence of the thermal conductivity for both \( a \)- and \( c \)-directions. The corresponding \( B \)-phase (e.g., \( H = 0, T < T_c \)) has a gap function with an equatorial node line and polar second order node points. The anisotropic paramagnetic limiting of the upper critical field suggests that its \( d \)-vector is pinned along the \( c \)-axis by an anisotropy potential due to spin–orbit coupling. This interpretation is however in conflict with a nearly isotropic constant Knight shift for larger fields (Kitaoka et al., 2000). Further investigations, notably field-angle dependent thermal conductivity experiments are necessary to confirm the node structure of the \( E_{2u} \) model.

The underlying normal heavy fermion state of \( \text{UPt}_3 \), which was the first system where heavy quasiparticles have directly been seen in dHvA oscillations (Taillefer and Lonzarich, 1988), has been described above. It was argued that the picture of heavy quasiparticle mass generation in 5f-metals has to be revised. This is due to a considerably different degree of localisation of 5f-electrons in different orbital states as opposed to the simple LDA picture which assumes complete delocalisation for all 5f-orbitals.

### 4.1.4. Small moment AF order

\( \text{UPt}_3 \) crystallizes in the hexagonal \( \text{MgCd}_3 \) structure (\( D_{6h} \) point group) and has two inequivalent U-sites with \( C_{3v} \) site symmetry. As in most other U-based superconductors the SC phase is embedded in an antiferromagnetic phase with very small moments (\( \mu = 0.035 \mu_B \)). The commensurate AF order of hcp \( \text{UPt}_3 \) with moments parallel to the hexagonal \( b \)-axis is indicated in fig. 26. The \( D_{6h} \) six-fold symmetry of the paramagnetic phase is reduced to an orthorhombic \( D_{2h} \) symmetry by the AF order with modulation vector \( \mathbf{Q} = (\frac{1}{2}, 0, 0) \). Although the AF intensity shows mean field behaviour \( \sim (T_N - T) \) (Aeppli et al., 1988), AF Bragg peaks retain a finite width pointing to imperfect AF ordering with a correlation length of \( \xi_{\text{AF}} \sim 150 \text{ Å} \). Only below 50 mK \( \xi_{\text{AF}} \) starts to diverge (Koike et al., 1998) and true long range magnetic order develops at 28 mK according to specific heat measurements (Schuberth et al., 1992). Still in the imperfect ordered regime the Bragg intensity starts to decrease below the superconducting
Fig. 26. Crystal structure and AF magnetic structure of UPt$_3$ below $T_N = 5.8$ K. The hexagonal axis defines the $c$ direction and the moments are oriented along $b \perp a$ (using orthogonal $a$, $b$, $c$ lattice vectors where $a = b = 5.764$ Å and $c = 4.884$ Å). Hexagonal planes are separated by $c/2$. The AF order leads to a doubling of the unit cell along $b$ (moment direction) resulting in orthorhombic ($D_{2h}$) symmetry of the AF crystal.

transition (Aeppli et al., 1988) which signifies a coupling of superconducting and AF order parameters, this has also been observed in URu$_2$Si$_2$, UPd$_2$Al$_3$ and UNi$_2$Al$_3$ (Metoki, 2001). Thus the small moment AF itself is a rather unconventional magnetic state but in the following discussions its influence on the superconducting state will be treated as ordinary AF order though with small moment.

4.1.5. The superconducting state, coupled with AF order
The exceptional nature of the superconducting state was already obvious from the two specific heat jumps first observed by Fisher et al. (1989) of comparable size indicating two SC transitions at $T_{c1,2}$. Later it was shown (Trappmann et al., 1991) that under hydrostatic pressure the two transitions merge at a critical pressure of $p_c \simeq 3.8$ kbar (fig. 27). It is important to note that above $p_c$ the two transitions do not separate again. Together with the pressure dependence of the AF moments observed in neutron diffraction (Hayden et al., 1992) this gives a clue about the nature of the two superconducting transitions as well as the role of AF order. If the two transitions would correspond to two SC phases whose order parameters belong to one-dimensional representations with accidentally close transition temperatures then one would expect simply a crossing of $T_{c1,2}(p)$ at $p_c$, there would be no symmetry reason why the two transitions should merge above $p_c$ as is actually seen in fig. 28. The observation (Hayden et al., 1992) that the magnetic moment vanishes at about the same critical pressure (fig. 28) then suggests a natural scenario: at large hydrostatic pressure $p > p_c$ which preserves hexagonal symmetry the SC order parameter belongs to a two-dimensional hexagonal representation, $E_1$ or $E_2$, i.e., it is a complex 2-component vector $\eta = (\eta_1, \eta_2) = (|\eta_1|, |\eta_2|e^{i\Phi})e^{i\phi}$ rather than a complex scalar $\eta e^{i\phi}$ as for one-dimensional representations associated with a single $T_c$. Here $\phi$ is the overall Josephson-phase of the order parameter and $\Phi$ is an intrinsic phase which is a novel feature of the 2-component order parameter. At pressures $p < p_c$ the AF moments appear, since they are aligned in the $b$-direction they reduce hexagonal symmetry to orthorhombic symmetry. The AF order parameter acts as a symmetry breaking field (SBF), any small coupling between the AF and SC order parameters will then lead to a splitting of $T_c$ into $T_{c1,2}$ which are associated with two SC order parameters of different symmetry with respect to the orthorhombic group. Therefore the small moment magnetism with
its easy-plane anisotropy is a lucky coincidence which helps to unveil the complex vector nature of the SC order parameter. This scenario may generically be described within the Landau theory characterised by a free energy density (Machida et al., 1989; Hess et al., 1989; Joynt, 1990)

$$f_L(\eta, M_Q) = \alpha_0(T - T_c)\eta \cdot \eta^* + \beta_1(\eta \cdot \eta^*)^2 + \beta_2|\eta \cdot \eta^*|^2 - \gamma_0 M_Q^2(\eta_x^2 - \eta_y^2).$$

(69)

Here $M_Q = M_Q\hat{y}$ is the AF order parameter which has already reached saturation in the SC temperature range, furthermore a shift of the critical temperature $T_c \sim M_Q^2$ has already been included in $T_c$. There are two fourth order terms characterized by the Landau parameters $\beta_1$, $\beta_2$ as compared to only one in the case of a scalar SC order parameter. The last term is a phenomenological coupling of the SBF to the E-type SC order parameter. This type of theory is the same for each of the even or odd two-dimensional E-representations. The different $k$-dependence of the respective SC-gap function enters only via the parameters of the Landau free energy in eq. (69) where $\alpha_0, \beta_1 > 0$. The ratio $\beta_2/\beta_1$ selects the SC ground state in the two-dimensional manifold of possible order parameters $\eta$. First we discuss the decoupled case $\gamma_0 = 0$ where a single transition at $T_c$ into a state belonging to that manifold takes place. Then all possible SC ground states have $|\eta_1| = |\eta_2| \equiv |\eta|$. For $-1 < \beta_2/\beta_1 < 0$ the ‘unitary’ state with intrinsic phase $\Phi = 0$ is stable whereas for $\beta_2/\beta_1 > 0$ the ‘non-unitary’ state with $\Phi = \pm \pi$ and $\eta \times \eta^* \neq 0$ is the stable one, which is the case in UPt$_3$. The SBF splits the degenerate
E-representations through the last term in eq. (69). Whereas at low temperature the stable state is determined by the fourth order terms, close to \( T_{c1} \) the second order terms \( \sim \gamma_0 > 0 \) dominate and favor a single component state, only below \( T_{c2} \) will the second component be nonzero. One has two phases \((A) \eta = \eta(1, 0)\) below \( T_{c1} \) and \((B) \eta = \eta(1, a i)\) below \( T_{c2} \). The \( T_{c1,2} \)-splitting and the difference in amplitudes \((a < 1)\) in the \( B \)-phase are determined by the Landau parameters according to

\[
\Delta T_c = T_{c1} - T_{c2} = \left( 1 + \frac{\beta_1}{\beta_2} \right) \lambda_0 T_c,
\]

\[
a = \frac{1 - \frac{\beta_1}{\beta_2} \lambda_0}{1 + \frac{\beta_1}{\beta_2} \lambda_0},
\]

\[
\lambda_0 = \frac{\gamma_0 M_{Q}^2}{\alpha_0 T_c}.
\]

Using the somewhat smaller transition temperatures \( T_{c1,2} \) of van Dijk (1994) with \( \Delta T_c = 0.054 \text{ K} \) (fig. 29) one obtains for the dimensionless coupling parameter of SC and AF order **Fig. 28. Reduction of the squared AF moment in UPt\(_3\) with hydrostatic pressure \( p \) (above), it vanishes at a critical pressure of \( p_c \approx 3.8 \text{ kbar} \). Simultaneously the SC \( T_c \) splitting is reduced and also vanishes at \( p_c \) staying zero for larger pressures (Hayden et al., 1992). This supports the idea that a two-fold degenerate SC order parameter \((e.g., E_{2u})\) is split by an AF symmetry breaking field according to eqs. (69), (70).**
parameters $\lambda_0 = 0.038$. The smallness of this parameter is primarily due to the small AF moment in UPt$_3$. If it would be only slightly larger the lower $B$-phase of UPt$_3$ would be completely suppressed and only one SC transition would be observable. Furthermore one has $\beta_1/\beta_2 = (T_2 - T_1)/\lambda_0 T_c \approx 2$ and the amplitude ratio $a = 0.86$ which will be set to 1 in the following. The ratio of the SC specific heat jumps at $T_{c1,2}$ is given by

$$\frac{\Delta_2 C}{\Delta_1 C} = \frac{T_2}{T_{c1}} \left( 1 + \frac{\beta_2}{\beta_1} \right).$$

(71)

Here $\Delta_i C (i = 1, 2)$ are both counted from the normal state. Using the experimental values this also leads to a ratio $\beta_1/\beta_2 = 2$. This type of phenomenological theory can be extended to include also the pressure dependence (Thalmeier et al., 1991). From the two different slopes of $T_{c1,2}$ vs. pressure one may infer a magnetic Grüniesen parameter $\Omega_M = -(\partial \ln M^2_Q/\partial \epsilon_p) = -385$ which is in reasonable agreement with $\Omega_M = -260$ as determined directly from the pressure dependence of Bragg intensities which are $\sim M^2_Q$. This gives further support that the AF order provides the SBF that leads to the observed splitting into $(1, 0)$ and $(1, i)$ superconducting phases at zero field.
4.1.6. The critical field curves and Ginzburg–Landau theory

Naturally the two superconducting transitions at \( T_{c1,2} \) will lead to two different critical field curves \( H_{c}^{1,2} \) which have been investigated by experimental methods mentioned before. The most obvious feature is a crossing of the critical field curves at a tetracritical point for all field directions. As a consequence there are three distinct SC regions in the \( H–T \) plane with phases \( A \) (high \( T \), low \( H \)), \( B \) (low \( T \), low \( H \)) and \( C \) (low \( T \), high \( H \)). This important result has first been obtained by Bruls et al. (1990) with ultrasonic experiments. In fig. 29 we show the phase diagram for \( H \perp c \) and \( H \parallel c \) obtained by van Dijk (1994) using thermal expansion measurements. Its most important properties may be summarized as follows: (i) The existence of a tetracritical point \( (T_1, H_1) \). Purely thermodynamical analysis of its vicinity (Yip et al., 1991) leads to the conclusion that if all four phase boundaries meeting at \( (T_1, H_1) \) are of second order, relations between their slopes and the specific heat jumps across them exist. They were confirmed by van Dijk (1994) which proved the second order nature of transitions. The change of slopes leads to pronounced kinks in the phase boundaries for \( H \parallel a \). (ii) Defining the upper critical field \( H_{c2}(T) \) as the upper phase boundary in fig. 29 for \( a, c \) directions one notices a reversal of the \( a-c \) anisotropy ratio \( H_{c2}^{a}(T)/H_{c2}^{c}(T) \) as function of temperature. This ratio is \( < 1 \) for \( T/T_c > 0.4 \) and \( > 1 \) for \( T/T_c < 0.4 \). This reversal has been advanced as a major argument for spin triplet pairing by Sauls (1994). For the explanation of the structure of the \( A, B, C \) phase diagram an appropriate Ginzburg–Landau theory for a SC 2-component vector order parameter has to be constructed (Machida et al., 1989; Hess et al., 1989). It is obtained by adding all appropriate hexagonal invariants formed from \( \eta_j \) and its covariant gradient operators \( D_j = \partial_j - i A_j \) with \( A \) denoting the vector potential corresponding to the field \( B \) given in units of \( \phi_0/2\pi \) where \( \phi_0 = \frac{hc}{2e} \) is the flux quantum. The homogeneous terms of the GL free energy density are again given by eq. (69) to which we have to add the gradient terms

\[
f_G(\eta, M_Q) = K_1(|D_x \eta_1|^2 + |D_y \eta_2|^2) + K_2(|D_x \eta_2|^2 + |D_y \eta_1|^2) + K_3(D_{1x} \eta_1 D_{1y}^* \eta_1^* + D_{2x} \eta_2 D_{2y}^* \eta_1^* + c.c.) + K_4(|D_x \eta_1|^2 + |D_x \eta_2|^2)
- \kappa M_Q^2 (|D_x \eta_1|^2 - |D_y \eta_1|^2 + |D_x \eta_2|^2 - |D_y \eta_2|^2).
\]

The expression for the total GL free energy density \( f_{GL} = f_L + f_G \) is valid for both \( E_1 \) and \( E_2 \) type order parameters of odd and even parity but in the whole section we restrict discussion to the most likely \( E_{2u} \) case. The last term in eq. (72) with \( K_M = \kappa M_Q^2 \) describes the coupling of the SBF to the gradients of the SC order parameter. In a conventional GL theory for a scalar order parameter one has only one GL parameter \( K = \frac{hc}{2m^*} \) determined by the effective electron mass. For the vector order parameter \( \eta \) one has altogether 5 parameters. It is therefore obvious that a GL theory for a multicomponent superconductor has little real predictive power, but it may be used as a convenient frame for qualitative discussion. In weak coupling BCS theory the coefficients \( K_i \) can be expressed as angular averages of Fermi velocities (Sauls, 1994) which leads to the conclusion that in the \( E_{2u} \)-case one has \( K_2, K_3 \ll K_1 \). In this case the terms mixing \( \eta_x \) and \( \eta_y \) can be neglected and the solution of the linearized GL-equations corresponding to eqs. (69), (72) lead to two parallel critical field curves without the term
$\sim K_M$ for all directions of the field. The tetracritical point in this model is therefore entirely due to the coupling of gradient terms $D_{x,y}\eta_i$ to the SBF where the sign of $\kappa$ is negative, i.e., it has to be opposite to that of $\gamma$ in the homogeneous coupling term of eq. (69) in order to obtain a crossing at $T_c$. Using eq. (72) and the quadratic part of eq. (69) one obtains the critical field curves

$$H_{c2}^1 = \frac{\phi_0}{2\pi} \frac{\alpha_0(T_{c1} - T)}{\sqrt{(K_1 + K_M)K_4}}.$$  

$$H_{c2}^2 = \frac{\phi_0}{2\pi} \frac{\alpha_0(T_{c2}^0 - T)}{\sqrt{(K_1 - K_M)K_4}}.$$  

(73)

Here $T_{c2}^0 > T_{c2}$ is the second transition temperature without inclusion of quartic terms of eq. (69) (Machida et al., 1989). For fields parallel to the $c$-axis similar expressions hold with $\sqrt{(K_1 \pm K_M)K_4}$ replaced by $(K_1 \pm K_M)$. Ignoring $K_M$ for the moment the $a$–$c$ anisotropy of $H_{c2}$ is given by a slope ratio $H_{c2}^1/H_{c2}^{1u} = \sqrt{K_4/K_1} = \sqrt{(v_z^c)/(v_0^a)} = \sqrt{m_a/m_c}$. The experimental slope ratio 1.64 is indeed exactly equal to the root of the mass anisotropy $m_a/m_c = 2.7$ as obtained from transport measurements. The linearized version of the $E_{2u}$ model has however no natural explanation for the slope changes ($B \perp c$) at the tetracritical point. On the other hand it explains the basic observation of the three ($A, B, C$) phases in the $B$–$T$ plane for all field directions. In addition to the low- (or zero-) field phases ($A, B$) discussed before a high field phase ($C$) with an order parameter $\eta = |\eta|(0, 1)$ appears which is stabilized by the gradient coupling term $\sim K_M$.

4.1.7. The superconducting gap function

A detailed understanding of thermodynamics and transport properties requires the $k$-dependent gap functions of the three phases. Their node structure determines the low temperature behaviour of thermodynamic and transport coefficients. The most commonly proposed triplet $f$-wave gap function with $E_{2u}$ symmetry is given by

$$d(k) = \eta_1\left[k_x\hat{z}(k_x^2 - k_y^2) + \eta_2(2k_x\hat{z}k_xk_y)\right].$$  

(74)

The orientation of the $d$-vector is assumed to be $k$-independent and pinned along the hexagonal $c$-axis ($\hat{z}$) by an anisotropy potential acting on the (Kramers degeneracy) pseudo spin of the heavy quasiparticles which has its origin in the spin–orbit coupling of 5f-electrons. This pinning effect leads to a large paramagnetic limiting effect for $B \parallel d, \hat{z}$ and to no effect for $B \perp d, \hat{z}$ which was suggested (Graf et al., 2000) as origin of the crossover in the $a$–$c$ upper critical field anisotropy mentioned before. This is claimed as strong evidence for triplet pairing (Graf et al., 2000) and as argument against the singlet $d$-wave ($E_{1u}$) gap function. Defining $d(k) = \Delta(k)\hat{z} = \Delta(\vartheta, \varphi)\hat{z}$ where $\vartheta, \varphi$ are the polar and azimuthal angles of $k$ one has explicitly the following $E_{2u}$ gap functions (without normalization) for the $A, B$ ($a = 1$) and $C$ phases ($\Delta_0 = |\eta_{1,2}|$)

$$A(1, 0): \quad \Delta(\vartheta, \varphi) = \Delta_0 k_c(k_x^2 - k_y^2) = \Delta_0 \cos \vartheta \sin^2 \vartheta \cos(2\varphi),$$

$$B(1, i): \quad \Delta(\vartheta, \varphi) = \Delta_0 k_c(k_x + ik_y)^2 = \Delta_0 \cos \vartheta \sin^2 \vartheta \exp(2i\varphi),$$

$$C(0, 1): \quad \Delta(\vartheta, \varphi) = 2\Delta_0 k_xk_y = \Delta_0 \cos \vartheta \sin^2 \vartheta \sin(2\varphi).$$  

(75)
An angular plot of these gap functions is shown in fig. 30 which also exhibits their node structure described in the caption. In the above equation the equal amplitude approximation \( a = 1 \) was made for the \( B \)-phase which leads to a gap with full rotational symmetry around \( c \).

As an effect of the SBF which leads to \( a < 1 \) the rotational symmetry is slightly broken, one has to multiply the \( B \)-phase gap function by a factor \( (1 - \epsilon^2 \sin^2 (2\varphi))^{1/2} \) with four-fold symmetry around \( c \) and \( \epsilon^2 = 1 - a^2 = 0.26 \). The node structure is unchanged, it consists of an equatorial line node and two second order node points at the poles. Both contribute to the linear behaviour of the quasiparticle DOS \( N(E) \) for \( E \ll \Delta_0 \). It was calculated (Yang and Maki, 2001) including the effect of impurity scattering which quickly leads to a residual DOS \( N(0) \). Indeed in UPt_3 large residual \( \gamma \) values are usually obtained by extrapolation from temperatures above the magnetic ordering at 28 mK. Finally one should note that the presence of a node line is not required by symmetry if one assumes the strong spin–orbit coupling case (Blount, 1985), it is rather a result of a special choice of the \( E_{2u} \) order parameter in the allowed subspace. Forming a linear combination of all possible \( E_{2u} \) representations there are in general only point nodes. However, as argued above and in sect. 2, the weak spin–orbit coupling case is more realistic in the effective quasiparticle pseudo spin picture for UPt_3 and then the appearance of node lines is natural. It is obvious from fig. 30 and eq. (75) that the non-unitary \( B \)-phase has fewer nodes than the unitary \( A \)- or \( C \)-phases. This general feature of non-unitary phases means that they are prefered as stable low temperature states in unconventional SC because fewer nodes lead to a larger condensation energy.

The ratio of fourth order Landau coefficients \( \beta_1 / \beta_2 \simeq 2 \) was taken as evidence for the weak coupling nature of superconductivity. Therefore the question arises whether it is possible to calculate the symmetry of the order parameter within a microscopic weak coupling Hamiltonian starting from an on-site effective quasiparticle interaction which is repulsive. Such
attempts have originally been made without the inclusion of the orbital degeneracy of U-5f states and within one band models with the static susceptibility tensor used as an input. However, not surprisingly the gap functions obtained are sensitive to the input function and also the $E_{2u}$ gap function is not the favored one (Norman, 1991, 1994) in such models. Attempts to include orbital degrees of freedom to remedy this situation have been made (Norman, 1994). As mentioned before it has now become clear that 5f-states in UPt$_3$ have also a dual nature, partly localised in 5f$^2$ configurations and partly itinerant states which have heavy masses due to renormalisation by low lying CEF excitations of the 5f$^2$-states. It is therefore possible that, as in the case of UPd$_2$Al$_3$, exchange of localised 5f$^2$ excitation modes play an essential role for the formation of unconventional Cooper pairs in UPt$_3$.

4.1.8. Low temperature transport properties

The quasiparticle energies vanish at the nodes of the superconducting gap. Therefore the low temperature ($T \ll T_c$) behaviour of thermodynamic and transport coefficients may give information on the node structure. Specific heat analysis has proved of little use due to the presence of the huge 28 mK peak caused by the onset of true long range magnetic order of the small moments. Thermal conductivity measurements for $0.1 < T/T_c < 0.3$ (Lussier et al., 1996) have been more successful. The temperature dependence in this asymptotic regime requires the presence of node lines and/or second order node points, furthermore comparison of the anisotropy ratio $\kappa_c(T)/\kappa_a(T)$ shown in fig. 31 with the calculated ones (Norman and Hirschfeld, 1996) gives good agreement for the $E_{2u}$ gap function of the $B$-phase in eq. (75). Extrapolation for zero temperature leads to a ratio $\sim 0.5$ contrary to the older $E_{1g}$ model which extrapolates to zero. This difference is due to the presence of second order node points with $\Delta(\vartheta) \sim \vartheta^2$ or $(\pi - \vartheta)^2$ at the poles in the $E_{2u}$ case. Finally for $T/T_c < 0.1$ one reaches

![Fig. 31. Anisotropy ratio $\kappa_c/\kappa_b$ of thermal conductivities as function of the reduced temperature. Data from (Lussier et al., 1996).](image)
the gapless regime where the pair breaking energy scale \( \sqrt{\hbar \Gamma kT} > kT \) with \( \Gamma \) denoting the normal state impurity scattering rate. In this case a behaviour \( \kappa/T \sim \alpha + \beta T^2 \) is predicted (Graf et al., 1996) in good agreement with observations (Suderow et al., 1997).

4.1.9. NMR Knight shift results
The Knight shift (KS) in the superconducting regime is a direct measure of the change \( \delta \chi_s \) in the spin susceptibility of the SC condensate. It can be measured by \( \mu \)SR or NMR experiments. In the latter the NMR resonance frequency is shifted due to the contact interaction of nuclear and electronic spin moment (sect. 2). Ideally when spin–orbit coupling can be neglected, the KS should drop to zero for a singlet SC state and should be unchanged for a triplet state. Spin–orbit coupling complicates the interpretation and may lead to only a partial reduction for a singlet state and an observable anisotropic reduction in the triplet state (Mineev and Samokhin, 1999). In the latter case for a rigid \( \mathbf{d}(\mathbf{k}) \) with respect to \( \mathbf{H} \) one has for the susceptibility tensor \((i, j = x, y, z)\):

\[
\chi_{sij} = \chi_n \int \frac{d\Omega}{4\pi} \left[ (P_{ij}^\perp(\mathbf{k}) + Y(\mathbf{k}, T) P_{ij}^\parallel(\mathbf{k})) \right],
\]

where \( \chi_n \) is the normal state spin susceptibility (assumed isotropic for simplicity). Furthermore \( P_{ij}^\parallel(\mathbf{k}) = d_i^*(\mathbf{k})d_j(\mathbf{k})/|\mathbf{d}(\mathbf{k})|^2 \) and \( P_{ij}^\perp(\mathbf{k}) = [\delta_{ij} - P_{ij}^\parallel(\mathbf{k})] \) are the projectors to the direction parallel and perpendicular to \( \mathbf{d}(\mathbf{k}) \), respectively, and \( Y(\mathbf{k}, T) \) is related to the Yoshida function of sect. 2.

In the strong spin–orbit coupling case with the \( \mathbf{d} \)-vector pinned along \( c \) one would expect a suppression of the KS for \( \mathbf{H} \) applied parallel to the \( c \) direction due to the effect of \( Y(\mathbf{k}, T) \) but not for \( \mathbf{H} \perp c \), i.e., for \( \mathbf{H} \) in the hexagonal plane. The Knight shift should therefore be quite anisotropic for pinned \( \mathbf{d} \)-vector. However it was found (Tou et al., 1996, 1998;
Kitaoka et al., 2000) that for large fields (in the C-phase) there is almost no change for both field directions. This was interpreted as evidence that the weak spin–orbit coupling case is realized in UPt$_3$. In this case the d-vector may be rotated by the field. For large fields it is always perpendicular to $\mathbf{H}$ and therefore no KS reduction will take place. On the other hand for low fields (in the B-phase) it may be pinned by the SBF (parallel to $a$) and therefore rotation of $\mathbf{H}$ in the hexagonal ab-plane will produce an anisotropic Knight shift as shown in fig. 32. The weak spin–orbit coupling scenario has also been theoretically investigated (Ohmi and Machida, 1996, 1996a). On the other hand this interpretation seems to be in conflict with the anisotropic paramagnetic limiting which leads to the reversal for the upper critical field anisotropy as explained before. This discrepancy has so far not been resolved.

4.1.10. Magnetothermal properties in the vortex phase

The $a$–$c$ anisotropy of the zero-field thermal conductivity in the B-phase has provided a major argument for the $E_{2u}$ SC order parameter with its second order nodal points at the poles. These node points (and also the equatorial node line) have, however, not been seen directly until now. As explained in sect. 2 magnetothermal conductivity or specific heat measurements in the vortex phase as function of field angles can provide such direct evidence. Due to the comparatively low $T_c$ of UPt$_3$ experiments have to be done at 50 mK or below, so far they have not been performed. Calculations of the field angle dependence for the B-phase (Maki and Thalmeier, 2003) and the C-phase (Thalmeier and Maki, 2003a) have been carried out however. For simplicity we discuss only the former since the modulus of the gap function in the B-phase is isotropic in the hexagonal plane according to eq. (75). This means that specific heat and $c$-axis thermal conductivity $\kappa_{zz}$ depend only on the polar field angle $\theta$. They are given in the low temperature and superclean limit of sect. 2 by $\langle \tilde{v} \rangle = (\langle v_a v_c \rangle)^{1/2}$

![Fig. 33. Angular dependence of specific heat (dot-dashed) and thermal conductivity $\kappa_{ii}$ ($i = x, z$) on polar field angle $\theta$. Cusps at $\theta = 0$ are due to polar node points of the gap in eq. (75). While $\kappa_{zz}$ is independent of $\phi$, $\kappa_{xx}$ is shown for $\phi = \frac{\pi}{4}$ (Thalmeier and Maki, 2003a).](image)
\[
\frac{C_s(\theta)}{\gamma T} = \frac{1}{\sqrt{3}} \tilde{v} \sqrt{eH I_B(\theta)}, \\
\frac{\kappa_{zz}(\theta)}{\kappa_n} = \frac{2}{3} \tilde{v}^2 \frac{\Delta^2 (eH) I_B(\theta) F_{zz}^B(\theta)}{\Delta^2 (eH) I_B(\theta) F_{zz}^B(\theta)}, \\
I_B(\theta) = \alpha \sin \theta + \frac{2}{\pi} E(\sin \theta),
\]

where \( F_{zz}^B(\theta) = \sin \theta \) and \( E(\sin \theta) \) is the complete elliptic integral. The \( \theta \)-dependent factors in the above equation are shown in fig. 33. Sharp cusps for \( \theta = 0, \pi \) are indeed seen in these functions which, if observed in the corresponding specific heat \( C_s(\theta) \) and thermal conductivity \( \kappa_{zz}(\theta) \), would constitute a direct and unambiguous proof for the polar node points in the \( B \)-phase of UPt\(_3\). Likewise the equatorial line node of the \( B \)-phase will lead to a flat minimum (without a cusp) for \( \theta = \frac{\pi}{2} \) in \( \kappa_{xx}(\theta) \). Observing these features in UPt\(_3\) as ultimate proof for the node structure is an experimental challenge.

4.2. Magnetic exciton mediated superconductivity in UPd\(_2\)Al\(_3\)

Among the U-based HF superconductors UPd\(_2\)Al\(_3\) (Geibel et al., 1991a) is a rather special case. In this HF compound with a moderate \( \gamma = 140 \text{ mJ/mol K}^2 \) there is also AF order below \( T_N = 14.3 \text{ K} \) with almost atomic size local moments (\( \mu = 0.85 \mu_B \)) in contrast to the small moments in other U-compounds. The entropy release is \( \Delta S(T_N) = 0.67 R \ln 2 \) per mole, much larger than in the itinerant SDW sister compound UNi\(_2\)Al\(_3\). The AF order coexists with superconductivity below \( T_c = 1.8 \text{ K} \). This suggests that in addition to the heavy itinerant quasiparticles nearly localised 5f-electrons should be present. They result from the dominating 5f\(^2\) configuration of the \( \text{U}^{4+} \) ion (Grauel et al., 1992). This dual nature of 5f-electrons is even more obvious than in UPt\(_3\) as is seen from various experimental investigations like susceptibility (Grauel et al., 1992), Knight shift (Feyerherm et al., 1994) and optical measurements (Dressel et al., 2002). The former exhibit a pronounced \( a-c \) axis anisotropy (Grauel et al., 1992) shown in fig. 34 with a much larger \( \chi_{\perp}(T) \) whose \( T \)-dependence is very reminiscent of CEF-effects.

Fig. 34. Magnetic susceptibility of UPd\(_2\)Al\(_3\) for field parallel to \( a \)- and \( c \)-axis. The pronounced maximum for \( \chi_{\perp}(T) \) is a typical signature of CEF split localised 5f-states (Grauel et al., 1992). The full line is a fit using a \( \text{U}^{3+} \) CEF level scheme containing singlet ground state and first excited state (33 K).
A CEF scheme with two low lying singlets split by $\delta$ was proposed for the 5f$^2$ localized states. Later investigations (Shiina, 2001) indicate that the first excited state is rather a doublet with $\delta = 6$ meV. Since the ground state is a singlet the local 5f-moment magnetism must be of the induced type caused by the mixing with the excited doublet via inter-site exchange. A direct confirmation of this dual nature of 5f-electrons in UPd$_2$Al$_3$ was obtained from inelastic neutron scattering (INS) (Mason and Aeppli, 1997) which found excitations that originate in local CEF transitions of energy $\delta$ and disperse into bands of `magnetic excitons’ due to intersite exchange. This band extends up to 8 meV and along the $c$ direction the modes are propagating with little damping. Later high resolution INS experiments (Sato et al., 1997, 1997a; Bernhoeft et al., 1998; Bernhoeft, 2000) have shown that below $T_c$ a resonance like structure in the dynamical structure function of localised moments appears which is linked to the superconducting quasiparticles. Complementary tunneling experiments probe the response of the itinerant quasiparticles and their superconducting gap. In a breakthrough experiment this has been achieved the first time for a HF superconductor using an epitaxially grown UPd$_2$Al$_3$–AlO$_x$–Pb tunneling device (Jourdan et al., 1999). Typical strong coupling features in the tunneling DOS have been observed which appear at an energy related to the excitations of local moments seen in INS. Together both experiments strongly suggest that the magnetic excitons identified in INS are the bosonic ‘glue’ which binds the electrons together to Cooper pairs (Sato et al., 2001). This is a new mechanism for superconductivity distinctly different from both the electron–phonon and spin fluctuation mechanism known sofar. The pairing potential is mediated by a propagating boson (the magnetic exciton) as in the former but depends on the spin state of conduction electrons as in the latter. It is the main purpose of this section to present the evidence for this important new mechanism for unconventional superconductivity. Before considering this in detail we summarize some essential physical facts known about UPd$_2$Al$_3$.

4.2.1. AF structure and superconducting properties

The AF magnetic structure of UPd$_2$Al$_3$ consists of FM ordered hexagonal planes with moments ($\mu = 0.83\mu_B$) in [100] direction and stacked antiferromagnetically along the $c$-axis (Krimmel et al., 1992; Kita et al., 1994). It corresponds to an AF wave vector $\mathbf{Q} = (0, 0, \frac{1}{2})$ (in r.l.u.) and is shown in fig. 35. As expected from the large $a$–$c$ anisotropy of the susceptibility in fig. 34 the AF structure does not change in applied magnetic fields $\mathbf{H}$ along c. For fields in the easy $ab$-plane the hexagonal in-plane anisotropy is much smaller and moment reorientation can be seen. The magnetic $B$–$T$ phase diagram has been determined by Kita et al. (1994) and no effects of the superconducting transition at $T_c = 1.8$ K on the magnetic structure has been found supporting the idea of two separate superconducting (itinerant) and magnetic (localised) 5f-subsystems.

The superconducting state of UPd$_2$Al$_3$ has been investigated by thermodynamic and transport measurements (Caspar et al., 1993; Hessert et al., 1997; Hiroi et al., 1997), NMR (Tou et al., 1995; Matsuda et al., 1997), $\mu$SR experiments (Feyerherm et al., 1994) and tunneling studies (Jourdan et al., 1999). Despite this great effort the symmetry of the order parameter is not reliably known sofar.

The upper critical field exhibits flattening for low temperatures which has been interpreted as a Pauli limiting effect and hence evidence for singlet pairing (Hessert et al., 1997). The
angular dependence \( H_{c2}(\theta) \), where \( \theta \) is the polar angle, changes dramatically from \( T = 1.5 \) K to \( T = 0.5 \) K which was interpreted as a gradual change from d-wave to s-wave character of the order parameter when temperature is lowered. Within this picture the effect of background AF order was ignored which may not be permissible because the critical fields for localized 5f-spin reorientation are of the same order as \( H_{c2} \) (Grauel et al., 1992; Kita et al., 1994).

The Knight shift in the SC phase is a direct probe for the reduction of the spin susceptibility \( \chi_s(T) \) in the SC state (Tou et al., 2003). Ideally there should be a complete suppression of \( \chi_s(T) \) for singlet pairing and no effect for triplet pairing (sects. 2 and 4.1). This picture is however complicated by the effect of spin–orbit coupling which may lead to intermediate cases. The result for \( \Delta K_s(T) \) as obtained from \( ^{27}\text{Al} \) NMR experiments (Tou et al., 1995) is shown in fig. 36. Although a reduction is clearly visible, large (anisotropic) residual Knight shift values remain. This is attributed to the localized 5f-susceptibility which cannot be independently determined. Therefore these results and similar ones from \( \mu \)SR experiments (Feyerherm et al., 1994) are difficult to interpret. It is not possible to say with certainty how strong
the Knight shift caused by the itinerant quasiparticles is actually reduced in the SC phase. But the conclusion concerning singlet pairing seems unaffected by this ambiguity.

The $^{27}\text{Al NMR}$ relaxation rate $T_{1}^{-1}$ was found to exhibit a $T^{3}$ behaviour in the SC state over four orders of magnitude down to $0.1T_{c}$ (Tou et al., 1995) where the gap amplitude will be constant. This means that $\Delta(k)$ should have a node line implying a low quasiparticle energy DOS $N_{s}(E) \sim |E|$. This conclusion was confirmed by $^{105}\text{Pd NMR/NQR}$ experiments (Matsuda et al., 1997) which also observed a $T^{3}$ behaviour of the relaxation rate and the absence of a coherence peak immediately below $T_{c}$ was noticed, both facts are naturally explained by the existence of a node line in $\Delta(k)$.

Commonly in HF compounds the low temperature behaviour of the specific heat is not easily explained by simple models that involve only the topology of gap nodes. Very often residual $\gamma$-values exist where they should not due to node points or lines in $\Delta(k)$. This is ascribed to the presence of a residual density of states induced by impurity scattering. Extracting the true low temperature behaviour of the electronic specific heat $C_{s}$ is also complicated by the presence of nuclear terms due to the hyperfine splitting which lead to a low temperature upturn $\sim \alpha T^{-2}$. This situation has also been encountered in UPd$_{2}$Al$_{3}$. Originally the existence of a residual $\gamma$-value was concluded in Caspary et al. (1993) but later measurements extending to lower temperatures (Sato, 1993) have shown that the data can be well fitted without the residual $\gamma$ but assuming instead a dominant $T^{2}$ term in $C_{S}(T)$ which is compatible with the node line hypothesis for UPd$_{2}$Al$_{3}$. It is not known yet whether the field dependence of $C_{S}(T,H)$ is dominated by field induced $\gamma(H) \sim \sqrt{H}$ expected in the presence of nodes.

Thermal conductivity measurements for parallel heat current and field lying in the hexagonal plane (Hiroi et al., 1997) have shown a dominating temperature dependence $\kappa(T) \sim T^{2}$ and a linear behaviour $\kappa(T,H) \sim H$ at low temperatures. This is again in agreement with the existence of node lines in $\Delta(k)$. However their exact position on the FS is not yet known. This has to be investigated by field-angle dependent thermal conductivity measurements which has been successful in this task in other unconventional superconductors as described in sect. 2.

4.2.2. Electronic structure, Fermi surface and effective mass

In LDA type electronic band structure calculations for intermetallic U-compounds all 5f-electrons are treated in the same way and their degree of (de-)localisation is not much different. This is in contradiction to the behaviour observed in UPd$_{2}$Al$_{3}$ just as described above. Nevertheless LDA calculations give usually reasonable results for the Fermi surface topology. This can be understood in the context of model calculations as a result of many body correlations. For example within the Anderson lattice model it may be shown that the Fermi volume comprises both the f-electrons and conduction electrons (‘Luttinger’s theorem’), even in the Kondo limit when the f-electron is localised (Fazekas, 1999). Therefore it looks as if the Fermi wave vector, and more generally the Fermi surface is the same as if both type of electrons were delocalised. This is indeed assumed in LDA calculations for UPd$_{2}$Al$_{3}$ which we discuss now. FS sheets and associated angle dependent de Haas–van Alphen (dHvA) frequencies have first been calculated by Sandratskii and Kübler (1994), Knöpfle et al. (1996) and compared to the experimental dHvA results (Inada et al., 1995). Similar calculations (Inada et al., 1999) obtain the same main FS sheets but differ in the positions of smaller sheets.
The larger sheets are also those with experimentally found heavy masses, which the LDA calculation naturally cannot explain. The largest sheet which also has the strongest 5f-admixture is shown in fig. 37 in the AF Brillouin zone. It has the shape of a corrugated cylinder if one ignores the hexagonal in-plane anisotropy. In the subsequent model calculations we assume this simplified FS for UPd$_2$Al$_3$ and neglect all other sheets. The LDA approach to the 5f-electronic structure does not include the different degree of localisation evident from experiments discussed before. Progress in the treatment of partial localisation of 5f-electrons has recently been made for a number of U-compounds including UPt$_3$ (Zwicknagl et al., 2002) (sect. 4.1) and UPd$_2$Al$_3$ (Zwicknagl et al., 2003). Due to the strong spin–orbit coupling 5f-electrons occupy total angular momentum orbitals $|j = 5/2, j_z\rangle$. The two localised 5f-electrons are put into $j_z = \pm 5/2$ and $j_z = \pm 1/2$ localized states and the remaining $j_z = \pm 3/2$ states are included in the LDA basis of band electrons. This treatment is justified by the larger hybridization of the $j_z = \pm 3/2$ orbitals as compared to $j_z = \pm 1/2, \pm 5/2$ already on the LDA level. It has been shown within model calculations (Efremov et al., 2004) that intra-atomic correlations strongly increase the orbital dependence of the effective hybridization, leading to
Table 5

<table>
<thead>
<tr>
<th>FS sheet</th>
<th>( m^*/m ) (exp.)</th>
<th>( m^*/m ) (theory)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \zeta )</td>
<td>65</td>
<td>59.6</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>33</td>
<td>31.9</td>
</tr>
<tr>
<td>( \beta )</td>
<td>19</td>
<td>25.1</td>
</tr>
<tr>
<td>( \epsilon_2 )</td>
<td>18</td>
<td>17.4</td>
</tr>
<tr>
<td>( \epsilon_3 )</td>
<td>12</td>
<td>13.4</td>
</tr>
<tr>
<td>( \beta )</td>
<td>5.7</td>
<td>9.6</td>
</tr>
</tbody>
</table>

Notation for FS sheets and experimental values from Inada et al. (1999). Theoretical values from Zwicknagl et al. (2003).

the dual character of the 5f-electrons (sect. 2). Although two of the 5f-electrons are localised, the Fermi surface obtained from this calculation shows good agreement with the experimental results of the dHvA experiments. They are presented in fig. 37 together with the theoretical calculations.

This is reassuring for the dual 5f model approach, however the FS is also in good agreement with standard LDA calculations as discussed before. The decisive advantage of the former is that it also provides a basically parameter free explanation for the mass enhancement which cannot be obtained within LDA. The enhancement is due to the coupling of the delocalised with the localised 5f-electrons characterised by a matrix element \( \alpha = 2a_5/M \). As explained below the latter are split into two low lying CEF singlet states with an excitation energy \( \delta \) which is known from INS. Then the global mass enhancement factor \( (m^*/m_b) \) (independent of the FS sheet) with respect to the band mass \( m_b \) may be calculated according to eqs. (65), (66) as in the case of UPt3. The comparison of experimental and theoretical total mass enhancement \( m^*/m \) is presented in table 5.

4.2.3. The dual model for UPd2Al3 and induced moment AF

The dual model for UPd2Al3 which comprises both localised \( 5f^2 \) electrons with total angular momentum \( \mathbf{J} \) and itinerant heavy 5f-electrons created by \( c_{k\sigma}^\dagger \) is described by the model Hamiltonian

\[
H = H_c + H_{CEF} + H_{ff} + H_{cf},
\]

\[
H = \sum_{k\sigma} \epsilon_{k\sigma} c_{k\sigma}^\dagger c_{k\sigma} + \delta \sum_i |e_i\rangle\langle e_i| - \sum_{\langle ij\rangle} J_{ff}(ij)\mathbf{J}_i\mathbf{J}_j - 2I_0(g - 1) \sum_i s_iJ_i,
\]

where

\[
\epsilon_{k\sigma} = \epsilon_{\perp}(k_{\perp}\sigma) - 2t_{||}\cos k_{\|}
\]

is a model for the heavy conduction band energies whose FS is a corrugated cylinder along the hexagonal \( c \)-axis. Here \( t_{||} \) is the effective hopping along \( c \) which determines the amount of corrugation. The form of the dispersion \( \perp c \) is not important. The localised \( 5f^2 \)-electrons show a CEF splitting \( \delta = 6 \text{ meV} \) into a singlet ground state \( |g\rangle \) and an excited singlet \( |e\rangle \) at \( \delta \). The remaining terms describe a superexchange \( J_{ff} \) between localised and an on-site
exchange \( I = I_0(g - 1) \) between itinerant and localised 5f-electrons. The total effective inter-site exchange has therefore an additional RKKY contribution:

\[
J(q) = J_{ff}(q) + I_0^2(g - 1)^2 \chi_e(q).
\]

(80)

Here \( \chi_0(q) \) is the conduction electron susceptibility. \( J(q) \) may be fitted to the experimentally observed magnetic excitations.

4.2.4. Induced moments and magnetic exciton dispersion in UPd\(_2\)Al\(_3\)

First we consider the magnetism of the localised 5f moments without dynamic effects of coupling to itinerant 5f-electrons. In a nonmagnetic singlet ground state system with \( \langle g|J|g\rangle \equiv 0 \) the moments have to be induced via nondiagonal matrix elements \( \langle e|J_x|g\rangle = -i \langle e|J_y|g\rangle = \frac{1}{4} \alpha \) between ground state and excited state singlets. In UPd\(_2\)Al\(_3\) the maximum of \( J(q) \) is at the AF wave vector \( Q = (0, 0, \frac{1}{2}) \) with \( J_e \equiv J(Q) \). In the resulting AF state the CEF ground state will then be a superposition of \( |g\rangle \) and \( |e\rangle \). This type of magnetism is well known for Pr-metal and its compounds where one has two CEF split singlets from the Pr\(_{4+}\)(4f\(^2\)) configuration (Jensen and Mackintosh, 1991). This induced moment AF is only possible if the control parameter

\[
\xi = \frac{\alpha^2 J_e}{2 \delta}
\]

(81)
exceeds a critical value, i.e., \( \xi > \xi_c = 1 \). Then the Néel temperature \( T_N \) and saturation moment \( \langle J \rangle_0 \) oriented along the \( a \)-axis (\( x \)-direction) are given by

\[
T_N = \frac{\delta}{2 \tanh^{-1}(\frac{1}{\xi})} \quad \text{and} \quad \langle J \rangle_0 = \frac{1}{2} \frac{\alpha}{\xi} \left( \xi^2 - 1 \right)^{1/2}.
\]

(82)

When \( \xi \) is only slightly larger than \( \xi_c = 1 \) \( (T_N/\delta \ll 1) \) the saturation moment will be \( \sim \alpha \exp(-\delta/2T_N) \) and hence decreases exponentially for small \( T_N \). This is the major difference from the usual local moment AF where the saturation moment is independent of \( T_N \).

The signature of singlet–singlet induced moment magnetism is the existence of a paramagnetic excitation that grows soft on approaching \( T_N \). The excitation spectrum is obtained from the dynamical local moment susceptibility

\[
\chi_{ij,\lambda\mu}(\tau) = -\langle T \{ J_{i\lambda}(\tau) J_{j\mu}(0) \} \rangle,
\]

(83)

where \( i, j = \text{lattice site}, \lambda, \mu = \text{AF sublattices (A, B)} \) and \( \alpha, \beta = x, y, z \) denotes Cartesian components. Within RPA its Fourier transform is obtained as

\[
\chi(q, \omega) = [1 - u(\omega)J(q)]^{-1} u(\omega).
\]

(84)

Here \( \chi(q, \omega) \) and the single ion dynamical susceptibility \( u(\omega) \) are tensors in both sublattice (A, B) and transverse (xy) Cartesian coordinates. The poles of eq. (84) determine the collective excitations of 5f-local moments. In the paramagnetic phase they are given by the magnetic exciton dispersion

\[
\omega_E(q) = \delta \left[ 1 - \frac{\alpha^2 J(q)}{2 \delta} \tanh \frac{\beta}{2 \delta} \right].
\]

(85)
The physical origin and nature of magnetic excitons is illustrated in the inset of fig. 38 and described in the caption. When $T_N$ is approached the exciton energy at $Q$ (PM zone boundary or AF zone center) becomes soft as a precursor to the appearance of the AF moment according to

$$\omega_{E}(Q, T) = \frac{1}{2} \left( \frac{\delta}{T_N} \right)^2 \left( \xi - \frac{1}{\xi} \right) (T - T_N).$$

Below $T_N$ an induced staggered moment leading to two magnetic sublattices appears. Its saturation value is given by eq. (82). Then $Q = (0, 0, \frac{1}{\xi})$ becomes the center of the new AF Brillouin zone $\left[ \frac{\pi}{2c}, \frac{-\pi}{2c} \right]$. In reality the softening of $\omega_{E}(Q, T)$ will be arrested at a finite value at $T_N$ by Curie type contributions to the static susceptibility which contribute to the AF instability but not to the exciton dispersion. Therefore a magnetic excitation gap at $Q$ will appear. Below $T_N$ the dispersion is somewhat modified due to the effect of the molecular field, however for $\xi$ only marginally above the critical value this modification is not important. This is indeed the case for the dual model of UPd$_2$Al$_3$ where $T_N/\delta = 0.22$ and hence $\xi = 1.015$. The magnetic exciton dispersion including the molecular field and exchange anisotropy has been derived (Thalmeier, 2002) and compared to the experimental results obtained in INS experiments (Mason and Aeppli, 1997). In this work the excitations were measured up to 10 meV in the whole BZ and it was found that well defined propagating modes exist along the hexagonal $c^*$-axis. For wave vector in the hexagonal plane $\perp c^*$ the line width of excitations was found to be much bigger. No excitation gap at the AF zone center $Q$ could be identified in these early experiments. This conclusion had to be revised in later high resolution experiments around $Q$ as discussed below. The comparison to the theoretical calculation using a parametrized exchange function $J(q)$ was given in Thalmeier (2002) and is shown in fig. 38. There the extended BZ was used which allows one to plot only the acoustic mode. It should be mentioned that it is not clear whether the dip at the K-point is realistic because the line width of magnetic excitons becomes rather large.

Fig. 38. Magnetic exciton dispersion of UPd$_2$Al$_3$ in the AF BZ. The diamonds are data from Mason and Aeppli (1997). The solid line is a calculation using a generalized version of eq. (85) with an appropriate model for the exchange function $J(q)$ (Thalmeier, 2002). The inset illustrates the principle of magnetic (Frenkel-)exciton propagation: A CEF singlet–singlet excitation ($\delta$) propagates between lattice sites $i, j$ through the action of intersite exchange $J(i, j)$ and thereby acquires a dispersion $\omega_{E}(q)$. 
The damping of magnetic exciton modes has two sources: (i) Intrinsic dynamical effects in the localised moment system beyond RPA, e.g., damping by thermal fluctuations in the singlet occupation. (ii) Extrinsic damping due to the coupling to conduction electrons described by the last term in eq. (78). For low temperatures the latter is the dominating part. In addition it leads to a renormalized exciton mode frequency. Both effects can be described in extending the previous RPA approach including the last coupling term in eq. (78). This leads to coupled RPA equations (Buyers and Holden, 1985), the solution for the localised dynamical susceptibility is given by

\[
\chi(q, \omega) = \frac{u(\omega)}{1 - J_{ff}(q)u(\omega) - I^2 u(\omega) \chi_e(\omega)}
\]

with

\[
u(\omega) = \frac{\alpha(S)}{\delta - \omega} \quad \text{and} \quad \chi_e(\omega) = \frac{1}{i\omega - \Gamma},
\]

where \(u(\omega)\) is the single ion local moment susceptibility and \(\chi_e(\omega) \equiv \chi_e(Q, \omega)\) is the low frequency conduction electron susceptibility at \(q = Q\) and \(\Gamma\) is a phenomenological damping rate. If one neglects its imaginary part and uses \(\chi_e(q) \sim \Gamma^{-1}\) for \(q \approx Q\) then, with eqs. (88), (80) the poles of eq. (87) again lead to the undamped magnetic exciton dispersion \(\omega_E(Q)\) of eq. (85). Inclusion of the imaginary part of \(\chi_e(\omega)\) leads to a shift of the mode frequency and a damping. For a large coupling constant \(I\), a part of the spectral weight of the magnetic exciton is shifted to low energies leading to an additional quasielastic peak.

The INS intensity is then proportional to the localised 5f-dynamical structure factor \(S(q, \omega) = \frac{1}{\pi} (1 + n(\omega)) \text{Im} \chi(q, \omega)\) where \(n(\omega) = (\exp(\beta\omega) - 1)^{-1}\) is the Bose factor. The experimental high resolution magnetic INS intensity for wave vectors close to \(Q\) as obtained by Sato et al. (1997, 1997a), Bernhoeft et al. (1998) is shown in fig. 39. At the AF vector \(Q\) above \(T_c\) (2.5 K) one can observe a strongly broadened inelastic peak at around 1.5 meV which is interpreted as the (upward) shifted magnetic exciton energy. In addition, as a signature of the strong coupling to conduction electrons one observes a quasielastic peak around \(\hbar\omega \simeq 0\). The full line is a calculation of \(S(Q, \omega)\) using eq. (87) for \(\chi(q, \omega)\) and taking \(I\) and \(\omega_E(Q)\) as adjustable parameters. The INS intensity was also measured for other wave vectors close to \(Q\) along \(c^*\). Repeating the same fitting procedure one may determine the unrenormalized magnetic exciton dispersion \(\omega_E(q)\) which is shown in the inset of fig. 39. Obviously the magnetic excitons have a gap at the AF zone center of about 1 meV. As mentioned before in the original experiments (Mason and Aeppli, 1997) this gap was not identified due to insufficient resolution. In the singlet–singlet CEF model, even including a uniaxial anisotropy of the exchange, this relatively large gap cannot be explained. Its most probable origin is an arrested softening of \(\omega_E(Q, T)\) due to higher lying CEF states.

In fig. 39 another important feature is obvious: As temperature is lowered below \(T_c = 1.8\) K the quasielastic peak evolves into a low energy inelastic peak. This is due to the appearance of the SC quasiparticle gap function \(\Delta(k)\) which possibly may have node lines or points. Its influence on the conduction electron susceptibility may be described in a simple manner by shifting the diffusive pole at \(-i\Gamma\) in \(\chi_e(\omega)\) to \(-i\Gamma + \Delta_\text{av}\) where the real part corresponds
to the $k$-averaged SC gap. The calculated intensity in the SC phase (e.g., at $T = 0.15$ K) is then again shown as a full line from which the size of $\Delta_{av}$ may be extracted. The result $\Delta_{av} \simeq \omega_E(Q)$ shows that magnetic exciton energy and SC gap are almost degenerate, which means that there will be a strong mixing of magnetic excitons with SC quasiparticle excitations. This explains why the lower peak appears with a large intensity.

4.2.5. Magnetic exciton anomalies in quasiparticle tunneling spectra

The near degeneracy of SC gap and magnetic exciton energy suggests that the latter might play a role in the formation of Cooper pairs in UPd$_2$Al$_3$. Knowing only the INS results this would only be a hypothesis. However in a breakthrough experiment (Jourdan et al., 1999) the first SC quasiparticle tunneling in a HF system by using epitaxially grown UPd$_2$Al$_3$ was achieved. This experiment is complementary to INS measurements as it probes the itinerant 5f-electrons. The resulting tunneling spectra which ideally are proportional to the SC quasiparticle DOS are shown in fig. 40. The tunneling current is parallel to the $c$-axis and a real gap is seen in $dI/dV$ in this direction. Therefore, if node lines of $\Delta(k)$ exist they should be perpendicular to the $c$-axis at $k_z$ positions where the velocity $v_c(k_z)$ vanishes. The most striking result is the presence of typical ‘strong coupling anomalies’ around 1 meV which are well known from ordinary electron–phonon superconductors like Pb. These anomalies are connected to
the frequency spectrum of the exchanged boson which is responsible for the formation of Cooper pairs in UPd$_2$Al$_3$. The Debye energy of UPd$_2$Al$_3$ $k\theta_D = 13$ meV is much too large to be connected with the observed modulation around 1 meV, however it agrees perfectly with the magnetic exciton energy $\omega_E(Q)$ found by INS (inset of fig. 39). Furthermore the average gap energy and mode energy determined in INS are of the same order. This leads to the conclusion that UPd$_2$Al$_3$ is a magnetic exciton mediated strong coupling superconductor. This is the first time that a non-phononic mechanism for superconductivity has been proven in a direct way by identifying the non-phononic boson (magnetic exciton) that provides the ‘glue’ for the formation of Cooper pairs in UPd$_2$Al$_3$. These arguments can be made more quantitative by using Eliashberg theory to calculate the SC quasiparticle DOS. The retarded effective potential due to magnetic exciton exchange can be described by a factorized ansatz

$$V_E(q, \omega) = \frac{V_0 \omega_0}{(\omega - \omega_E(Q))^2 + \omega_0^2} f(k) f(k')$$

which exhibits a maximum at the observed exciton frequency $\omega_E(Q)$ with a width $\omega_0$ and strength $V_0$. Here $q = k' - k$ and $f(k)$ is the form factor which describes the $k$-dependence of the superconducting singlet gap function $\Delta(k) = \Delta f(k)$. For an isotropic conventional
e-p superconductor \( f(\mathbf{k}) \equiv 1 \). As explained in the next section in UPd$_2$Al$_3$ the gap function should have a node line perpendicular to the \( c \)-axis. For tunneling current along \( c \) the tunneling DOS can be determined from the isotropic Eliashberg equations using eq. (89) if one replaces \( f(\mathbf{k}) \) by the averaged constant \( \langle f(\mathbf{k})^2 \rangle_{FS}^{1/2} \) which may then be absorbed into the coupling strength \( V_0 \) that is adjusted to obtain the experimental \( T_c \). The result of this analysis is shown in fig. 41 which nicely explains the experimental results in fig. 40, especially:

(i) The width of the gap does not depend much on temperature but is rather filled up with increasing temperature. This is characteristic for a strong coupling superconductor, the ratio \( 2\Delta/T_c = 5.6 \) obtained here is rather high and agrees well with the ratio \( 2\Delta/T_c = 6 \) obtained previously from the analysis of INS spectra. (ii) In the calculated tunneling DOS of fig. 41 typical modulations above the gap due to the retardation of the potential in eq. (89) appear at a voltage that corresponds to the magnetic exciton energy at \( Q \) which is about 1 meV.

4.2.6. Possible symmetries of the superconducting order parameter

The existence of node lines of \( \Delta(\mathbf{k}) \) in UPd$_2$Al$_3$ was suggested by various thermodynamic properties discussed previously. Other evidence was proposed by Bernhoeft (2000) where it is claimed that the symmetry property \( \Delta(\mathbf{k} \pm \mathbf{Q}) = \pm \Delta(\mathbf{k}) \) has to be fulfilled to explain the large intensity of the low energy quasiparticle-like peak in fig. 39. This implies a node line orthogonal to the \( c \)-axis which is compatible with the appearance of a gap in the \( c \)-axis tunneling geometry as used for fig. 40. In the ansatz of eq. (89) the energy and momentum dependences are factorized and therefore the form factor \( f(\mathbf{k}) \), i.e., gap function symmetry is put in by hand. Considering the presence of a strong AF ordered moment \( \parallel x \)-axis the appropriate symmetry group is orthorhombic D$_{2h}$. The simplest spin singlet even parity irreducible
This gap function is independent of $k_x$ and $k_y$. A more general $\Delta(k)$ with the same symmetry can be obtained by multiplying eq. (90) with any fully symmetric function $f_{\Gamma_1^+}(k_x, k_y)$. $\Delta(k)$ has the required behaviour under the transformation $k \rightarrow k \pm Q$ and it has a node line at the AF zone boundary $k_z = \pm \frac{1}{2}Q_z = \pm \frac{\pi}{2c}$. A theoretical investigation of possible gap functions based on a nonretarded weak coupling theory of the magnetic exciton mediated pair potential was undertaken in Thalmeier (2002). It was found that due to the CEF-anisotropy and the action of the AF local moment order the degeneracy of odd parity triplet states is lifted and one of them would be favored against the above even parity singlet state. However this may be due to the neglect of retardation which is not a good approximation for UPd$_2$Al$_3$ as evident from the previous discussion. As mentioned before Pauli limiting of $H_c^2$ and Knight shift reduction below $T_c$ favor an even parity $A_{1g}$ (singlet) state. Recently a strong coupling theory has been presented by McHale et al. (2004).

As in the borocarbides the gap function will be modified by the background magnetic order due to the reconstruction of Bloch states close to the magnetic Bragg planes $k_z = \pm \frac{1}{2}Q_z$. The AF modified gap function for the even parity state is given by eq. (117) which in the present case reduces to

$$\Delta(k) = \Delta \cos k_z \left( \frac{\cos^2 k_z}{\lambda_{AF}^2 + \cos^2 k_z} \right)^{1/2}.$$  \hspace{1cm} (91)

Here the dimensionless interaction parameter $\lambda_{AF} = (\langle J \rangle_0)/\langle 0 | W \rangle$ describes the strength of the AF influence on $\Delta(k)$. It is interesting to note that despite the presence of a node line the above order parameter is fully symmetric ($A_{1g}$) because it is invariant under all symmetry transformations of the magnetic unit cell. Thus in a strict sense UPd$_2$Al$_3$ cannot be called a superconductor with unconventional gap symmetry, although the novel magnetic exciton mechanism certainly is unconventional. Various possible nodal structures in addition to eq. (90) have been discussed by Thalmeier and Maki (2002) in the context of magnetothermal properties and experiments were performed by Watanabe et al. (2004).

4.2.7. UNi$_2$Al$_3$: a possible triplet superconductor

The HF superconductor UNi$_2$Al$_3$ (Geibel et al., 1991) ($\gamma = 120$ mJ/K$^2$ mol) is isostructural to its hexagonal sister compound UPd$_2$Al$_3$ (fig. 35) but it has rather different physical properties. It exhibits an incommensurate SDW below $T_m = 4.6$ K ($\mu = 0.2\mu_B$) with a modulation wave vector $Q_m = (0.39, 0, 0.5)$, like in UPd$_2$Al$_3$ the moments are pointing to n.n. along the a direction of the hexagonal plane. Superconductivity sets in below $T_c = 1.2$ K and coexists with magnetic order. The $T$-dependence of $1/T_1$ in NMR (Kyogaku et al., 1993) and the small entropy release of $\Delta S = 0.12 R \ln 2$ at $T_m$ (Tateiwa et al., 1998) points to an itinerant character of magnetic order in contrast to UPd$_2$Al$_3$ ($\Delta S = 0.67 R \ln 2$) which exhibits local moment magnetism of the induced type with large ordered moment. This view is supported by INS experiments (Gaulin et al., 2002) which do not show any evidence for propagating collective
modes in the SDW state unlike the magnetic exciton mode discussed previously for UPd$_2$Al$_3$. The magnetic excitation spectrum in UNi$_2$Al$_3$ consists of quasielastic spin fluctuations for all wave vectors and its energy width of about 6 meV corresponds to the coherence temperature $\sim 80$ K associated with the $\gamma$ value. For temperatures not too close to $T_m$ the spin fluctuations are centered around odd multiples of the commensurate AF wave vector $Q = (0, 0, 0.5)$ and also extending along ridges in $q_x$-direction.

From the spin fluctuation spectrum of UNi$_2$Al$_3$ observed in INS which is located around an AF Bragg point one might expect it should be a textbook example of spin fluctuation mediated superconductivity. However as in the case of UPt$_3$ the spin singlet pairing predicted by AF spin fluctuation type theories (Miyake et al., 1986) is probably not realised. According to the $^{27}$Al-Knight shift measurements of (Ishida et al., 2002) which did not observe any drop in the spin susceptibility below $T_c$, UNi$_2$Al$_3$ should have a spin triplet SC order parameter. This is supported by the observed lack of Pauli paramagnetic limiting effect on $H_{c2}(T)$ (Sato et al., 1996). As in the case of UPd$_2$Al$_3$ the position of gap nodes is not known with certainty, but NMR results suggest the presence of a node line in UNi$_2$Al$_3$ as in UPd$_2$Al$_3$. In the simplest scenario then the latter might correspond to the singlet gap function of eq. (90) while the former has one of the triplet gap functions.

\[ d(k) = \Delta \hat{d} \sin k_z. \]  

(92)

With the possibilities $\hat{d} = \hat{x}, \hat{y}, \hat{z}$ due to the AF orthorhombic symmetry. These gap functions have all an equatorial node line at $k_z = 0$. Confirmation of the node structure has to await the results of angle resolved magnetotransport or specific heat measurements in the vortex phase and for a confirmation of the spin state the Knight shift measurements on high quality single crystals are necessary.

4.3. Ferromagnetism and superconductivity in UGe$_2$

The possibility of coexisting ferromagnetism (FM) and superconductivity was first considered by Ginzburg (1957) who noted that this is only possible when the internal FM field is smaller than the thermodynamic critical field of the superconductor. Such a condition is hardly ever fulfilled except immediately below the Curie temperature $T_C$ where coexistence has been found in a few superconductors with local moment FM and $T_C < T_C$ such as ErRh$_4$B$_4$ and HoMo$_6$S$_8$. If the temperature drops further below $T_C$ the internal FM molecular field rapidly becomes larger than $H_{c2}$ and SC is destroyed. The reentrance of the normal state below $T_C$ has indeed been observed in the above compounds. The only compound known sofar where local moment FM coexists homogeneously with SC for all temperatures below $T_C$ is the borocarbide compound ErNi$_2$B$_2$C (Canfield and Bud’ko, 1996) (sect. 5). The competition between FM and superconductivity becomes more interesting if FM order is due to itinerant electrons which also form the SC state. In Hartree–Fock approximation the Stoner condition for the reduced interaction constant $\lambda = I N(0) = 1 - S^{-1} \geq 1$ ($S =$ Stoner parameter, $I =$ exchange constant) of conduction electrons determines the existence of itinerant FM order. If the interaction is slightly above the critical value $\lambda_c = 1$ one has weak FM order such as in ZrZn$_2$ with large longitudinal ferromagnetic spin fluctuations. Fay and Appel (1980) have shown
that in this case p-wave superconductivity may actually be mediated by the exchange of FM spin fluctuations and coexist with the small FM moments. According to this theory p-wave superconductivity should exist on both (FM and PM) sides of the critical value $\lambda_c = 1$ for some range of the interaction parameter $\lambda$. Until recently this remained only a theoretical scenario. The discovery of unconventional superconductivity under pressure in the itinerant FM UGe$_2$ (Saxena et al., 2000) and later for FM URhGe (Aoki et al., 2001) and the 3d FM ZrZn$_2$ (Pfleiderer, 2001) under ambient pressure has opened this field to experimental investigation.

It became quickly clear however that the physics of UGe$_2$ is not as simple as suggested by the FM spin fluctuation model. Firstly the maximum of SC $T_c$ occurs at a pressure where the FM moment per U is still 1$\mu_B$ as compared to 1.5$\mu_B$ at ambient pressure thus $\lambda_c$ should still be far from the critical value where spin fluctuations are important. Secondly the SC phase diagram is not approximately symmetric around the critical value as expected but the region of SC lies completely inside the FM phase. This problem has been further investigated by resistivity measurements (Huxley et al., 2001) where an additional phase within the FM region appeared at lower temperature witnessed by a resistivity anomaly. Its critical temperature $T_x(p)$ precisely hits the maximum of the SC $T_c(p)$ curve as seen in fig. 43. The $x$-phase has been associated with a combined CDW/SDW transition. This is also suggested by electronic structure calculations (Shick and Pickett, 2001) which show nesting features of the Fermi states. In addition the main FS sheet consists mostly of majority spin ($m_s = \uparrow$, $m_l = 0$) electrons. Therefore the SC pairs must have triplet character. It is suggestive that the unconventional Cooper pairing then appears close to the quantum critical point of the alleged CDW/SDW

Fig. 42. Conventional orthorhombic unit cell of UGe$_2$ with lattice parameters given by $a = 3.997$ Å, $b = 15.039$ Å and $c = 4.087$ Å. U atoms (large spheres) form zig-zag lines along $a$. 
transition and therefore \( T_c \) should rather be associated with the \( T_x \) transition instead of the FM transition at \( T_C \). However so far no indication of a CDW/SDW order parameter below \( T_x \) has been identified in neutron diffraction (Kernavanois et al., 2001) and the subject remains open.

Various theoretical scenarios for UGe\(_2\) have been formulated. In the context of Ginzburg–Landau theories the symmetries of possible SC order parameters and their coupling to the FM have been discussed, e.g., Machida and Ohmi (2001), Mineev (2002). However, as already mentioned in the context of UPt\(_3\) the predictive power of such theories is limited. The traditional approach of microscopic theories invoking FM spin fluctuations has been extended beyond Hartree–Fock with the inclusion of mode coupling terms in order to explain the asymmetric phase diagram around the critical coupling strength (Kirkpatrick et al., 2001). The new aspect of the \( T_x \) transition line was introduced by Watanabe and Miyake (2002) who, in order to comply with the experimental evidence from Kernavanois et al. (2001), interpret it as a crossover line into a region of enhanced SDW/CDW fluctuations rather than an ordered phase. When \( T_x(p) \) approaches zero these fluctuations then would mediate formation of a triplet p-wave SC state.

4.3.1. Electronic structure and band magnetism

The itinerant FM UGe\(_2\) crystallizes in the base-centered orthorhombic ZrGa\(_2\) type structure (Cmmm) which is shown in fig. 42. It may be viewed consisting of antiphase zig-zag U chains running along the \( a \)-axis. This is also the easy axis for the FM moments (\( \mu = 1.43 \mu_B \) per formula unit) below the Curie temperature of \( T_C = 52 \) K at ambient pressure. Magnetization and susceptibility are extremely anisotropic (Onuki et al., 1992) corresponding to almost Ising like behaviour of U-moments. The specific heat coefficient is \( \gamma = 32 \) mJ/mol K\(^2\) which corresponds to an enhancement factor of 2.7 as compared to the band mass (Shick and Pickett, 2001). This points to some degree of mass renormalization by dynamic correlation effects. Individual masses observed in dHvA experiments exhibit an enhancement factor of \( m^*/m \simeq 15–25 \) compared to the free electron mass. Therefore UGe\(_2\) is an itinerant 5f metal with sizable correlation effects but has an order of magnitude smaller effective masses than real U-heavy fermion metals.
The electronic band structure of \( \text{UGe}_2 \) has first been calculated within the LDA + U approach (Shick and Pickett, 2001) where \( U \simeq 0.7 \text{ eV} \) is a strongly screened on-site Coulomb interaction that was fitted to reproduce the proper FM ground state moment of 1.43\( \mu_B \). On the other hand this value is too small to reproduce the observed photoemission spectrum. The calculated Fermi surface has a main sheet which mostly consists of \((m_l = 0, m_s = \uparrow)\)-states. Bands crossing the Fermi energy have mostly U-5f majority spin character, indicating almost complete polarisation, i.e., strong FM. The LDA + U Fermi surface also displays a nesting feature for \( \mathbf{Q} = (0.45, 0, 0) \) (in r.l.u.) which is due to bands that have little dispersion along \( b \), i.e., perpendicular to the zig-zag U chains. The proper easy a axis for FM moment and a strong magnetocrystalline anisotropy of the moment due to spin–orbit coupling was also correctly found. These calculations describe the ambient pressure strong band ferromagnetism in \( \text{UGe}_2 \) which is still far from the superconducting pressure region with reduced moments.

A more recent theoretical investigation of the collapse of the FM state under hydrostatic pressure and the simultaneous appearance of another instability has been given in Yaresko and Thalmeier (2004). In this study a relativistic LSDA calculation was performed for isotropically compressed lattice constants simulating the application of hydrostatic pressure \( p \). The experimental lattice constants corresponding to \( p = 0 \) are given in fig. 42. In agreement with the experimental results a FM ground state with the \( a \)-axis as easy axis is obtained. However the U spin \((-1.39\mu_B)\) and orbital \((1.94\mu_B)\) magnetic compensate partly leading to a total moment of only 0.55\( \mu_B \) significantly smaller than the experimental value \((1.43\mu_B)\). This was the main reason for performing the above mentioned LDA + U calculation. However the LSDA calculation reproduces quite well the angular dependence of the cross section of a major FS sheet with an area of \( F \simeq 9 \text{ kT} \). The calculated cyclotron masses are about 8 m and smaller than the experimental values of 15.5 m. This confirms that \( \text{UGe}_2 \) has itinerant 5f-electrons with only moderately strong correlations.

The hydrostatic pressure simulation shows that FM U moments decrease continuously with pressure until at compression factor \( x = 0.94 \) only a nonmagnetic solution is obtained (fig. 44). The magnetocrystalline anisotropy energy \( E_{M\parallel} - E_{M\perp} \) also decreases upon compression. On the other hand the DOS \( N(0) \) first increases to a maximum at \( x \sim 0.98 \) and then decreases again. In order to check whether a tendency to a CDW/SDW instability exists which might be related to the empirical observation of the \( T_x \) phase boundary inside FM the staggered static susceptibility within LSDA has been calculated (Yaresko and Thalmeier, 2004). It is given by

\[
\chi_0(q, 0) = \sum_{i,j,k} \left[ f(\epsilon_{ik}) - f(\epsilon_{jk+q}) \right] \frac{|M_{ik,jk+q}^\alpha|^2}{\epsilon_{ik} - \epsilon_{jk+q}},
\]

(93)

where \( \epsilon_{ik} \) are the band energies, \( M_{ik,jk+q}^\alpha \) are the transition matrix elements \((\alpha = \parallel, \perp M)\) and \( f(\epsilon) \) is the Fermi function. The \( q_z \) wave-number dependence of \( \chi_{0\perp} \) and \( \chi_{0\parallel} \) has been calculated as function of the lattice compression. Whereas \( \chi_{0\perp}(q_z) \) shows monotonic decrease with compression \( \chi_{0\parallel}(q_z) \) is very sensitive to lattice constant change, it develops a strong enhancement at small wave vector at the same intermediate compression \( x = 0.98 \) where \( N(0) \) also is at its maximum. This is an indication that for an intermediate compression (smaller
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Fig. 44. Left panel: Calculated pressure dependence of spin \( M_S \), orbital \( M_L \) and total moment \( M_S + M_L \). Right panel: Pressure dependence of calculated longitudinal susceptibility. The steep drop at \( q_z = 0 \) is caused by the loss of intra-band transitions due to finite \( q_z \) resolution. The enhancement of \( \chi_{0\parallel} \) under pressure for small \( q_z \) suggests the evolution of a SDW instability within the FM phase (Yaresko and Thalmeier, 2004).

than the critical compression \( x = 0.94 \) where FM vanishes) UGe\(_2\) has a tendency to develop a long wavelength longitudinal SDW-like instability. It is possible that the tendency towards instability for the susceptibility associated with an unconventional SDW is even stronger. However it should be noted that the susceptibility enhancement does not correspond to any obvious nesting in the FS sheets obtained in LSDA calculations. This raises the question whether the observed nesting feature in the LDA + U FS of Shick and Pickett (2001) has a real physical significance to the alleged hidden order phase.

4.3.2. Coexistence of FM order and superconductivity under pressure

The experimental phase diagram of FM collapse under pressure and simultaneous appearance of superconductivity is shown in fig. 43. The critical pressure for disappearance of FM order is \( p_{c2} = 16–17 \) kbar. The SC phase appears between \( p_{c1} = 10 \) kbar and \( p_{c2} = 16 \) kbar which is also the critical pressure for the FM-PM transition. The critical temperature \( T_x(p) \) of the \( x \)-phase hits the maximum of \( T_c(p) \) at the optimum pressure \( p_m = 12.5 \) kbar. As mentioned before the nature of the order parameter in the \( x \)-phase remains elusive. The coincidence of maximum \( T_c \) with vanishing \( x \)-phase order parameter suggests that the collective bosonic excitations of the \( x \)-phase which supposedly become soft at \( p_m \) mediate superconductivity and not quantum critical FM spin fluctuations which are absent due to the persisting large FM
molecular field. Associated with the $T_x(p)$ phase boundary is an abrupt first order change of the magnetic moment $\mu(p)$ around $p_m$ whose different size distinguishes the FM2 region ($\mu(0) \simeq 1.45\mu_B$) from the FM1 region ($\mu(p_m) \simeq 1\mu_B$) (Pfleiderer and Huxley, 2002). The connection between SC and the $x$-phase is also witnessed by the anomalous upper critical field behaviour around $p_m$ which displays a reentrance behaviour as function of pressure seen in fig. 45. Apparently close to the $T_x$ line the superconducting state is strongly stabilized as shown by the dramatically increased value of $H_c(0)$. The existence of phases with ‘hidden order parameters’ which show up in thermodynamic and transport coefficients but do not appear as spin- or charge-density modulations in neutron diffraction is not uncommon for U-compounds. As discussed in the next section URu$_2$Si$_2$ has a similar phase. Two kinds of proposals for such hidden order may be considered: (i) If the U 5f-electrons have a partly localised character as in the latter case the hidden order may be of quadrupolar or more generally multipolar character. (ii) If 5f-electrons are strongly itinerant, as apparently in UGe$_2$, one may have an unconventional density wave order parameter which has been introduced in sect. 2. These phases have $\mathbf{k}$-dependent gap functions which do not belong to identity representations as in the CDW/SDW case. This type of order has also been proposed for the yet unidentified $x$-phase in UGe$_2$ and its coexistence behaviour with FM and unconventional superconductivity was studied (Varelogiannis et al., 2003).

4.3.3. Theoretical scenarios for superconductivity in UGe$_2$

From the previous discussion of experimental evidence the superconducting mechanism via ferromagnetic fluctuations can be ruled out since the ferromagnetic polarization at $p_m$ where $T_c$ is largest is still 65% of the maximum value at ambient pressure (Pfleiderer and Huxley, 2002). Therefore models based on the FM quantum critical point scenario that follow the original work of Fay and Appel (1980) are not relevant for UGe$_2$. On the other hand the phe-
nomenological classification of possible SC order parameters based on Landau expansion of
the free energy is simplified by the remaining large spin polarisation of conduction electrons,
only spin triplet pairing is possible in order to avoid the effect of the large exchange field.

4.3.4. Symmetry properties of gap states and Ginzburg–Landau theory
Triplet pair states are characterised by the vector gap function \( \mathbf{d}(\mathbf{k}) \) defined in sect. 2. Due to
the constraint of equal (pseudo-)spin pairing (i.e., \( \Delta_+ \equiv 0 \)) the \( \mathbf{d} \)-vector is confined to the
\( bc \)-plane perpendicular to the FM moment \( \mathbf{M}_0 \) which is parallel to the easy \( a \)-axis. The gap
function can be written as (Machida and Ohmi, 2001; Fomin, 2001)

\[
\mathbf{d}(\mathbf{k}) = f(\mathbf{k}) \eta = f(\mathbf{k}) (\hat{b} + i \hat{c}).
\]

Here \( \eta = (\eta_x, \eta_y, \eta_z) = (0, 1, i) \) is the vector order parameter and \( f(\mathbf{k}) \) the orbital part which
transforms as a representation of the (approximate) tetragonal group. As in the case of UPt3
it is assumed that the pseudo spins describing the Kramers degeneracy of quasiparticle states
are only weakly coupled to the orbital momentum, then \( f(\mathbf{k}) \) may belong to any tetragonal
representation which have typical point or line node structures. Presently there is no experi-
mental information on the nodes. The equal spin pairing state is nonunitary, i.e., it breaks time
reversal symmetry. This property is directly enforced by the nearly complete FM polarisation.
Therefore the Cooper pairs have a net spin moment \( \mathbf{S} = i \mathbf{d}(\mathbf{k}) \times \mathbf{d}(\mathbf{k})^* \) which aligns with the
ordered FM moment. In a Ginzburg–Landau expansion of the free energy it is sufficient to
consider only the aligned component \( \eta_+ = \eta_y + i \eta_z \) and neglect \( \eta_- = \eta_z \) because of the large
\( \mathbf{M}_0 \) even around the critical pressure. The total GL functional should then only be expanded
in terms of \( \eta_+ \) and should also not contain \( \mathbf{M}_0 \) explicitly except in the vector potential. One
obtains (Machida and Ohmi, 2001) for \( \mathbf{H} \) along the \( a \)-axis (\( \parallel \mathbf{M}_0 \)),

\[
f_{\text{GL}} = a_0 (T - T_c) |\eta_+|^2 + \frac{1}{2} \beta |\eta_+|^4
+ K_1 \left( \frac{d\eta_+}{dy} \right)^2 + \left( \frac{2\pi}{\phi_0} \right)^2 K_2 (M_0 + \mu_0 H)^2 y^2 |\eta_+|^2
\]

(95)

because in this case the magnetic induction in the FM state is given by \( \mathbf{B} = (\mu_0 H + \mathbf{M}, 0, 0) \).
Similar expressions hold for \( \mathbf{H} \) along \( c \)- and \( b \)-axis. Minimization of eq. (95) leads to the crit-
ical field curves close to \( T_c \). The term \( M_0^2 |\eta_+|^2 \) only renormalizes \( T_c \). Then one obtains the
upper critical fields \( H_{c2}^0 \sim (T_c - T) \) and \( H_{c2}^{b,c} \sim (T_c - T)^{1/2} \). The remarkable root singularity
for \( H_{c2}^{b,c} \) at \( T_c \) is due to the presence of a finite FM moment. Experimentally the critical field
curves are quite anomalous close to the optimal pressure \( p_m \) although different exponents for
different field directions are not observed, the \( H_c^{c2} \)-curve, however, does show an anomalous
strong upturn around \( 0.5 T_c \) (Kobayashi et al., 2001), see fig. 45. In any case a proper the-
ory of \( H_{c2} \) has to include the influence of the hidden order parameter close to maximum \( T_c \)
(fig. 43). The symmetry considerations of this section gave some constraints one should ex-
pect to hold for the SC order parameter, however the simple Landau theory approach only
indicates that \( H_{c2} \) should behave anomalous, without being able to give a quantitative account
close to the optimum \( T_c \).
4.3.5. **Microscopic approaches**

Aside from the FM QCP scenarios now known to be irrelevant for UGe$_2$ (Pfleiderer and Huxley, 2002) little theoretical investigation has been undertaken sofar with the exception of Watanabe and Miyake (2002). This theory starts from the assumption that $T_s(p)$ represents the phase boundary of a coupled (conventional) CDW-SDW transition. Because of the strong background FM polarisation these two order parameters have to appear simultaneously. Inversely, if one is close to the critical pressure $p_m$ their critical fluctuations will be coupled to fluctuations in the FM polarisation by a mode–mode coupling term in the free energy, which couples the amplitudes of FM, CDW and SDW fluctuations at the commensurate nesting vector $Q$ of a n.n.n. tight binding model used for the majority bands of UGe$_2$. The strong coupling theory for this mechanism has been used to calculate $H_{c2}$ and apparently qualitative anomalies like those close to $p_m$ (fig. 45) are obtained.

However, no evidence for CDW or SDW formation central to this theory has been seen below $T_s$ sofar. On the other hand in specific heat measurements a pronounced anomaly $\Delta C(T_s)$ at 1.15 GPa suggests that $T_s(p)$ is a real phase line (Tateiwa et al., 2003). This leads one to the natural suggestions that one should look for more general particle–hole pairing ('unconventional density wave') discussed in sects. 2 and 4.4 as an alternative for the phase below $T_s$ since they do not lead to charge or magnetic superstructures (Varelogiannis et al., 2003).

4.4. **A case of ‘hidden order’ in URu$_2$Si$_2$**

This moderate HF compound ($\gamma = 110$ mJ/mol K$^2$) has mystified both experimentalists and theorists alike since the discovery of AF order at $T_m$ and another still unidentified ('hidden order') phase at $T_0$ which both seem to appear at the same temperature $T_0 = T_m = 17.5$ K, at least for annealed samples. In addition the compound becomes a nodal superconductor below $T_c \simeq 1.2$ K (Palstra et al., 1985; Schlabitz et al., 1986; de Visser et al., 1986). The simple tetragonal AF order with wave vector $Q = (0, 0, \pi_c)$ has tiny moments $\mu \simeq 0.02 \mu_B$ along $c$–axis (Broholm et al., 1987; Walker et al., 1993) which are of the same order as in UPt$_3$. However, unlike in UPt$_3$, very large thermodynamic effects, e.g., in specific heat ($\Delta C/T_0 \simeq 300$ mJ/mol K$^2$), thermal expansion, etc., occur which are hard to reconcile with the small ordered moments. The pronounced anomalies at $T_0$ were interpreted as evidence for the presence of a second ‘hidden order’ parameter which cannot be seen in neutron or X-ray diffraction. Many different types of models for hidden order have been proposed, where the 5f-electrons of URu$_2$Si$_2$ are considered as essentially localized (Santini and Amoretti, 1994), or itinerant (Ikeda and Ohashi, 1998) or, of dual nature (Okuno et al., 1998; Sikkema et al., 1996). In the former case local quadrupoles of the CEF states are supposed to show staggered order below $T_0$ akin to the many examples of such order in 4f-compounds. Quadrupolar order does not break time reversal symmetry and cannot be directly seen by neutron diffraction. The small dipolar moments are considered as an unrelated secondary order parameter with accidentally the same transition temperature. In the itinerant models the order parameter is due to an unconventional pairing in the particle–hole channel leading to an unconventional SDW which has vanishing moment in the clean limit and also does not break time reversal invariance. The small staggered moments may then be induced in the condensate
due to impurity scattering. Finally in the dual models one assumes a localised singlet–singlet system in interaction with the itinerant electrons to cause induced moment magnetism with small moments but large anomalies.

In all models it was previously taken for granted that both the primary ‘hidden’ order parameter and AF order coexist homogeneously within the sample. However, hydrostatic and uniaxial pressure experiments (Amitsuka et al., 1999, 2002) have radically changed this view, showing that the order parameters exist in different parts of the sample volume; the tiny AF moment is not intrinsic but due to the small AF volume fraction under ambient pressure. Applying hydrostatic pressure or lowering the temperature increases the AF volume fraction and hence the ordered moment until it saturates at an atomic size moment of 0.4$\mu_B$/U. This means that the evolution of AF arises from the increase of AF volume with pressure rather than the increase of the ordered moment $\mu$ per U-atom. This interpretation is supported by the observation of a comparatively weak increase of $T_0$ with pressure (Amitsuka et al., 1999).

4.4.1. **Electronic structure and 5f-states**

Inelastic neutron scattering (Park et al., 2002) using the time-of-flight method has shown that the valence state in URu$_2$Si$_2$ is U$^{3+}$ corresponding to 5$f^2$, judging from the observation of the $^3$H$_4 \rightarrow ^3$F$_2$ transition of this configuration. In tetragonal (D$_{4h}$) symmetry the ninefold degenerate $^3$H$_4$ multiplet should be split by the CEF potential into five singlets and two doublets. There is indeed an indication of four strongly broadened CEF transitions at energies ranging from $\delta = 5$ meV to 159 meV. However, the data are not sufficient to determine the CEF potential and states. There is a transition at 49 meV which might be associated with the $\Gamma_3$–$\Gamma_5^1$ transition from the $\Gamma_3$ ground state which has been invoked in the model of Santini and Amoretti (1994). Earlier, low energy triple axis INS (Broholm et al., 1991) has shown that the transitions have a considerable dispersion. In addition the $c$-axis susceptibility exhibits clear CEF anomalies from which a CEF level scheme has been derived (Santini and Amoretti, 1994). However the result is not unique and the overall splitting obtained is by a factor two smaller as compared to the above INS results. In this analysis the low energy group of CEF states consists of three singlets shown in table 6.

The clear evidence for CEF states supports the view of mostly localised 5f-states in URu$_2$Si$_2$. This raises the question of the origin of HF behaviour in this compound. The specific heat coefficient in the low temperature ordered (magnetic + hidden) phase is reduced to

<table>
<thead>
<tr>
<th>Symmetry</th>
<th>CEF-state</th>
<th>Energy (meV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma_3$</td>
<td>0 = $\frac{1}{\sqrt{2}}(</td>
<td>2\rangle +</td>
</tr>
<tr>
<td>$\Gamma_1^1$</td>
<td>1 = $\epsilon(</td>
<td>4\rangle +</td>
</tr>
<tr>
<td>$\Gamma_2$</td>
<td>2 = $\frac{1}{\sqrt{2}}(</td>
<td>4\rangle -</td>
</tr>
</tbody>
</table>
only a moderate $\gamma = 64$ mJ/mol K$^2$, 40% of the paramagnetic value and only one tenth that of typical HF values. Magnetoresistance measurements have shown that URu$_2$Si$_2$ is a compensated metal without open orbits (Ohkuni et al., 1999). Three of the closed Fermi surface sheets have been determined by dHvA experiments (Ohkuni et al., 1999). Their moderately heavy masses are in the range $m^* = 8$–25 m which are, however, still larger by a factor ten as compared to the LDA band masses. For a nonmagnetic 5f singlet ground-state system as given in table 6 there is no Kondo mechanism to generate heavy quasiparticles. It is tempting to assume that rather the same mechanism as recently proposed for UPt$_3$ (Zwicknagl et al., 2002) and UPd$_2$Al$_3$ (Zwicknagl et al., 2003) is important: mass renormalization through virtual excitations within a pair of low lying 5f singlet states which leads to (White and Fulde, 1981)

$$\frac{m^*}{m_b} = 1 + N(0) \frac{2\alpha^2 I^2}{\delta}, \quad (96)$$

where $\alpha = \langle 0|J_z|1 \rangle$ is a dipolar matrix element, $I$ is the one-site exchange and $\delta$ the singlet–singlet splitting. This is a simplified version of the mass renormalisation in eqs. (65), (66) with $\alpha \equiv 2a_5 f|M|$. For several singlet pairs as in the present case one has a sum of such contributions. As discussed for UPt$_3$ for realistic parameters of U-compounds this mechanism may easily lead to a mass enhancement factor $\frac{m^*}{m_b} \simeq 10$ which would be an appropriate value for URu$_2$Si$_2$.

4.4.2. Phase transitions, field and pressure dependence

The discrepancy between small AF moment in URu$_2$Si$_2$ and large thermodynamic anomalies has led to the postulation of a hidden (non-dipolar) order parameter. The crucial questions about its nature are: (i) Is the order primarily involving the localised 5f-CEF split states or the heavy itinerant conduction electrons. (ii) Does the hidden order parameter break time reversal invariance or not. In the former case it may induce AF as secondary order parameter, in the latter the two order parameters are unrelated and their appearance at the same temperature $T_m = T_0$ has to be considered as accidental.

The continuous phase transition at $T_0$ is clearly seen in a large specific heat anomaly which becomes more pronounced in a magnetic field (fig. 46). The magnetic entropy contained in this peak is orders of magnitude larger compared to other magnetic U-compounds and alloys if one scales it with the AF ordered moment (Amitsuka et al., 2002) which proves that it must be connected to a hidden order parameter. A similar behaviour is seen in thermal expansion (Mentink et al., 1997) along $a$, however, the tetragonal symmetry below $T_0$ is preserved and no superstructure evolves (Kernavanois et al., 1999). The shape of specific heat and thermal expansion anomalies and their sharpening in an external field is reminiscent of antiferroquadrupolar (AFQ) phase transitions due to localised f-electrons known in the 4f-Ce-hexaborides and $Tm$-intermetallics. A quadrupolar order parameter is even under time reversal and is expected to be in competition with the dipolar AF order.

Pressure experiments have indeed found this competition. It is evident from the comparison of the pressure dependent ordered AF moment, and the position and intensity of $^{29}$Si-NMR satellites. While the former increases continuously with pressure, the splitting which is proportional to the local 5f-moment is pressure independent, however, the split satellite intensity
which is a measure of the AF volume fraction also increases with pressure (fig. 47). Analysis of these results show that the AF moment per magnetic U-site $\mu_{AF} \simeq 0.25\mu_B$ is not small. It is the small AF volume fraction at ambient pressure which leads to the small overall ordered moment of $0.03\mu_B$. As pressure increases the ordered moment also increases due to the increase in AF volume which continuously replaces the volume fraction with the hidden order. The replacement is complete at $p \simeq 1$ GPa. Somewhat above at $p_c$ another phase transition takes place in the complete AF-phase leading to a sudden increase of the on-site U-moment, however we note that this has not been observed in magnetic X-ray diffraction experiments.

The observed moderate increase of $T_m$ with pressure (Amitsuka et al., 1999) is compatible with the observation of a pressure independent U-moment. Subsequent uniaxial pressure experiments (Yokoyama et al., 2002) have shown that only the [100] or [110] uniaxial pressure...
in the tetragonal plane leads to the increase of the AF volume fraction or destruction of the hidden order, whereas the [001] uniaxial pressure has little effect. This anisotropic behaviour under pressure suggests that the hidden order parameter is associated with the tetragonal four-fold symmetry plane.

4.4.3. **Theoretical models: localised vs. itinerant**

We shall not recount the many attempts to explain the broken symmetry states of URu$_2$Si$_2$ but rather concentrate on two typical examples that seem to be compatible with the recent important results of the pressure investigations just described.

4.4.3.1. **AFQ order of local induced quadrupole moments.** The competitive behaviour of AF and hidden order points to even time reversal symmetry of the latter. If one assumes that the localised 5f electrons are involved in the hidden order then the already mentioned AFQ order in the singlet CEF level scheme is the most obvious choice. This model was proposed by Santini and Amoretti (1994) and Santini (1998) and is also the most well studied one. It is an entirely localised 5f-model and is defined by the Hamiltonian

\[ H = \sum_i \sum_{k,q} B^0_k O^{\Gamma_3}_k(i) + K \sum_{i \in A, j \in B} Q(i) Q(j) - \sum_i g \mu_B J_z(i) H \]  

(97)

whose terms represent CEF potential in Stevens operator representation, quadrupolar interactions and Zeeman energy, respectively. $K$ is an effective quadrupolar inter-site exchange mediated by conduction electrons, similar as in 4f-systems (Thalmeier and Lüthi, 1991). The CEF parameters $B^0_k$ are obtained from fitting to the susceptibility which leads to the CEF states in table 6. Since the ground state is a singlet, quadrupole order can only appear as induced order, i.e., nondiagonal matrix elements $\langle \Gamma_3 | Q | \Gamma_1 \rangle \neq 0$ must be present. This is possible for quadrupole operators $Q_{\Gamma_3} = (J^2_z - J^2_x)$ or $Q_{\Gamma_5} = (J_x J_y + J_y J_x)$. Naively this would be compatible with uniaxial pressure results which show that the hidden order parameter is most sensitive to strains within the tetragonal $xy$-plane. Mean field calculations were performed with eq. (97) based on the above CEF scheme under the assumption of two sublattice AFQ ordering. They give a qualitatively correct behaviour of the specific heat, thermal expansion, nonlinear and anisotropic susceptibility and magnetization. For example, the thermal expansion exhibits step like anomalies at $T_0(H)$ which increase with field strength. This is reproduced by the model since in tetragonal symmetry $(i = a, c)$ $\alpha_i = g_i \frac{\partial \langle O^0_2 \rangle}{\partial T}$ where $Q = O^0_2 = 3J_z^2 - J(J + 1)$ and $g_i$ is an effective magnetoelastic coupling constant. Close to $T_0$ this is proportional to the growth rate $\frac{\partial \langle Q \rangle^2}{\partial T}$ which leads to jumps in thermal expansion coefficient $\alpha_a = (1/a)(\partial a/\partial T)$ etc. reminiscent of those in the specific heat.

In principle the AFQ order parameter may also lead to a lattice superstructure with the AFQ vector $Q$ via magnetoelastic coupling terms. This has not been found (Kernavanois et al., 1999) but may simply be too small to observe as it is the case in other AFQ compounds like CeB$_6$. More disturbing is the fact that in the AFQ state an external field in the $ab$-plane should also induce a field dependent staggered magnetization which has not been observed either.
In the AFQ hidden order model there is no natural connection to an AF order parameter. For this purpose a dipolar interaction term has to be introduced (Santini, 1998) and the resulting magnetic ordering temperature $T_m$ is unrelated to $T_0$, if they are equal this has to be interpreted as accidental. In the light of the new pressure experiments it seems that they are close at ambient pressure, slightly favoring AFQ order. For $p > 0$ $T_0(p)$ and $T_m(p)$ cross at the critical pressure $p_c$, stabilizing the AF local moment phase for $p > p_c$. At ambient pressure local stress around crystal imperfections might already lead to a small AF volume fraction which then increases upon applying external pressure. Such a situation may be phenomenologically described by a Landau free energy ansatz (Shah, 2000)

$$f_L = \alpha_0(T - T_0(p))Q^2 + \beta Q^4 + \alpha'_0(T - T_m(p))m^2 + \beta' m^4 + \gamma_0 m^2 Q^2,$$

(98)

where the last term is a coupling term for AF ($m$) and AFQ ($Q$) order parameters. For $\gamma_0 > 0$ the two order parameters are in competition. If $T_m(p)$ and $T_0(p)$ are close the inclusion of inhomogeneities might then lead to a phase separation where $m$ and $Q$ order parameters exist in macroscopically separate volume fractions. The Landau approach has been extended by including the strain coordinates to analyze neutron diffraction results under uniaxial pressure (Yokoyama et al., 2003).

Although the AFQ scenario for the hidden order parameter seems most attractive for the explanation of macroscopic anomalies at $T_0$ there is no direct experimental proof. This would require the observation of either (i) the induced AF magnetic order in an external field or (ii) direct signature of 5f-orbital order in resonant X-ray scattering as, e.g., in the AFQ phase of CeB$_6$ (Nakao et al., 2001).

4.4.3.2. Hidden order as unconventional density wave. A complementary view of hidden order in URu$_2$Si$_2$ starts from a completely itinerant view of 5f-electrons by describing them within a multiband extended Hubbard model (Ikeda and Ohashi, 1998, 1999). Naturally this approach is unable to account for the CEF signatures in specific heat, susceptibility and similar quantities. In this model the large jump $\Delta C(T_0)$ is due to a condensation of electron–hole pairs (sect. 2). Unlike conventional CDW or SDW states however the pair states belong to nontrivial anisotropic representations of the symmetry group, similar to Cooper pairs in unconventional superconductors. The symmetry classification of unconventional electron–hole pair states and their stability analysis is given in (Gulacsi and Gulacsi, 1987; Schulz, 1989; Ozaki, 1992). In full generality this has only been done for n.n. tight binding models with perfect nesting property of 2D conduction bands $\epsilon_{k+Q} = -\epsilon_k$ and $Q = (1/2, 1/2, 0)$. In the low energy corner of the $U$–$V$ phase diagram ($U, V$ = on-site and n.n. Coulomb interaction, respectively) the stable state is an unconventional triplet particle–hole pair condensate (‘d-SDW’). In contrast to a common SDW where the gap function $\Delta_s(k) = \text{const}$ is momentum independent, in the d-wave case one has $\Delta^d_{\pm}(k) = \Delta^d(k)\sigma_z$ with

$$\Delta^d(k) = i\Delta^d_0(\cos k_\perp - \cos k_\parallel).$$

(99)

The order parameter is purely imaginary because it is connected with persistent commensurate spin currents around lattice plaquettes. This state breaks spin rotational symmetry but not time reversal symmetry because the latter inverses both current and spin direction leaving the order
parameter invariant. The staggered magnetic moment in the s- or d-wave case is given by

\[ M^Q_{zd} = \sum_{k\sigma} \sigma \langle c^\dagger_{k\sigma} c_{k+Q\sigma} \rangle = \sum_k \frac{\Delta^{d,s}(k)}{2E(k)} \tanh \frac{E(k)}{2T}, \]

where \( E_k = [\epsilon_k^2 + \Delta_k^2]^{1/2} \) is the quasiparticle energy which is fully symmetric in both s, d cases. Therefore \( M^Q_{zd} > 0 \) for conventional s-SDW but \( M^Q_{zd} = 0 \) for d-SDW case since the sign change in eq. (99) as function of \( k \) makes the integral in eq. (100) vanish for d-SDW (this notation convention is a misnomer since \( M^Q_{zd} = 0 \) means that there is precisely no ‘spin density’ in the d-wave case).

Despite the vanishing staggered moment in the d-SDW state there is a large BCS-like specific heat anomaly similar to the conventional s-SDW case. Therefore the d-SDW state would naturally explain the most prominent property of the hidden order phase in URu2Si2. The tiny moments would then immediately appear at the hidden order transition \( T_m \equiv T_0 \) around impurities, either through the creation of a s-SDW component by a proximity effect (Ikeda and Ohashi, 1998, 1999) or through anisotropic exchange interactions of the condensate with magnetic impurities (Virosztek et al., 2002). In the latter case the temperature dependence of the staggered moment can be explained. The phase separation of hidden order and staggered micromagnetism is then again seen as a consequence of the sample defects and inhomogeneous impurity concentration.

4.4.4. High field phase diagram and metamagnetism

Besides pressure, magnetic fields also destroy the hidden order as can be seen from the progressive \( T_0(B) \)-reduction in fig. 46. The corresponding critical field line forms the boundary to the hidden order phase (I) in fig. 48. Assuming the AFQ hidden order model of URu2Si2 there is an important difference to other known AFQ compounds which have a degenerate CEF ground state like cubic \( \Gamma_8 \) in CeB6. There \( T_0(B) \) first increases with field strength due to the lifting of orbital degeneracy. Since in URu2Si2 \( T_0(B) \) decreases monotonously this is a strong argument for the induced AFQ order in a singlet–singlet system. At low temperatures the hidden order phase is destroyed at a field \( B_0(0) = 36.1 \) T. For \( B > B_0(0) \) \( M(B) \)

![Fig. 48. B–T phase diagram of hidden order (HO) phase (I), metamagnetic phase (III) and high field phase (IV). Open symbols correspond to resistivity anomalies and filled symbols to specific heat maxima. For zero field \( T_0 = 17.5 \) K (HO) (Jaime et al., 2002).](image-url)
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exhibits multistep metamagnetic behaviour (Sugiyama et al., 1999) which may in principle be explained by the AFQ model as a field induced crossing of singlet CEF levels (Santini, 1998) complemented by the effect of inter-site exchange interactions. Originally it was thought that the metamagnetic transition extends to temperatures much higher than $T_0$ (Sugiyama et al., 1999). Recent pulsed field experiments (Jaime et al., 2002) have shown however that the metamagnetic phase is much more confined (region III in fig. 48). Alternatively this region has also been interpreted as reentrant hidden order phase (Harrison et al., 2003). Finally above $40T$ one reaches a moment of $1.5\mu_B/U$ which is much larger than the zero field value at high pressure, hence at high field not only the AF volume fraction is maximal but also the magnetic ground state of $U^{4+}$ has a larger moment. The structure of magnetic phases in the high field regime is still unknown.

4.4.5. Collective excitations in the ordered phase

Inelastic neutron scattering has perhaps provided the most convincing evidence that the localised 5f-electron picture is a good starting point for URu$_2$Si$_2$. In this case the dipolar CEF transition from the singlet ground state $|0\rangle'$ to excited singlet $|2\rangle'$ with dipolar matrix element $\alpha = \langle 0' | J_z | 2' \rangle$ should disperse into a collective ‘magnetic exciton’ band. Here the primes indicate that the corresponding singlet states in table 6 are modified due to the presence of AF order which mixes the unprimed states with higher CEF states, without this mixing obviously $\alpha$ would vanish. These magnetic excitons have indeed been found (Broholm et al., 1991) as well propagating modes in a large part of the tetragonal BZ. They may be described by a similar expression as magnetic excitons for UPd$_2$Al$_3$. The relatively large matrix element $\alpha = 1.2$ obtained from fitting to experimental results (Broholm et al., 1991) is not in contradiction to the small overall AF moment, since the latter is only an effect of the small AF volume fraction. The $|0\rangle'$ to $|1\rangle'$ singlet excitation has no dipolar matrix element, it is the excitation whose softening is governed by the quadrupolar matrix element $\alpha_Q = \langle 0' | Q | 1' \rangle$ leading to the induced AFQ (hidden) order at $T_0$. This quadrupolar mode does not directly appear in the dipolar response function and the INS cross section. It has been suggested that its presence can be seen indirectly through an influence on the dipolar excitation (Santini et al., 2000).

4.4.6. The superconducting state

As in other U-HF compounds the superconducting state in URu$_2$Si$_2$ with $T_c = 1.4$ K is embedded in the AF phase and here in addition in the hidden order phase with $T_0 \gg T_c$. Although there are a number of signatures for an unconventional pair state, it has attracted much less attention than the hidden order phase, possibly because there is no direct evidence for a multicomponent SC order parameter (Thalmeier, 1991). The evidence for gap anisotropy is obtained from various low temperature power laws for specific heat (Hasselbach et al., 1993) ($\sim T^2$) and NMR relaxation rate (Matsuda et al., 1996) ($\sim T^3$) which suggests the presence of line nodes. A plot of $C/T$ vs. $T$ is shown in fig. 9 in comparison with theoretical curves (Hasselbach et al., 1993) obtained for gap functions allowed in the Landau theory of tetragonal superconductors (Volovik and Gorkov, 1985). The region of experimental linear $T$ behaviour in $C/T$ is surprisingly large. This cannot be explained within a Landau approach which is restricted to the vicinity of $T_c$. In addition the reduced specific heat jump $\Delta C(T_c)$ as com-
pared to a constant gap supports the existence of a gap anisotropy with line nodes. This is also the conclusion from point contact spectroscopy (Hasselbach et al., 1992). The upper critical field curves of URu$_2$Si$_2$ show two anomalies (Keller et al., 1995): First the anisotropy ratio $B_{c2}^a/B_{c2}^c = 4$ is large, it cannot be fully explained by the anisotropy of Pauli limiting fields given by $\sqrt{\chi_c/\chi_a} \simeq \sqrt{5}$, orbital effects due to effective mass anisotropy must be involved. Furthermore, a distinct upward curvature of $B_{c2}^a$ is observed, similar to critical fields in the borocarbide superconductors where it was associated with two band effects or alternatively anisotropic Fermi velocities. The effect of AF on $B_{c2}$ should be negligible due to its small volume fraction.

As discussed in sect. 2 the observation of the dHvA effect far in the vortex phase ($B \ll B_{c2}$) is a sure signature of nodal superconductivity. Although oscillations of the three Fermi surface sheets in URu$_2$Si$_2$ have been seen below $B_{c2}$ (Ohkuni et al., 1999) the amplitude falls off quite rapidly with $B$, especially for field along $c$. Therefore, these experiments are not able to confirm the existence of nodes in $\Delta(k)$.

4.5. Superconductivity in the non-Fermi liquid state of UBe$_{13}$ and U$_{1-x}$Th$_x$Be$_{13}$

This cubic compound was discovered rather early (Ott et al., 1983, 1984) as a superconducting HF system. The U atom in this structure is embedded in an icosahedral cage of 12 Be-atoms. A global understanding of the normal state and symmetry breaking in both superconducting and magnetic state is still elusive. Firstly 'pure' UBe$_{13}$ crystals do not have the highest quality as compared to, e.g., UPt$_3$ so that the symmetry of the anisotropic SC gap functions has not been identified, furthermore the Th-doped crystals U$_{1-x}$Th$_x$Be$_{13}$ show a perplexing variety of SC and possibly also magnetic phases whose microscopic origin and order parameter symmetries are not understood. The $T-x$ phase diagram of U$_{1-x}$Th$_x$Be$_{13}$ has been investigated with a variety of methods and the most detailed results have been obtained using the thermal expansion method (Kromer et al., 1998, 2000, 2002) (fig. 50). The most important question concerns the nature of the low temperature phase (C) at intermediate doping ($0.02 < x < 0.045$). This phase may either be a nonunitary SC phase with condensate magnetic moments due to unconventional Cooper pairs (Sigrist and Rice, 1989) or a phase with coexisting anisotropic SC and a SDW type phase suggested by Kromer et al. (1998) on experimental grounds and already proposed in a theory by Kato and Machida (1987). In the former case one has to assume a crossing of two SC phases with two independent order parameters as function of the Th-concentration (Sigrist and Ueda, 1991) and in the latter a crossing of a SC and SDW phase line. Already in the normal state UBe$_{13}$ is a rather anomalous metal, e.g., non-Fermi-liquid (nFl) behaviour has been observed and attributed to a multichannel Kondo effect.

4.5.1. Normal state and nFl properties of UBe$_{13}$

The 5f-electron level in UBe$_{13}$ is close to the Fermi level as seen from photoemission results. Therefore the HF-behaviour which sets in below a fluctuation temperature of $T^* \simeq 8$–25 K does not correspond well to the Kondo picture which requires 5f states to lie sufficiently removed from the Fermi energy. Nevertheless the Kondo model in its single and multichannel
version has been employed for this HF compound. The specific heat coefficient is strongly enhanced with $\gamma \approx 1 \text{ J/mol K}^2$. Furthermore resistivity, specific heat and thermal expansion reveal the presence of a second low energy scale with $T_m \approx 2 \text{ K}$ where these quantities exhibit an additional maximum anomaly. This proves that at $T_m$ a coherent Landau–FL state has not yet evolved and that the superconducting transition at $T_c \approx 0.9 \text{ K}$ in UBe$_{13}$ happens within a strongly anomalous nFI state. In fact if superconductivity is suppressed by a strong magnetic field the $\gamma$ value shows a roughly logarithmic increase with decreasing temperature typical for a nFI state. No saturation of $\gamma(T)$ is observed down to lowest temperatures (Helfrich, 1996). In most other U-HF compounds superconductivity is embedded in a weakly AF state. Despite a number of attempts no long range AF order has been found in UBe$_{13}$, $\mu$SR measurements set an upper limit of $10^{-3} \mu_B$ for the moments on U-sites.

4.5.2. The 5f-ground state of U
The observed normal state nFI behaviour raises the still controversial question of the magnetic ground state of U-atoms in UBe$_{13}$. The high temperature susceptibilities do not allow to distinguish between a 5f$^3$- and 5f$^2$-state of U. In the former case proposed in Felten et al. (1986) a Kramers degenerate magnetic $\Gamma_6$ CEF ground state and two excited $\Gamma_8$ quartets at 180 K and still higher energy were inferred from analysis of specific heat which exhibits a CEF Schottky anomaly around 80 K. In this case the HF state would be due to a conventional Kondo mechanism involving the magnetic $\Gamma_6$ localised doublet. However this picture is in conflict with the pronounced nFI anomalies mentioned above. Indeed the nFI behaviour was taken as direct evidence for a 5f$^2$ configuration with a nonmagnetic $\Gamma_3$ doublet ground state which has a nonvanishing quadrupolar moment (Cox, 1987, 1988) whose fluctuations scatter conduction electrons. The effective Hamiltonian is of the two-channel Kondo type which leads to an overcompensation of quadrupolar moments and therefore to typical logarithmic nFI anomalies in specific heat and susceptibility at low temperatures (Schlottmann and Sacramento, 1993; Zvyagin, 2000). The $\Gamma_3$ quadrupolar model would predict a positive nonlinear magnetic susceptibility in $\chi_3$ with considerable anisotropy at low temperatures, instead the opposite, namely almost isotropic, negative $\chi_3(T)$ was found (Ramirez et al., 1994). This suggests that the ground state is a magnetic Kramers doublet $\Gamma_6$. To explain nFI behaviour one either has to invoke a magnetic multichannel Kondo effect or closeness to a quantum critical point in UBe$_{13}$.

4.5.3. The superconducting state in UBe$_{13}$
The $T_c$ values for superconductivity which occurs in the nFI state depend considerably on the type of sample. There are two classes with ‘high’ $T_c$ ($\approx 0.9 \text{ K}$) and ‘low’ $T_c$ ($\approx 0.75 \text{ K}$) which, however, are not much different in their impurity content. Low temperature ‘power law’ dependencies give conflicting information on the node structure of the gap and no firm conclusion on whether spin singlet or triplet pairing is realized in UBe$_{13}$ can be drawn. Therefore, these results will not be discussed further here although an axial order parameter with point nodes seems to be favored. The most direct evidence obtained so far for unconventional superconductivity is connected with the giant ultrasonic absorption anomaly observed directly
below $T_c$ (Golding et al., 1985; Müller et al., 1986) which was attributed to collective modes or domain-wall damping due to a multicomponent order parameter (Sigrist and Ueda, 1991).

The $B$–$T$ phase diagram of UBe$_{13}$ presented in fig. 49 has quite anomalous appearance: The upper critical field $B_{c2}(T)$ shows an inflection point around 0.45 K, furthermore deep in the superconducting regime another anomaly line $B^*(T)$ starting at $T_L = 0.7$ K has been identified both by specific heat (Helfrich, 1996) and thermal expansion measurements (Kromer et al., 1998, 2000). This line might be connected to the onset of magnetic correlations which, however, do not lead to long range order. This interpretation is supported by recent thermal expansion results (Kromer et al., 2000) who have shown that the line can be followed as function of Th-doping and eventually, according to this picture, long range SDW order appears above a Th concentration of $x_{c1} = 0.02$.

4.5.4. Superconducting phase diagram of Th-doped crystals
The $T$–$x$ superconducting phase diagram of the thorated U$_{1-x}$Th$_x$Be$_{13}$-crystals ($x \leq 0.10$) whose most recent version (Kromer et al., 2002), is shown in fig. 50 has attracted enormous interest because it was taken as strong evidence for unconventional superconductivity. Mainly two observations favored an interpretation in terms of exclusively SC phase transitions into SC phases denoted $A$, $B$ and $C$ in fig. 50: (i) At $x_{c1} = 0.02$ a cusp-like increase of $T_c$ into $T_{c1}$ occurs with seemingly different $T_c$ pressure coefficients below and above $x_{c1}$, suggesting that SC phases below $T_c$ ($A$, $x < x_{c1}$) and $T_{c1}$ ($B$, $x > x_{c1}$) are different. (ii) Below $T_{c2}$ for $x_{c1} < x < x_{c2}$ with $x_{c2} = 0.045$ a second superconducting transition into phase $C$ takes place which was inferred from the sudden increase of slope in $H_{c1}$ (Rauchschwalbe et al., 1987). Furthermore in phase $C$ magnetic moments $\mu \simeq 10^{-3}$ per U site appear according to $\mu$SR results (Heffner et al., 1990). This lead to the suggestion that the SC order parameter in the
Fig. 50. Superconducting phase diagram of \( U_{1-x}Th_xBe_{13} \) for \( x < 0.10 \) (Kromer et al., 2002). Full lines and symbols: thermodynamic phase boundaries; broken lines and open symbols: lines of anomalies. \( T_L \) and \( T_{\alpha}^{\min} \) denote line of anomalies from minimum in the thermal expansion coefficient \( \alpha(T) \), \( T_{\max}^{\alpha} \) is the line of anomalies from the maximum in \( C(T) \) and \( \alpha(T) \). A, B and C denote distinct SC phases in the \( T_c \)-crossing model.

The \( C \) phase is nonunitary, i.e., Cooper pairs carry a magnetic moment which appears around impurity sites.

These observations lead to a scenario within the Ginzburg–Landau theory approach based on the basic assumption of a crossing of \( T_c(\Gamma_1, x) \) and \( T_c(\Gamma_5, x) \) of two different superconducting order parameters at the critical concentration \( x_{c1} \) (Sigrist and Ueda, 1991). They belong to cubic \( O_h \) representations \( \Gamma_1 \) (fully symmetric) and \( \Gamma_5 \) (three-fold orbital degeneracy), though different pairs of representations are also possible. Below \( x_{c1} \) and hence also for stoichiometric \( UBe_{13} \), \( T_c(\Gamma_5, x) > T_c(\Gamma_1, x) \) and the unconventional \( \Gamma_5 \) (A) SC state is stable. Above \( x_{c1} \) the opposite inequality holds and the conventional \( \Gamma_1 \) (B) SC phase is stable immediately below \( T_{c1} \). For lower temperature when the size of the SC order parameter increases the fourth order mixing terms in the GL functional favor a mixed \( \Gamma_1 \oplus \Gamma_5 \) SC state (C) which becomes stable below the transition at \( T_{c2} \). For a specific region in the parameter space of the GL functional the mixed state may be nonunitary and therefore lead to Cooper pairs with magnetic moments. For further increase of Th-concentration enhanced pair breaking eventually leads again to a decreasing \( T_c(\Gamma_1, x) \) and a second crossing at \( x_{c2} \) appears.

However, the results of recent detailed thermal expansion experiments including additional Th-concentrations have challenged this \( T_c(\Gamma_5, x) - T_c(\Gamma_1, x) \) crossing interpretation (Kromer, 2000; Kromer et al., 2000, 2002). Firstly, investigation of additional Th dopings has shown that pressure dependence of \( T_c \) and \( T_{c1} \) is not really different outside the critical region around \( x_{c1} \). More importantly, however, a new line of anomalies \( T_L \) (dashed line in fig. 50)
has been found that starts at the aforementioned anomaly for UBe$_{13}$ in fig. 49 ($B^* = 0$) and continues until it merges with $T_{c2}$ for $x > x_{c1}$ and then even beyond $x_{c2}$. For $x = 0$ $T_L$ was interpreted as onset temperature of magnetic correlations. The thermal expansion anomaly at $T_L$ becomes ever sharper when $x$ approaches $x_{c1}$ which suggests increasing magnetic correlation length until finally at $x_{c1}$ at the real phase transition line $T_{c2}$ a true SDW state has evolved which continues beyond $x_{c2}$. In this picture the phases $A$ and $B$ are the same superconducting phases and $C$ is characterised by coexistence of the superconducting $B$-phase with the SDW state which also is responsible for the observed small moments (10$^{-3}$µB), there is no need to assume an exotic nonunitary SC order parameter in the $C$-phase. The evolution of a SDW state within the SC phase has already been proposed by Kato and Machida (1987). However in this picture the increased slope of $H_{c1}$ below $T_{c2}$ has no obvious explanation. Furthermore the sudden drop of flux creep rate by several orders of magnitude below $T_{c2}$ in phase $C$ (Dumont and Mota, 2002) can be explained in the crossing scenario where the nonunitary nature of $C$ provides an efficient mechanism for flux pinning (Sigrist and Agterberg, 1999). Indeed the drop in the flux creep rate is not observed for pure UBe$_{13}$ below $T_L$ (Mota and Cichorek, 2003).

In thermal expansion investigations it also became clear that the additional temperature scale $T_{\text{max}}$ is continuously reduced with increasing Th-concentration. Amazingly it hits $T_{c1}$ exactly at its maximum and (observed for $B > B_{c2}$) vanishes at $x_{c2}$ as shown by the dashed $T_{\text{max}}$ line in fig. 50. If this energy scale is due to magnetic excitations it suggests a close connection to the mechanism of Cooper pairing.

The discussion in this section has necessarily been rather qualitative as there is no developed microscopic theory for this complex behaviour where indeed SC and perhaps SDW order evolve in an incoherent nonstoichiometric HF metal with pronounced nFl behaviour.

5. Rare earth borocarbcide superconductors

The superconducting class of layered transition metal borocarbides RNi$_2$B$_2$C was discovered (Nagarajan et al., 1994; Cava et al., 1994) and investigated rather recently. Here R stands either for nonmagnetic Y, Lu and Sc or for lanthanide elements in a magnetic R$^{3+}$ state. Several excellent reviews are available already (Hilscher and Michor, 1999; Müller and Narozhnyi, 2001), mostly focusing on the material physics and chemistry of these compounds. The crystal structure shown in fig. 51 is body centered tetragonal (space group I4/mmm). It consists of R-C rock salt type planes separated by Ni$_2$B$_2$ layers built from NiB$_4$ tetrahedra and stacked along the c-axis. More general structures with more than one R-C layer are possible (Hilscher and Michor, 1999) but will not be discussed further. The nonmagnetic borocarbides have relatively high $T_c$’s around 15 K as seen in fig. 51. There is evidence that the superconducting mechanism is primarily of the electron–phonon (e-p) type although this cannot explain the large anisotropy of the SC gap. At first sight the layered structure is similar to the high-$T_c$ cuprates. However, unlike the copper oxide planes the Ni$_2$B$_2$ planes show buckling (fig. 51, left panel), as a consequence the electronic states at the Fermi level in the borocarbides do not have quasi-2D $d_{x^2−y^2}$ character and, therefore, have much weaker
Fig. 51. Left: Tetragonal crystal structure (I4/mmm) of RNi$_2$B$_2$C. Low temperature AF2 magnetic structure with [110] easy axis is indicated by arrows. \(a = b = 3.52 \text{ Å}\) and \(c = 10.53 \text{ Å}\) for \(R = \text{Ho}\). Reciprocal lattice vectors are given by \(2a^*\), \(2b^*\) and \(2c^*\) where \(a^* = \frac{2\pi}{a}\), etc. Right: Magnetic (\(T_{IC}\): incommensurate magnetic structure, \(T_N\): simple AF structure) and superconducting (\(T_c\)) transition temperatures in Kelvin for the RNi$_2$B$_2$C series.

correlations excluding the possibility of AF spin-fluctuation mediated superconductivity. The nonmagnetic borocarbides serve as a kind of reference point to separate the fascinating effects of AF and SC order parameter coupling in the magnetic RNi$_2$B$_2$C. However, the former have their own peculiarities which are not yet completely understood. Foremost, despite their alleged electron–phonon nature, LuNi$_2$B$_2$C and YNi$_2$B$_2$C have strongly anisotropic gap functions and low energy quasiparticle states as evident from specific heat and thermal conductivity. Furthermore an anomalous upturn in \(H_{c2}\) has been observed. The magnetic RNi$_2$B$_2$C are an excellent class of materials to study the effects of competition of magnetic order and superconductivity for the following reasons: The \(T_c\)'s are relatively high and their size relative to \(T_N\) varies systematically across the R-series. Especially interesting are the cases of RNi$_2$B$_2$C with \(R = \text{Dy, Ho and Er}\) where \(T_c\) and \(T_N\) (or \(T_{IC}\)) are not too different, leading to strong competition of the magnetic and SC order parameters. Furthermore the superconducting condensate and magnetic moments are carried by different types of electrons, namely itinerant 3d-electrons for the Ni$_2$B$_2$ layers and localized R$^{3+}$ 4f-electrons for the R-C layers, respectively. Finally they are well separated and their coupling which is of the local exchange
type can be treated in a controlled perturbative way somewhat akin to the situation in the well
known classes of Chevrel phase (Fischer, 1990) and ternary compound (Fischer and Maple,
1982) magnetic superconductors.

The antiferromagnetic molecular field establishes a periodic perturbation characterized by
a length scale of the order of the Fermi wavelength \( k_F^{-1} \ll \xi_0 \). This implies that the spatial
extent of the Cooper pairs extends over many periods of the staggered molecular field. The
latter is therefore effectively averaged to zero and does not suppress superconductivity via an
orbital effect. The system is invariant under the combined operation of time inversion followed
by a translation with a lattice vector which allows to form Cooper pairs in a spin singlet state
with vanishing (crystal) momentum in the antiferromagnetic lattice. This pair-state can be
considered as a natural generalization of the pairing in time-reversed states encountered in
usual nonmagnetic superconductors to which it reduces in the limit of vanishing staggered
magnetization. The magnetic order does not lead to depairing (Fulde and Keller, 1982).

For detailed investigation it is necessary to gain a clear understanding of the magnetic
phases of RNi_2B_2C. A theory of the rich metamagnetic phases in the \( B-T \) phase diagram of
HoNi_2B_2C and DyNi_2B_2C is a necessary prerequisite to comprehend the interaction effects
of the two order parameters in these compounds.

The investigation of alloy series of magnetic R^{3+} dissolved in the nonmagnetic borocar-
bides allows one to study the pair breaking effects due to incoherent exchange scattering and
its associated de Gennes scaling of \( T_c \). One also observes ‘inverse’ de Gennes scaling (Müll-
er and Narozhnyi, 2001), i.e., the suppression of \( T_c \) by nonmagnetic impurities like Lu and Y in
the magnetic RNi_2B_2C superconductors which shows quite similar behaviour as function of
the impurity concentration although its underlying physics is very different.

5.1. Physical properties of the nonmagnetic borocarbides

The nonmagnetic YNi_2B_2C and LuNi_2B_2C compounds with comparatively high \( T_c \) of 16.5 K
and 15.5 K serve as reference systems for the more difficult systems RNi_2B_2C with both
magnetic and superconducting phases. The electron–phonon nature of superconductivity in
YNi_2B_2C and LuNi_2B_2C is inferred from a substantial s-wave character of the order parameter
as witnessed by the appearance of a moderate Hebel–Slichter peak in the \(^{13}\text{C} \) NMR relaxation
rate (Mizuno et al., 1999). On the other hand the gap function is strongly anisotropic as can
be seen both from temperature and field dependence of thermodynamic and transport quanti-
ties. At low temperatures one observes a specific heat \( C_s \sim T^3 \) (Hilscher and Michor, 1999)
and thermal conductivity \( \kappa_{xx} \sim T \) (Boaknin et al., 2001) and \( \kappa_{zz} \sim T^\alpha (\alpha \sim 2–3) \) indicating
the presence of gap nodes (Izawa et al., 2002a). More precisely, within experimental accuracy,
there must be at least a gap anisotropy of \( \Delta_{\text{min}} / \Delta_{\text{max}} \leq 10^{-2} \) (Izawa et al., 2002a). For
an electron–phonon superconductor this would be the largest anisotropy ever observed. This
conjecture is also supported by a \( \sqrt{H} \) field dependence (Nohara et al., 1997) of the low tem-
perature specific heat and the linear \( H \) dependence of the thermal conductivity \( \kappa_{xx} \) for \( \mathbf{H} \) along
[001] (Boaknin et al., 2001). Since in the latter case the heat current is perpendicular to the
vortices this proves that quasiparticles must be present in the inter-vortex region. This is also
required to explain the observation of dHvA oscillations far in the vortex phase. Experimental
evidence therefore demands a nodal gap function for borocarbides and the $s + \gamma$ wave model proposed in Maki et al. (2002) fulfills the requirements. In addition it explains recent results on ultrasonic attenuation which also confirmed the existence of gap nodes in the cubic plane (Watanabe et al., 2003). In the following we will address the question of electron–phonon coupling strength, isotope effect and positive curvature in the upper critical field. Furthermore power law behaviour of the electronic specific heat and thermal conductivity as function of temperature and in addition their field-angle dependence $C_{ij}(T, H)$ and $\kappa_{ij}(T, H)$ for $T \ll T_c$ in the vortex phase allows to discuss the nodal structure of the SC gap.

5.1.1. Evidence for electron–phonon superconductivity

The classical argument in favor of the e-p mechanism is the observation of an isotope effect characterized by the isotope exponent for a specific atom with mass $M$ as given by $\alpha_M = -d \ln T_c / d \ln M$. However, this cannot be applied easily to complex layered superconductors, as evident from the existence of an isotope effect in the nonphononic cuprate superconductors with nonoptimal doping. A boron isotope effect has been found in YNi$_2$B$_2$C ($\alpha_B = 0.2$) and LuNi$_2$B$_2$C ($\alpha_B = 0.11$) much smaller than the BCS value $\alpha_M = 0.5$ and a nonphononic origin for $\alpha_B$ originating in the influence of boron on the charge density in the B$_2$Ni$_2$ layers has, therefore, been suggested (Drechsler et al., 2001).

The most direct support for the phonon mediated Cooper pairing is due to scanning tunneling spectroscopy (Martinez-Samper et al., 2003) which has shown the existence of a strong coupling signature in the tunneling DOS due to a soft optical phonon close to the FS nesting wave vector $Q$. The e-p coupling constant was derived as $\lambda = 0.5–0.8$ which is compatible with the value $\lambda = 0.53$ obtained from resistivity data.

Additional support for the electron phonon mechanism comes from the comparison of thermodynamic ratios $\Delta(T_c)/\gamma T_c$ and $\Delta(0)/kT_c$ calculated in strong coupling theory and the phonon spectra measured by inelastic neutron scattering (Hilscher and Michor, 1999). The strong coupling corrections of the above ratios to their BCS values are related to the logarithmic moments $\tilde{\omega}$ of the phonon DOS $F(\omega)$ or more precisely of the $\alpha^2 F(\omega)$ Eliashberg function. These moments may be directly obtained from the INS phonon DOS or indirectly from the above ratios. A comparison shows a reasonable agreement between the moments with a $T_c/\tilde{\omega}$ ratio characteristic for moderately strong coupling electron phonon superconductivity. The moments are, however, much smaller than the Debye energies obtained from the phonon specific heat which again indicates that certain low lying optical phonon modes play a special role for the coupling. By varying temperature it has been found that these are soft optical phonon modes along the $[q_x, 0, 0]$ direction which involve primarily the vibration of the heavy Y or Lu atoms. The softening occurs at $q_x \sim 0.55$ (in r.l.u. of $2\pi/a$) which corresponds to a nesting vector of the Fermi surface (Dugdale et al., 1999) where the $q$-dependent electronic susceptibility which determines the renormalized phonon frequency becomes strongly enhanced. The special role of these optical phonons for the strong coupling effects is underlined by the moment ratio $(\tilde{\omega}_{YNi_2B_2C}/\tilde{\omega}_{LuNi_2B_2C}) \sim 1.5 \sim (M_{Lu}/M_{Y})^{1/2}$ rather than being equal to the square root of the unit cell masses which would only be 1.16.
5.1.2. Anomalous $H_{c2}$-behaviour

The upper critical field in nonmagnetic LuNi$_2$B$_2$C and YNi$_2$B$_2$C compounds shows a peculiar positive (upward) curvature below $T_c$. This phenomenon is also observed in various other layered superconductors. Within the standard Ginzburg–Landau description of $H_{c2}$ for isotropic single band superconductors the slope is determined by the Fermi velocity and only a negative curvature is possible. A positive curvature can be obtained by assuming a strong anisotropy of the Fermi velocity (Drechsler et al., 2001) which may be simplified to a two band model with two Fermi velocities. The upper critical field for the two band model may be calculated within the linearized version of Eliashberg theory (Shulga et al., 1998). In this way a fit for the experimental $H_{c2}$-curves using a two band Fermi velocity ratio of $v_{F1}: v_{F2} = 0.97: 3.7$ (LuNi$_2$B$_2$C) and $0.85: 3.8$ (YNi$_2$B$_2$C) can be obtained. It also has to be assumed that there is strong e-p coupling in the $v_{F1}$ band and sizable coupling between $v_{F1}$ and $v_{F2}$ bands. It should be noted that the presence of magnetic impurity scattering may change the picture (Shulga and Drechsler, 2001) because an increasing scattering rate decreases the positive curvature. Thus the extraction of a unique set of parameters for the two band model from the positive $H_{c2}$ curvature seems difficult. As expected for uniaxial crystal structures, there is an $a-c$ anisotropy of $H_{c2}$ but surprisingly there is also a strong four-fold anisotropy within the tetragonal $ab$-plane (Metlushko et al., 1997) which cannot be due to FS effects on the level of the effective mass approximation, it rather should be taken as another indication of the large gap anisotropy in the $ab$-plane.

5.1.3. Specific heat and thermal conductivity results

The low temperature dependence of the specific heat is apparently described by a power law behaviour $C_s \sim T^n$ with $n$ between 2 and 3 (Hilscher and Michor, 1999). Thermal conductivity $\kappa_{xx}$ (Boaknin et al., 2001) clearly exhibits $T$-linear behaviour for $T \ll T_c$ suggesting the presence of nodal lines or second order node points as introduced below which would be compatible with $n = 2$ for the specific heat. Furthermore the investigation of field (and field-angle) dependence of $C_s(T, H)$ and $\kappa_i(T, H)$ ($i, j = x, y, z$) is a powerful method to obtain information on the quasiparticle spectrum and hence on the anisotropy properties of the gap function (sect. 2). In a conventional superconductor with isotropic gap the quasiparticles at low temperature are confined to the vortex core where they form closely spaced bound states with an energy difference $\Delta^2/E_F$ much smaller than $kT$. Therefore, they can be taken as a cylinder of normal state electrons of diameter $\xi_0$ (coherence length) which gives a field independent contribution $\xi_0^2 \gamma T$ per vortex to the linear specific heat. Then in the vortex phase $\gamma(H)$ is predicted to scale with the number of vortex lines which is $\sim H$. However in superconductors with nodes the quasiparticles with momenta in the node direction can tunnel into the inter-vortex region and then their energy is Doppler shifted by the supercurrents around the vortex by an amount $\delta E = m v_F \cdot \mathbf{v}_s$. As shown by Volovik (1993) this leads to a finite residual quasiparticle DOS at zero energy given by

$$N_s(0)/N_n = K \sqrt{\frac{H}{H_{c2}}},$$

(101)
where \( K \) is a constant and \( N_n \) the normal state DOS. As found later the DOS also depends on the relative angle of the field with respect to the nodal positions of the gap function. The DOS in eq. (101) leads then to a \( \sqrt{H} \) behaviour of the specific heat (Nohara et al., 1997) and also for \( \kappa_{zz}(H) \) for temperatures larger than the quasiparticle scattering rate. Thermal conduction perpendicular to the vortex lines (\( H \parallel c \)) as given by \( \kappa_{xx}(H) \) that starts immediately above \( H_{c1} \) for \( T \ll T_c \) can only result from the presence of extended quasiparticle states outside the vortex cores and hence is a direct proof for the presence of nodal regions in the gap. To draw similar conclusions from the specific heat it is necessary to observe the \( \sqrt{H} \) behaviour for \( H \ll H_{c2} \) and in connection the infinite slope of \( \gamma(H) \) for \( H \rightarrow 0 \). The Doppler shift picture of magnetothermal properties is only an approximation. A comparison with exact quasiclassical calculations has been given in Dahm et al. (2002) and Nakai et al. (2004).

5.2. Theoretical analysis of nonmagnetic borocarbides

We briefly discuss the electronic structure and associated FS topology which is also important for the magnetic borocarbides. Our main topic here is the explanation of the quasi-unconventional SC low temperature behaviour of nonmagnetic Lu- and Y-borocarbides which may be understood within a hybrid s + g wave model for the SC gap function.

5.2.1. Electronic structure of the borocarbides

The crystal structure of the borocarbides is of a layered type which might one lead to expect quasi 2D features in the electronic bands. Calculations by various groups (Mattheis, 1994; Pickett and Singh, 1994; Rosner et al., 2001) have shown however that they have definitely 3D character which is also suggested by the rather isotropic resistivity (Fisher et al., 1997). Conduction states are composed of wide Ni–B–N–sp bands and narrow (~ 3 eV) Ni–3d bands centered about 2eV below the Fermi level which is close to a local peak of the DOS in LuNi2B2C and YNi2B2C. The DOS peak generally decreases when Lu is replaced by progressively lighter R atoms which leads to a correspondingly smaller \( T_c \). This also agrees with the observation that substitution of Ni with Co (hole-doping) or with Cu (electron-doping) decreases \( T_c \) (Gangopadhyay and Schilling, 1996; Schmidt and Braun, 1997) because the Fermi level moves away from the local peak position. Unlike in the cuprates all Ni 3d-states in the borocarbides contribute to conduction bands at the Fermi surface which therefore consists of many sheets. In LuNi2B2C the main sheet exhibits an important nesting feature (Dugdale et al., 1999). FS lobes along [110] direction in the tetragonal plane are connected by a nesting vector \( \mathbf{Q} = (0.55, 0, 0) \). This in turn leads to a peak structure in the staggered susceptibility \( \chi(\mathbf{q}) \) (Rhee et al., 1995) at \( \mathbf{q} = \mathbf{Q} \). In the RNi2B2C the effective magnetic RKKY interaction \( J(\mathbf{q}) \) is proportional to \( \chi(\mathbf{q}) \) and therefore \( a \)-axis incommensurate magnetic order with modulation vector \( \mathbf{Q} \) is seen in Gd, Tb, Ho, Er and Tm-borocarbides which coexists with \( c \)-axis spiral order in HoNi2B2C (sect. 5.3). Further evidence of the importance of the nesting at \( \mathbf{Q} \) comes from the presence of a pronounced Kohn anomaly at wave vector \( \mathbf{Q} \) in the phonon dispersion of LuNi2B2C (Dervenagas et al., 1995). Finally the previous conjecture of strongly different Fermi velocities in the two band model has been confirmed in electronic structure calculations for YNi2B2C (Rosner et al., 2001).
5.2.2. Nodal structure of the superconducting gap and impurity effects
As explained before, thermodynamics and transport behaviour points to an extremely anisotropic or nodal gap function in the nonmagnetic borocarbides. A gap function compatible with reported experiments was proposed in Maki et al. (2002). It is a hybrid $s + g$ wave gap which is fully symmetric ($A_{1g}$) under the tetragonal group $D_{4h}$ and has the form

$$\Delta(k) = \frac{1}{2} \Delta \left( 1 - \sin^4 \phi \cos(4\phi) \right),$$

(102)

where $\vartheta, \phi$ are the polar and azimuthal angle in $k$-space, respectively. This gap function has four second order node points at $(\vartheta, \phi) = (\frac{\pi}{2}, 0), (\frac{\pi}{2}, \pi)$ and $(\frac{\pi}{2}, \pm \frac{\pi}{2})$ (fig. 52) which dominate the quasiparticle DOS for $E \ll \Delta$ where

$$\frac{N_s(E)}{N_n} = \frac{\pi |E|}{4 \Delta}. \quad (103)$$

The form of $\Delta(k)$ implies a fine tuning of $s$ and $g$ amplitudes. At its second order node points there is no sign change of the gap function and therefore the derivative is also zero. The resulting linear quasiparticle DOS then leads to the observed low temperature specific heat

$$\frac{C_s}{\gamma T} = \frac{27}{4\pi \zeta(3)} \frac{T}{\Delta}, \quad (104)$$

where $\gamma$ is the Sommerfeld constant. The presence of nodal points in the hybrid $s + g$ wave gap function has no intrinsic symmetry reason but is due to a ‘fine-tuning’ of $s$- and $g$-wave amplitudes in eq. (102). For YNi$_2$B$_2$C this is realised to an astonishing degree of $\Delta_{\text{min}}/\Delta_{\text{max}} \leq 0.01$. There is presently no microscopic explanation for this fine tuning but a phenomenological justification for the stability of an $s + g$ wave order parameter for a wide range of pair potentials has been given (Yuan and Thalmeier, 2003).

Fig. 52. Polar plot of the anisotropic SC gap function $\Delta(k)$ of the $s + g$ model. Nodal points are along [100] and [010] directions (Maki et al., 2002).
The nodal $s + g$ wave gap function has a surprising behaviour when normal impurity scattering is taken into account (Yuan et al., 2003). For nodal gap functions belonging to a single nontrivial representation, like a $d$-wave gap there is a strong difference between scattering in the Born limit (small phase shift) and unitary limit (phase shift $\pi/2$). In the latter resonance scattering leads to the appearance of a residual zero field DOS resulting in a finite specific heat coefficient for $T < \Gamma$ ($\Gamma = \text{quasiparticle scattering rate}$). In the hybrid $s + g$ wave case, quite the opposite behaviour, i.e., gap opening by impurity scattering, is observed which is almost identical for Born limit and unitary limit. The quasiparticle DOS including impurity scattering is given by

$$\frac{N_s(E)}{N_n} = \text{Im} \left( \frac{i \tilde{\omega}_n}{\sqrt{\tilde{\omega}_n^2 + \tilde{\Delta}_k^2}} \right)_{\tilde{\omega}_n \rightarrow E + i\delta},$$

where $\tilde{\omega}_n = \omega_n + i \Sigma_0$ and $\tilde{\Delta}_k = \Delta_k + \Sigma_1$ are the Matsubara frequency and gap function renormalised by impurity scattering which leads to diagonal and nondiagonal self energies $\Sigma_0$ and $\Sigma_1$, respectively. For $\omega \rightarrow 0$ one finds $\tilde{\Delta} \rightarrow \frac{1}{2} \Delta + \Gamma$ where $\Gamma$ is the scattering rate. In other words an energy gap $\omega_g$ immediately opens up for finite $\Gamma$ as can be observed in fig. 53 which scales approximately as $\omega_g(\Gamma) = \Gamma/(1 + \frac{2\Gamma^2}{\Delta})$. Thus the $s + g$ wave fine tuning is destroyed by impurity scattering and nodal quasiparticles are removed for temperatures $T < \Gamma$. This would lead to a crossover to exponential low temperature behaviour in the specific heat. Indeed this gap creation was observed in Pt-substituted Y(Ni$_{1-x}$Pt$_x$)$_2$B$_2$C with $x = 0.2$ by Nohara et al. (1997) as shown in fig. 55. This remarkable behaviour, which is exactly opposite to the $d$-wave case where residual states at the Fermi level are created, can be traced back to the different character of first order nodes ($d$-wave) where the gap function changes sign and second order nodes ($s + g$ wave) where the angular derivatives of $\Delta(\theta, \varphi)$ also vanish and no sign change of the gap occurs. In the first case the effect of impurity scattering averages out in the gap equation, in the second it does not, leading to a finite excitation gap $\omega_g$. 
5.2.3. Thermodynamics and transport in the vortex phase

As discussed in sect. 2 the Volovik effect leads to a momentum and position dependent Doppler shift of the quasiparticle energies caused by the supercurrent flowing around the vortices. The corresponding DOS change depends both on the field strength and field direction with respect to nodal positions. For the gap model of eq. (102) the residual field induced DOS is given by Won et al. (2003), Thalmeier and Maki (2003b)

\[ N_s(0) = \frac{C_s}{\gamma T} = \frac{\tilde{v} \sqrt{eH}}{2\Delta} I(\theta, \phi), \]

where \( \tilde{v} = \sqrt{v_a v_c} \) and \( I(\frac{\pi}{2}, \phi) = \max(\sin \phi, |\cos \phi|) \). This function has a cusp-like minimum at \( \phi = \pi \) when \( H \) is sweeping over the node points. Also the residual DOS exhibits naturally the experimentally observed \( \sqrt{H} \) behaviour. For the calculation of thermal conductivity in the vortex phase of the s+g wave superconductor one has to be aware that impurity scattering immediately opens a gap \( \omega_g \approx \Gamma \). To have nodal quasiparticles for transport available one must fulfil \( \omega_g \approx \Gamma < T \). On the other hand to have an appreciable oscillation amplitude one must still be in the low temperature limit \( \Gamma < T < \tilde{v} \sqrt{eH} \ll \Delta \). The angle dependence of the c-axis thermal conductivity in the leading order is then given by

\[ \frac{\kappa_{zz}}{\kappa_n} \approx \frac{2}{\pi} \frac{\tilde{v} \sqrt{eH}}{\Delta} I(\theta, \phi), \]

i.e., in this limit to leading order it is completely determined by the angular dependent residual DOS. To determine the proper nodal positions from the maxima/minima of \( \kappa_{zz}(\theta, \phi) \) it is important that the low temperature limit is reached.

The angular dependence of the c-axis thermal conductivity has been investigated in detail in (Izawa et al., 2002a). The geometry with heat current along \( c \) and \( H \) conically swept around \( c \) is shown in the inset of fig. 54. For the in-plane field (\( \theta = \frac{\pi}{2} \)) pronounced cusps in \( \kappa_{zz}(\frac{\pi}{2}, \phi) \) appear for \( \phi = 0, \pm \pi/2 \) in \( \kappa_{zz}(\frac{\pi}{2}, \phi) \) as is visible in fig. 54. This is a typical signature for the existence of point nodes in the gap in the direction of the tetragonal [100] and [010] axes. When the polar field angle \( \theta \) decreases the oscillations in \( \kappa_{zz}(\theta, \phi) \) as function of azimuthal field angle \( \phi \) are rapidly diminished. This behaviour is indeed predicted from eq. (106) as shown in fig. 54. A similar calculation for a \( d_{xy} \)-gap function \( \Delta(\phi) = \Delta(\psi) = \Delta \sin(2\phi) \) which has line nodes in the same directions may be performed (Thalmeier and Maki, 2003b).

In this case no cusp appears and the amplitude of \( \kappa_{zz}(\theta, \phi) \) oscillation in \( \phi \) is almost independent of \( \theta \). This speaks strongly in favor of the \( s + g \) wave order parameter as the correct model for YNi₂B₂C and possibly LuNi₂B₂C. It is the first confirmed case of a superconductor with (second order) node points in the gap functions. The only other known candidate is UP₃ whose \( E_{2u} \) gap function in the \( B \)-phase is supposed to have point nodes at the poles (sect. 4.1.1). Similar experiments as in fig. 54 have yet to be performed for this compound. The point nodes of YNi₂B₂C were found to lie along the tetragonal axis, this means that the \( s + g \) gap function in fig. 52 is rotated by 45 degrees compared to the one used in Maki et al. (2002). According to eq. (106) the \( \phi \) dependence of the specific heat is determined by the same angular function.
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Fig. 54. Left: Experimental c-axis thermal conductivity of YNi$_2$B$_2$C as function of azimuthal field angle $\phi$ for various polar field angles $\theta$ (Izawa et al., 2002). The inset shows the field geometry with $\mathbf{H}$ being swept around $c$ by varying $\phi$ and keeping $\theta$ constant. Right: Theoretical normalized $\phi$-dependence of angular function $I(\theta, \phi)$ which determines $\kappa_{zz}$ according to eqs. (106), (107) (Thalmeier and Maki, 2003b).

Fig. 55. Left panel: Cusp-like four-fold oscillations in $\kappa_{zz}(90^\circ, \phi)$ of YNi$_2$B$_2$C due to $s+g$ wave point nodes are completely destroyed by 5% Pt-doping (Kamata, 2003). Right panel: Transition from $\sqrt{H}$-behaviour of $\gamma(H)$ for the nodal SC at $x=0$ to linear in $H$ behaviour for $x=0.2$ when a large gap has opened (Nohara et al., 1997).

and cusps should also appear there. This was indeed observed (Park et al., 2002) (fig. 12). In the Pt-substituted compound Y(Ni$_{1-x}$Pt$_x$)$_2$B$_2$C already $x=0.05$ is sufficient to destroy the angular oscillations in $\kappa_{zz}(\theta, \phi)$ (Kamata, 2003) which is again due to the rapid gap-opening for $s+g$ wave and destruction of nodal regions by impurity scattering as shown in fig. 55.

In this section we have shown abundant experimental evidence that the nonmagnetic YNi$_2$B$_2$C and possibly LuNi$_2$B$_2$C have anisotropic gap functions with point nodes that can
be described by a hybrid $s + g$ wave order parameter. As a consequence the simple electron–phonon type pairing mechanism originally envisaged for the borocarbides certainly has to be supplemented, for example, by strongly anisotropic Coulomb interactions, to account for the strongly anisotropic gap function found there.

5.3. Magnetic borocarbides

The magnetic 4f-electrons of lanthanides and the superconducting 3d-electrons of Ni are well separated in the RC layers and Ni$_2$B$_2$ layers of RNi$_2$B$_2$C. The coupling between magnetic and superconducting order parameters is weak enough to be treated as a perturbation but still strong enough to lead to pronounced effects on the SC properties. The magnetic and superconducting transition temperatures $T_m$ and $T_c$ of the series are shown in fig. 51. We will first discuss some of the basic observations in the series with focus on the Ho and Er borocarbides where coexistence of both order parameters is most interesting. The size of $T_m$ and the influence of magnetism on $T_c$ are both governed by the de Gennes factor $d_G = (g - 1)^2 J (J + 1)$ where $g$ is the Landé factor and $J$ the total angular momentum of the $R^{3+}$ ion. The former suggests the RKKY interaction via Ni 3d conduction electrons of intervening Ni$_2$B$_2$ layers as the coupling mechanism for the 4f moments in RC layers. The magnetic structures exhibit a great variety which has been described and tabulated in Lynn et al. (1997) and Müller and Narozhnyi (2001) and the following general features are observed: (i) Both commensurate (C) and incommensurate (IC) magnetic order parameters are present. (ii) For Dy, Ho the commensurate AF structure consists of ferromagnetically ordered $ab$-planes stacked along the $c$-axis. (iii) In IC structures one has two possibly coexisting type of modulations: helical modulation of stacked FM planes along the $c$-axis or SDW type (both longitudinal and transverse) modulation within each $ab$-plane with a modulation vector $Q$ close to the FS nesting vector. (iv) The easy axis selected by the CEF potential is [110] (Ho, Dy), [100] (Er) or [001] (Tm).

As an example the low temperature HoNi$_2$B$_2$C commensurate AF2 structure is indicated in fig. 51. In the RKKY mechanism IC modulation vectors should result from the maxima of the static electronic susceptibility $\chi(q)$ which has been approximately calculated by Rhee et al. (1995). It exhibits a pronounced peak at the nesting vector $Q$ of the Fermi surface which is indeed quite close to observed $a$-axis magnetic modulation vectors $Q_a = (0.55, 0, 0)$ of the Gd, Tb, Ho and Er borocarbide compounds. On the other hand in HoNi$_2$B$_2$C only a flat maximum is observed for wave vectors $q = (0, 0, q_c)$ close to the AF vector $Q_{AF} = (0, 0, 0.5)$ so that the observed helically modulated structures along $c^*$ should not be associated with any FS nesting feature. For theoretical model calculations of stable magnetic structures it is a more sensible approach to parametrize the long range RKKY exchange interactions and compare the results with the experimentally determined field-angle dependent phase diagram at low temperature. This has been most successfully done for HoNi$_2$B$_2$C (Amici and Thalmeier, 1998) and ErNi$_2$B$_2$C (Jensen, 2002). Four of the RNi$_2$B$_2$C ($R = Dy, Ho, Er, Tm$) show coexistence of superconductivity and magnetic order. As mentioned before HoNi$_2$B$_2$C and DyNi$_2$B$_2$C (Winzer et al., 1999) display the most spectacular interaction between superconductivity and magnetic order as already suggested by close transition temperatures $T_m$ and $T_c$ (fig. 51). On the other hand in ErNi$_2$B$_2$C and TmNi$_2$B$_2$C the SC vortex state exhibits an anomalous behav-
The metamagnetism and IC-C lock-in transition in HoNi$_2$B$_2$C

The metamagnetism of HoNi$_2$B$_2$C has been investigated theoretically in considerable detail (Amici and Thalmeier, 1998; Amici et al., 2000). First we discuss the 4f-magnetism of HoNi$_2$B$_2$C. This problem has two related aspects. For zero field the first IC magnetic phase with $Q = (0, 0, 0.45)$ appears at $T_{IC} = 6$ K which has a helical modulation of the moment of FM $ab$-planes along $c$. At $T_N = 5$ K a lock-in transition occurs to the simple AF with wavevector $Q_{AF}$ and [110] easy axis which is the stable low temperature structure. In addition an $a$-axis modulation with wave vector $Q_a$ exists which will be neglected here. The application of a magnetic field $H$ at an angle $\theta$ with respect to the easy axis leads to the appearance of additional phases as witnessed by metamagnetic transition steps in the magnetization curve (Canfield et al., 1997) and shown in the inset of fig. 56. At these steps some of the FM layers with moments pointing roughly opposite to the field direction align with the easy axis closest to the field direction, this leads to a stacking of FM planes with a resulting net magnetic moment. Following the magnetization steps as function of $\theta$ one can construct the metamagnetic phase diagram (fig. 56). The observed magnetic high-field phases are listed in the caption of fig. 56.

The metamagnetism of HoNi$_2$B$_2$C is caused by two conflicting interactions, namely tetragonal CEF potential and the RKKY-interaction. Whereas the first tries to align the moments along the easy axis as determined by the CEF potential the second one prefers a helical modulation of moments with a wavevector that minimizes the exchange energy. The favored structure results from a compromise and may change with temperature and external field. The
CEF potential is obtained from inelastic neutron scattering experiments (Gasser et al., 1996; Cavadini et al., 2003) by fitting the observed energies and intensities of CEF transitions to the predictions of an extended point charge model of the tetragonal CEF which determines the parameters of the model Hamiltonian $H_{CEF}$. Then the CEF energies and wave functions may be obtained explicitly. For Ho$^{3+}$ ($J = 8$) the 17 singlet and doublet CEF states split into two groups: A group of 13 states with energies $> 10$ meV which is irrelevant in the interesting temperature range and four low lying tetragonal CEF states consisting of a ground state singlet $\Gamma_4$ (0 meV) and excited doublet $\Gamma_5^*$ (0.15 meV) and another singlet $\Gamma_1$ (0.32 meV). The interacting Ho 4f-states are then described by

$$H_M = \sum_i [H_{CEF}(J_i) - \mu_B g_i J_i B] - \frac{1}{2} \sum_{ij} J(i,j) J_i J_j,$$

(108)

where the first part describes the CEF potential and Zeeman energy ($g = 5/4$ for Ho$^{3+}$) and the second term the effective RKKY exchange interaction which is to be fitted empirically to reproduce the critical fields and temperatures of the metamagnetic phase diagram. In mean field (mf) approximation the dimensionless magnetization $m_i$ has to fulfil the self-consistency relations

$$m_i(B_i^f, T) = \text{Tr}(e^{\beta H_{mf}(J_i, B_i^f) J_i}); \quad B_i^f([m_l], T) = g \mu_B + \sum_j J(i, j) m_j,$$

(109)

here $i$ runs over all moments $m_i$ in the magnetic unit cell in a given magnetic structure and $B_i^f$ is the effective field experienced by the moment. Because the $ab$-planes are FM ordered (neglecting the $Q_a$-modulation) these mf-equations may be solved within a simplified quasi one-dimensional model where only structures are considered that correspond to a different stacking of FM planes along $c$. Structures with unit cell sizes of up to 29 layers have been considered. Of all structures that are solutions of eq. (109) the stable one at a given field and temperature has to minimize the Helmholtz free energy per volume

$$f_H = f_G(B) + \frac{B^2}{8\pi} - \frac{H \cdot B}{4\pi} = f_G(B) + 2\pi M^2 - \frac{H^2}{8\pi},$$

(110)

where $B = H + 4\pi M$ is assumed to be homogeneous throughout the sample with the homogeneous magnetization per volume given by $M = (g \mu_B / v_c) \bar{m}_i$. The bar indicates the average over all moments and $v_c$ is the chemical unit cell volume. The effective exchange parameters between 0th and $i$th FM layer, $J_i = \sum_{(j_i)} J(0, j)$ ($j_i$ runs over all the sites of the $i$th layer) are considered to be empirical parameters that are determined from the critical fields of the metamagnetic steps, from $T_{IC}$ and from the requirement that the 1D Fourier transform $J(q_z)$ of $J_i$ has a maximum close to the IC modulation vector $Q$. This empirically determined exchange function $J(q_z)$ is shown in the inset of fig. 57. In this way most of the metamagnetic phases observed in HoNi$_2$B$_2$C and the field angle have been explained within the CEF-RKKY-exchange model eq. (108) as shown in fig. 56 (Amici and Thalmeier, 1998). The exchange parameters derived theoretically in this work have been confirmed experimentally by Cavadini et al. (2003). However some discrepancies remain, especially it was found later
(Campbell et al., 2000; Detlefs et al., 2000) that in the high-field phase F3(\uparrow\uparrow\rightarrow) the ab-planes are no longer FM ordered so that the simplified one-dimensional model is insufficient for high fields, but it is completely adequate for studying the coexistence behaviour with superconductivity since \(H_c(0) \simeq 0.8T\) for HoNi\(_2\)B\(_2\)C is below the critical field where the \(\uparrow\uparrow\rightarrow\) phase appears.

The inter-layer exchange parameters \(J_i\) with \(i = 0-3\), or exchange function \(J(q_z)\) obtained in the analysis of metamagnetism can now be used to predict the \(B-T\) phase diagram for \(H\) along the easy axis \([110]\) without invoking any further empirical parameters. The solution of eq. (109) for finite temperature leads to the phase diagram shown in fig. 57. The first phase that appears at \(T_{IC}\) is determined by the maximum gain in exchange energy, it is a long wavelength commensurate helix phase H5 (FM moment of \(ab\)-plane rotating around the \(c\)-axis) with wave vector \(Q_{H5} = 0.802c^*\) which is not too far from the true IC wave vector \(Q = 0.9c^*\) observed in neutron diffraction. To approximate the true IC phase even better with high order C phases, one had to include still more interlayer interactions, the free energies of these phases are, however, extremely close. When temperature decreases below \(T_{IC}\) the moments \(m_i(T)\) increase and the CEF contribution to the free energy which grows with fourth power of the moment size becomes more important. Because it favours alignment with the easy axis directions \([110]\), etc. the helix phase becomes less favorable and finally at \(T_N = 5K\) a lock-in transition to the simple commensurate AF phase takes place. There are indications from magnetic X-ray diffraction that an intermediate lock-in to another C-phase above \(T_N\) may exist because a splitting of the IC satellite Bragg peak was observed. Such behaviour is well known from 1D ANNNI models that exhibit devil’s staircase behaviour (Bak and von Boehm, 1980). At temperatures below 4 K application of a magnetic field along the easy axis finally leads to a metamagnetic transition from the AF2 phase (\(\uparrow\downarrow\)) to the ferrimagnetic AF3 phase with unit cell (\(\uparrow\downarrow\uparrow\)) (fig. 57).
5.3.2. Weak ferromagnetism in ErNi$_2$B$_2$C

This compound becomes superconducting at $T_c = 11$ K and develops a transversely polarised SDW of Er moments below $T_{IC} = 6$ K with a propagation vector $\mathbf{Q} = (0.533, 0, 0)$ corresponding to the FS nesting vector and a moment $\mu = 7.8 \mu_B$ and an easy axis [100]. Magnetization measurements (Canfield and Bud’ko, 1996) have shown that below $T_{WFM} = 2.3$ K a remnant magnetization exists due to the transition to a weak ferromagnetic (WFM) state with an average moment of $\mu_{WFM} = 0.33 \mu_B$/Er which persist to lowest temperatures. Therefore ErNi$_2$B$_2$C is the first example of true microscopic SC/FM coexistence for all temperatures below $T_c$. The origin of the WFM phase has been investigated with neutron diffraction and its structure was determined (Kawano et al., 1999; Kawano-Furukawa et al., 2002). On lowering the temperature from $T_N$ the SDW exhibits a ‘squaring up’ witnessed by magnetic satellite peaks and finally breaks up into commensurate AF sections separated by antiphase boundaries as shown in fig. 58. In every second layer along $c$ the boundaries are located between the Er-bonds creating first disordered moments which finally below $T_{WFM}$ order ferromagnetically. Because only the $z = 0$ and equivalent layers of the ErNi$_2$B$_2$C bcc structure carry the FM moment one obtains an average FM bulk moment of $\mu_{WFM} = (2/40)M_{sat} = 0.39 \mu_B$ which is close to the moment from magnetization measurements. The overall WFM structure (fig. 58) of ErNi$_2$B$_2$C can be viewed as FM sheets in the $bc$-plane with easy axis $b$ and stacked along $a$. The large stacking distance along $a$ is the origin of the bulk WFM moment of $0.33 \mu_B$/Er. As in the case of HoNi$_2$B$_2$C metamagnetic transitions appear for applied field in the $ab$-plane. They have been explained by a similar model (Jensen, 2002) including dipolar interactions. The stable zero field structure has the observed modulation vector with $|\mathbf{Q}| = 11/20$.

5.4. Coexistence of superconductivity and magnetic order

Coexistence behaviour has been studied both for stoichiometric compounds Er, Tm, Dy and Ho borocarbides as well as for pseudo-quartenary compounds $R_{1-x}R’_xNi_2B_2C$ where $R$, $R’$ are different lanthanide atoms or compounds $R(Ni_{1-x}M_x)_2B_2C$ with $M$ denoting another transition metal atom. In the stoichiometric compounds the interaction of both order parameters
is most dramatically seen in the upper critical field $H_{c2}$-anomalies, especially for HoNi$_2$B$_2$C with an almost reentrant behaviour visible in fig. 59. Reentrance can finally be achieved by replacing Ni with Co around $x = 0.005$. According to sect. 5.2 this reduces the DOS at the Fermi level. As seen in fig. 59, this results in a lower $T_c$ and destabilizes the superconducting order parameter against magnetic order, finally for $x > 0.0075$ Co reentrance disappears and superconductivity exists only below the lock-in transition at $T_N$ which remains almost unchanged. The main effect of M substitution for Ni is therefore a $T_c$ tuning by variation of the DOS. Lanthanide substitution like $R' = Ho$ or Dy instead of $R = Lu$ or Y has a more subtle effect. For relatively small concentrations $x$ of the magnetic R ions $T_c$ is reduced as expected from the Abrikosov Gor’kov pair breaking theory by paramagnetic impurities in a nonmagnetic host superconductor, in this case LuNi$_2$B$_2$C. The pair breaking, like $T_m$ in the magnetic borocarbides, is then controlled by the de Gennes factor $dG$. This leads to a reduction of $T_c$ linear in $x \cdot dG$. For large concentrations ($x > 0.5$) of magnetic ions however this concept breaks down dramatically. In fact for $x = 1$ DyNi$_2$B$_2$C is superconducting and one observes a kind of ‘inverse’ de Gennes scaling of the transition temperature with $(1-x)dG$ corresponding to the concentration $(1-x)$ of nonmagnetic Lu impurities in the antiferromagnetic host superconductor DyNi$_2$B$_2$C which also act like pair breakers.

5.4.1. Coexistence of helical SDW, antiferromagnetism and superconductivity in HoNi$_2$B$_2$C

In this subsection we discuss a microscopic model (Amici, 1999; Amici et al., 2000) which successfully describes the most interesting coexistence behaviour found in HoNi$_2$B$_2$C. A more phenomenological approach based on GL theory has also been presented for Ho$_{1-x}$Dy$_x$Ni$_2$B$_2$C in Doh et al. (1999). Firstly these compounds should more appropriately be called superconducting magnets because the magnetic and superconducting energy scales given by $E_m = kT_m$ and $E_{SC} = k^2T_c^2/E_F$ differ strongly. If $T_m \simeq T_c$ as in the present case then $E_{SC}/E_m = kT_c/E_F \simeq 10^{-2}$. This is due to the fact that only a fraction $kT_c/E_F$ of con-

Fig. 59. Upper critical field curves for HoNi$_2$B$_2$C ($x = 0$) and Co-substituted compounds. For $x = 0$ one observes near reentrance around $T_N \simeq 5.3$ K. For $x = 0.005$ the reduced $T_c$ leads to real reentrance and for $x = 0.015$ finally $T_c$ falls below $T_N$ and ordinary $H_{c2}$ behaviour sets in Schmidt and Braun (1997).
duction electrons participates in the pair formation whereas the exchange energy of all localised spins is involved in the magnetic ordering. Thus \textit{local moment} magnetic order energetically completely dominates superconductivity whose influence on the former will therefore be neglected (Amici and Thalmeier, 2001). In fact in our previous discussion this was implicitly assumed by neglecting the effect of superconductivity on the RKKY interaction via the conduction electron susceptibility. This is justified because the latter would only be affected for $|q| \lesssim \xi_0^{-1}$ ($\xi_0 = SC$ coherence length), and we discuss the competition with AF order or IC phases with modulation vectors close to $Q_{AF}$.

The really important microscopic coupling is then caused by the appearance of the ordered 4f local moments. According to their magnetic structure they exert an additional periodic potential on the conduction electrons. For the AF or helix structure the local 4f moments are described by

$$\langle S_{R_i} \rangle = S(T) (\hat{a} \cos(Q \cdot R_i) + \hat{b} \sin(Q \cdot R_i)),$$

(111)

where $S(T)$ is the size of the ordered moments and $Q$ the helix wave vector with $Q = Q_{AF}$ in the AF case. The exchange interaction between the conduction and 4f-electrons then leads to an additional spin dependent periodic potential

$$H_{\text{ex}} = \frac{1}{2} I S \sum_k \left( c_{k+Q}^\dagger c_k^\dagger + c_{k-Q}^\dagger c_k^\dagger \right).$$

(112)

The exchange constant $I$ may be estimated from the RKKY expression $I^2 N(E_F) dG \simeq kT_m$ by using $N(E_F) = 4.8 \text{ eV}^{-1}$ resulting in $I \simeq 5 \text{ meV}$. The periodic exchange potential of eq. (112) has important effects on the conduction electron states, the original band energies $\epsilon_k$ and Bloch states $c_k$ are strongly modified close to the magnetic Bragg planes at $\pm Q$ and at $c^a \pm Q$. The modified magnetic bands and Bloch states are obtained by a unitary transformation (Herring, 1966) which mixes states $(k, \uparrow)$ and $(k + Q, \downarrow)$ where each pair is decoupled from the others. This leads to magnetic Bloch states given by

$$c_{k+} = u_k c_{k+}^\dagger + v_k c_{k+Q}^\dagger, \quad c_{k-} = u_{-k} c_{k-Q}^\dagger + v_{-k} c_{k+}^\dagger,$$

(113)

with the dispersion of the magnetic band given by

$$\tilde{\epsilon}_{k \pm} = \frac{1}{2} (\epsilon_k + \epsilon_{k\pm Q}) + \frac{1}{2} (\epsilon_k - \epsilon_{k\pm Q}) \left( 1 + \frac{I^2 S^2}{(\epsilon_k - \epsilon_{k\pm Q})^2} \right)^{1/2}.$$

(114)

The $(u_k, v_k)$ satisfy appropriate orthonormality conditions and fulfil the relation

$$u_k^2 - v_k^2 = \left( \frac{(\epsilon_k - \epsilon_{k\pm Q})^2}{(\epsilon_k - \epsilon_{k\pm Q})^2 + 4I^2 S^2} \right)^{1/2}.$$

(115)

Far away from magnetic Bragg planes $|\epsilon_k - \epsilon_{k\pm Q}| \gg IS$ this leads to $u_k \sim 1$ and $v_k \sim 0$ and $\tilde{\epsilon}_{k \pm} \sim \epsilon_{k \pm}$ so that the original band states and energies are recovered with $(k, \pm)$ reducing to $(k, \uparrow, \downarrow)$ Bloch states. An illustration of the reconstructed FS is shown in fig. 60. Although the effect of the exchange potential eq. (112) is confined to a small region of k-vectors with $\delta k \perp \leq IS/v_F$ counted from the magnetic Bragg planes, this modification proves important for
superconducting properties. From the previous consideration on superconducting and magnetic energy scales it is clear that the Cooper pairs have to be constructed from the magnetic Bloch states. This implies that the BCS pair Hamiltonian which is expressed in terms of nonmagnetic band states has to be transformed to the new magnetic basis states. For the AF structure this has been done in Zwicknagl and Fulde (1981) and for the more general helix phase in Morosov (1996, 1996a). Therefore, in addition to the modification eq. (114) of band energies, the nonmagnetic BCS pair potential \( V_{\mathbf{k}, \mathbf{k}'} \) will be replaced by an effective pair potential between magnetic Bloch states:

\[
\tilde{V}_{\mathbf{k}, \mathbf{k}'} = (u_{\mathbf{k}}^2 - v_{\mathbf{k}}^2) V_{\mathbf{k}, \mathbf{k}'} (u_{\mathbf{k}'}^2 - v_{\mathbf{k}'}^2).
\]

According to eq. (115) \( \tilde{V}_{\mathbf{k}, \mathbf{k}'} \) vanishes if \( \mathbf{k} \) or \( \mathbf{k}' \) are located on a Bragg plane. Naturally the BCS gap equations then lead to a modified gap function. To keep the discussion of magnetic effects simple we neglect the g-wave part in eq. (102) and assume an originally isotropic s-wave gap \( \Delta \). This should not influence the qualitative coexistence behaviour. Then the above equation leads to an additional \( \mathbf{k} \)-dependence originating in the exchange potential eq. (112) which is confined to the immediate vicinity of the Bragg planes. It is given by

\[
\Delta(\mathbf{k}, T) = \left( u_{\mathbf{k}}^2 - v_{\mathbf{k}}^2 \right) \Delta(T),
\]

\[
\Delta(T) = \int_0^{\omega_D} \mathrm{d}\epsilon \left( V \int_{\text{MFS}} \frac{dS'}{(2\pi)^3} \frac{(u_{\mathbf{k}}^2 - v_{\mathbf{k}}^2)^2}{|\nabla_{\mathbf{k}} \tilde{\epsilon}_{\mathbf{k}'}|} \right) \frac{\Delta(T) F(T)}{\sqrt{\epsilon^2 + \Delta^2(T)}},
\]

where \( F_k(T) = [1 - 2f_k(T)] \) and the inner integration is taken over the reconstructed magnetic Fermi surface (MFS) corresponding to \( \tilde{\epsilon}_k \). Obviously the modified gap function \( \Delta(\mathbf{k}, T) \) vanishes on node lines \( \perp \mathbf{c} \) lying on the Bragg planes and accordingly the self-consistent gap equation for \( \Delta(T) \) will lead to a reduced magnitude of the gap due to the factor in the numerator of the integral in brackets. However, note that no sign change in the modified gap function
\[ \Delta(k, T) \text{ occurs when } k \text{ crosses a node line, it still belongs to a fully symmetric } A_{1g} \text{ representation of the underlying AF magnetic structure. Since } (u_k^2 - v_k^2) \approx 1 \text{ almost everywhere except in the immediate vicinity of Bragg planes } (\delta k / k_F \lesssim ISN(E_F)) \text{ the reduction of the gap amplitude is not large and helix order may coexist with superconductivity. For the special AF case this is known already from Baltensperger and Strässler (1963) and the helix case with general modulation vector } Q \text{ is not very different because it only has two more Bragg planes than the AF case. The second equation has the formal appearance of the BCS gap equation, however, the expression in parentheses has to be interpreted as an effective electron–phonon interaction } V_e(T) \text{ or dimensionless } \lambda_e(T) = N(E_F) V_e(T) \text{ which itself depends on the temperature via } S(T). \text{ Its reduction from the nonmagnetic background value } \lambda = N(E_F) V \text{ is due to the reconstruction of electronic states confined close to the Bragg planes. Therefore it may be treated as a perturbation linear in the small parameter } IS(T)/E_F. \text{ One obtains}

\[ \delta \lambda(T) = \lambda - \lambda_e(T) = -\frac{V}{8\pi\hbar^2 v_r v_z} IS(T) \]  \hspace{1cm} (118)

for a FS piece without nesting features which is cut by a pair of Bragg planes with a FS radius \( k_r \) at the intersection and radial and parallel velocities \( v_r \) and \( v_z \), respectively. For a helical structure with two pairs of Bragg planes at \( \pm \frac{1}{2} Q \) and \( \pm \frac{1}{2} (c^* - Q) \) one has two add two corrections as in eq. (118) with different FS parameters. The main effect of the interaction of magnetic order and superconductivity described by eq. (112) has now been condensed into eq. (118). The physical consequences of the reduction of \( \lambda \) due to magnetic order are a reduced superconducting \( T_c \) and condensation energy which results in an anomaly in \( H_{c2}(T) \) that is directly correlated with the appearance of the local moment order parameter \( S(T) \). The upper critical field anomaly may approximately be calculated by using an appropriately modified BCS expression and including the magnetization response of the local moments:

\[ H_{c2}^{[110]}(T) = B_{c2}[\lambda_e(T), T] - M^{[110]}(T), \]
\[ H_{c2}^{[100]}(T) = B_{c2}[\lambda_e(T), T], \]  \hspace{1cm} (119)

where \( B_{c2}(\lambda_e(T), T) = B_{c2}^0(\lambda_e(T)) \cdot [1 - (T/T_c(\lambda_e(T)))^2] \) is the BCS expression modified due to the \( T \)-dependence of the effective e-p coupling. The magnetization for the [001] direction can be neglected because the 4f moments of CEF states are well constrained within the easy ab-plane. Using eqs. (118), (119) with an appropriate set of parameters for HoNi2B2C the calculated upper critical field is shown in fig. 61. It clearly shows a pronounced depression of \( H_{c2}(T) \) when magnetic order sets in. As discussed before there is no big difference in the effect of helical order (below \( T_C \)) and commensurate AF order (below \( T_N \)), the overall depression is simply controlled by the increase of \( S(T) \) when \( T \) decreases. At this point we mention again that in our discussion of coexistence behaviour the \( a^* \)-incommensurate magnetic modulation has been neglected. This is fully justified according to hydrostatic pressure experiments by Dertinger (2001) where it has been shown that: (i) while the \( a^* \) satellite intensity in ND decreases with pressure and vanishes at \( p > 0.7 \text{ GPa} \), (ii) the reentrance minimum in \( H_{c2} \) becomes even more pronounced. This proves that the \( a^* \)-modulation of moments plays no important role in the reentranent behaviour of SC, contrary to what has been claimed by many
previous investigations. The [110], [001] anisotropy of $H_{c2}$ is due to the different magnetization response in both directions. For field lying in the easy plane $H_{c2}(T)$ exhibits a plateau which is due to the appearance of the ferrimagnetic AF3 phase above the first critical field in fig. 57. This phase has a net magnetic moment that cannot coexist with superconductivity. If the nonmagnetic background e-p coupling $\lambda$ is reduced by about 10% the resulting reduction in $T_c$ and condensation energy leads to a reentrance behaviour of $H_{c2}(T)$ as shown by the dashed curve. These features described by the theory of Amici et al. (2000), namely depression, anisotropy (plateau) and reentrance behaviour have all been observed experimentally in HoNi$_2$B$_2$C, the latter under Co-substitution of Ni which reduces $T_c$ as shown in fig. 59.

5.4.2. Coexistence of superconductivity and weak ferromagnetism in ErNi$_2$B$_2$C
At the SDW transition of ErNi$_2$B$_2$C ($T_{IC} = 6$ K) a similar but much less pronounced dip in $H_{c2}$ as in HoNi$_2$B$_2$C is observed indicating the coexistence of both types of order. Even more exciting is the observation of WFM order below $T_{WFM} = 2.3$ K which coexists with SC ($T_c = 11$ K) to the lowest temperatures. This first confirmed example of microscopic FM/SC coexistence has therefore attracted a lot of attention. Established theories (Ginzburg, 1957) predict that thermodynamic coexistence in type II superconductors is only possible when the internal FM field $H_{int}$ is smaller than $H_{c2}$ (Fischer, 1990). In ErNi$_2$B$_2$C due to the large FM layer spacing (fig. 58) one estimates from $\mu$ a small $H_{int} \sim 0.5$ kG which is of the same order as $H_{c1}$. Despite its smallness $H_{int}$ does have a very peculiar effect: For applied fields $H$ with orientation $\theta_a$ close to the $c$-axis the perpendicular internal field along the $a$ direction rotates the effective field of the vortex phase to an angle $\theta_v$ towards the $a$-axis. Thus the vortex lattice will show a misalignment with the applied field by a small angle $\theta_v - \theta_a$. This effect has been found experimentally by Yaron et al. (1996). Theoretically this effect was considered in the context of a Ginzburg–Landau theory including the terms corresponding WFM order parameter and the coupling to SC (Ng and Varma, 1997). Since the London penetration depth $\lambda$ is 26 times larger than the FM layer spacing in fig. 58 (Kawano-Furukawa et al., 2001) it can still be...
justified to use an averaged FM moment density of saturation value $0.34\mu_B$. The tilt angle $\theta_v - \theta_a$ of vortices becomes larger at lower temperature in agreement with theoretical predictions. If the internal field $H_{\text{int}}$ is at least somewhat larger than $H_{c1}$ another exotic transition to a spontaneous vortex phase in zero applied field may take place. In this phase the internal field spontaneously forms a vortex lattice oriented along the a direction. Low field magnetization measurements and small angle neutron scattering experiments (Kawano-Furukawa et al., 2001) seem to support such a possibility but the issue is not settled.

6. Rare earth skutterudite superconductors

The new heavy fermion superconductor PrOs$_4$Sb$_{12}$ which has been discovered recently (Bauer et al., 2002) is potentially of similar interest as UPt$_3$ because it represents the second example of multiphase superconductivity (Izawa et al., 2003) with a critical temperature $T_c = 1.85$ K. This material belongs to the large class of RT$_4$X$_{12}$ skutterudite cage compounds ($R =$ alkaline earth, rare earth or actinide; $T =$ Fe, Ru or Os and $X =$ P, As or Sb) which have a bcc-filled skutterudite structure with tetrahedral space group $T_4^h$ (fig. 62). In this structure large voids formed by tilted $T_4X_{12}$ octahedrons can be filled with $R$ atoms. They are however rather loosely bound and therefore may have large anharmonic oscillations (‘rattling’) in the cage. In addition if equivalent equilibrium positions are present tunneling split states may exist. Both effects may lead to interesting low temperature elastic and transport phenomena, i.e., thermoelectric effects (Sales et al., 1996; Sales, 2003). Depending on the cage-filling atom this large class of compounds displays also a great variety of interesting effects of strong electron correlation. Mixed valent and heavy fermion behaviour, magnetic and quadrupolar order, non-Fermi liquid and Kondo insulating behaviour has been found, see Bauer et al. (2002), Sales (2003) and references cited therein. Recently the SC in non-stoichiometric skutterudites Pr(Os$_{1-x}$Ru$_x$)$_4$Sb$_{12}$ has been investigated.

![Fig. 62. Cubic crystal structure of filled skutterudite RT$_4$X$_{12}$. $R$ atoms: large circles, $X$ atoms: middle size circles, $T$ atoms: small circles located in the center of $TX_8$ octahedra (grey). For PrOs$_4$Sb$_{12}$ the lattice constant is $a = 9.3017$ Å.](image-url)
throughout the whole concentration range of $0 \leq x \leq 1$ (Frederick et al., 2003). While for $x = 0$ one has an unconventional HF superconductor the $x = 1$ compound PrRu$_4$Os$_{12}$ on the other hand is a conventional SC with $T_c \approx 1$ K. The type of SC changes at $x \approx 0.6$ where the transition temperature $T_c(x)$ has a minimum value of 0.75 K. Here we will focus, however, exclusively on the HF multiphase superconductivity in PrOs$_4$Sb$_{12}$ ($x = 0$).

6.1. Electronic structure and HF behaviour of PrOs$_4$Sb$_{12}$

The LDA + U band structure and Fermi surface of PrOs$_4$Sb$_{12}$ has been investigated by Sugawara et al. (2002) and compared with dHvA results. Three FS sheets, two of them closed and approximately spherical were identified both experimentally and in the calculation. Their dHvA masses are $m^* = 2.4$–7.6m, this is considerably higher than the calculated LDA + U band masses ranging from $m_b = 0.65$–2.5m. The observed dHvA masses however are still much too small compared to the thermal effective mass $m^* \approx 50m$. This places PrOs$_4$Sb$_{12}$ among the heavy fermion metals, the first one ever observed for a Pr-compound with its 4f$^2$ electronic configuration, the estimated quasiparticle bandwidth is $T^* \approx 10$ K (Bauer et al., 2002) which is in the same range as the lowest CEF splittings. Therefore, the low temperature behaviour of $C(T)$, $\chi(T)$ and $\rho(T)$ is determined by both CEF effects and heavy quasiparticle formation and it is difficult to separate them (Maple et al., 2002). The latter is rather anomalous for a HF system since the $A$-coefficient in $\rho(T) = \rho_0 + AT^2$ is smaller by two orders of magnitude compared to other HF systems. The susceptibility is largely dominated by localised 4f$^2$ CEF states and from the high temperature behaviour an effective Pr$^{3+}$ moment $\mu = 2.97\mu_B$ is obtained.

6.2. Pr CEF states and antiferroquadrupolar order

The CEF level scheme of Pr$^{3+}$ is determined by a CEF potential with tetrahedral $T_h$ symmetry. For a long time it was thought that it is equivalent to that of other cubic point groups like $T_d$ or $O_h$. However, it was shown recently (Takegahara et al., 2001) that this is incorrect. Due to the absence of two types of symmetry operations the CEF potential is rather given by

$$H_{CEF} = W \left[ x \left( \frac{O_4}{F(4)} \right) + (1 - |x|) \left( \frac{O_6^c}{F(6)} \right) + y \left( \frac{O_6^l}{F^l(6)} \right) \right],$$

where $O_4 = O^0_4 + 5O^4_4$, $O_6^c = O^0_6 - 21O^4_6$ and $O_6^l = O^2_6 - O^6_6$ are Steven’s operators and for Pr$^{3+}$ $F(4) = 60$, $F(6) = 1260$ and $F^l(6) = 30$. For the cubic groups $O_h$ and $T_d$ we have $y = 0$ and there is only one CEF parameter $x$, aside from the overall scale $W$, then $H_{CEF}$ reduces to the well known form. However in tetrahedral symmetry $T_h$ in general a second CEF parameter $y \neq 0$ appears. The consequences were analysed in detail in Takegahara et al. (2001), specifically the $\Gamma_1$, $\Gamma_2$ states of the cubic CEF case will be mixed into two inequivalent $\Gamma_1$ singlets of $T_h$ and likewise $\Gamma_4$, $\Gamma_5$ of $O_h$ will be mixed to become two inequivalent $\Gamma_4$ triplets of $T_h$ where the degree of mixing depends on $y$. The experimental determination of the Pr$^{3+}$ CEF level scheme has been performed under
the restriction \( y = 0 \) (i.e., assuming the cubic CEF potential), therefore, we keep the original notation of states. With this restriction the level scheme has been determined by fitting \( \chi(T) \) and also directly by inelastic neutron scattering (Maple et al., 2002). The latter leads to a level scheme \( \Gamma_3(0 \text{ K}), \Gamma_5(8.2 \text{ K}), \Gamma_4(133.5 \text{ K}), \Gamma_1(320 \text{ K}) \), which is close to the one obtained from \( \chi(T) \). The ground state is a nonmagnetic (non-Kramers) doublet with a quadrupole moment and quadrupole matrix elements connecting to the first excited triplet state. An alternative CEF level scheme has been proposed (Aoki et al., 2002; Tayama et al., 2003) from specific heat and magnetization experiments. It has a singlet ground state without magnetic nor quadrupole moment, namely \( \Gamma_1(0 \text{ K}), \Gamma_5(6 \text{ K}), \Gamma_4(65 \text{ K}) \) and \( \Gamma_3(111 \text{ K}) \). The splitting energy \( \delta \approx 6–8.2 \text{ K} \) of ground state to first excited state is also seen as a pronounced zero field Schottky anomaly sitting on top of the superconducting transition. Application of a field above the SC \( H_{c2}(0) = 2.2 \text{ T} \) suppresses the Schottky anomaly.

However, at larger fields of \( H \approx 4 \text{ T} \) a new and sharper one appears at lower temperatures possibly signifying the stabilization of a long range antiferroquadrupolar ordered phase (Aoki et al., 2001, 2002; Vollmer et al., 2003; Maple et al., 2003). This is summarized in the phase diagram shown in the left part of fig. 63. A detailed analysis of AFQ long range order in the high field region has been given by Shiina (2004). It is helpful to compare with other known AFQ ordered 4f systems, where Ce\(_{1-x}\)La\(_x\)B\(_6\) is certainly the most well studied one (Shiina

![Fig. 63. Left panel: Complete B–T phase diagram (Maple et al., 2002) with SC regime and region of field induced order parameter which is presumably of antiferroquadrupolar (AFQ) type. Data are obtained from resistivity (\( \rho \)), magnetization (\( M \)), specific heat (\( C \)) and thermal expansion (\( \alpha \)) measurements. Right panel: High magnetic field phase diagram (Aoki et al., 2003) with upper part of AFQ phase (full squares) and line of high field Schottky anomaly from the \( \Gamma_1–\Gamma_3 \) crossing. The inset shows calculation for tetrahedral CEF model.](image-url)
et al., 1997). It has a $\Gamma_8$ quartet as CEF ground state which carries both quadrupole and magnetic moment. Both spontaneous AFQ and AF order are therefore observed with $T_Q > T_N$ and we consider only the former. For $x = 0.5$ the AFQ phase diagram (Nakamura et al., 1997) is quite similar to that of fig. 63, namely the AFQ phase is only induced in finite field and $T_Q(H)$ increases with field. In the Ce$_1-x$La$_x$B$_6$ system the AFQ order has been identified by observing the induced secondary AF order which is induced by a homogeneous field $H$ in the ordered AFQ background. This can be done by neutron diffraction and NMR experiments. Recent neutron diffraction results (Kohgi et al., 2003) suggest that the CEF ground state is singlet $\Gamma_1$ and the AFQ order parameter is of $O_{yz} = J_y J_z + J_z J_y$ type. Additional high field measurements on $C(T, H)$ for different field directions (Rotundu et al., 2003) also give strong evidence for the $\Gamma_1$ ground state scenario and for a $\Gamma_1-\Gamma_5$ level crossing as origin of the field induced AFQ phase (fig. 63, right panel).

The existence of a nonmagnetic ($\Gamma_1$ or $\Gamma_3$) ground state in PrOs$_4$Sb$_{12}$ leads one to speculate about the origin of the observed HF behaviour since the usual Kondo lattice mechanism as in Ce-intermetallic compounds which demands a Kramers degenerate magnetic ground state cannot be at work here. In the case that $\Gamma_3$ were realised one might conjecture that instead of the exchange scattering one has aspherical Coulomb scattering from the quadrupolar degrees of freedom of the nonmagnetic $\Gamma_3$ ground state. As discussed for the isoelectronic U$^{4+}$-configuration in UBe$_{13}$ in sect. 4.5 this is described by a multichannel Kondo Hamiltonian for the quadrupolar pseudo-spin. It can lead to a partial screening of the $\Gamma_3$-quadrupole below a quadrupolar Kondo temperature $T^*$. At even lower temperatures logarithmic non-Fermi liquid anomalies in thermodynamic quantities should develop due to the partial screening. This cannot be confirmed due to the intervening SC transition. Therefore this quadrupolar HF mechanism for PrOs$_4$Sb$_{12}$ is only a conjecture. If a singlet $\Gamma_1$ ground state is realised as suggested by neutron diffraction and high field $C(T, H)$ results mentioned before one has to invoke another mechanism for heavy mass generation which is similar to the one implied in U-compounds like UPd$_2$Al$_3$ (Zwicknagl et al., 2003). It is due to mass renormalisation by the low lying $\Gamma_1-\Gamma_3$ excitation, i.e., contrary to the previous scenario it is caused by off-diagonal parts of the conduction electron-CEF level interaction.

6.3. The superconducting split transition

The SC specific heat jump is superposed on the Schottky anomaly from the lowest CEF excitation at $\delta$. Nevertheless its detailed analysis provides clear evidence for a split SC transition (Bauer et al., 2002; Vollmer et al., 2003) at $T_{c1} = 1.85$ K and $T_{c2} = 1.75$ K as shown in fig. 64. The total jump of both transitions amounts to $\Delta SC C/\gamma T_c \simeq 3$ which exceeds the BCS value 1.43 for a single transition considerably. It also proves that the SC state is formed from the heavy quasiparticles that cause the enhanced $\gamma$-value. A $T_c$-splitting of similar size also was clearly seen in thermal expansion measurements (Oeschler et al., 2003, 2004). The two superconducting transitions in fig. 64 are reminiscent of the split transition in UPt$_3$ (sect. 4.1, fig. 27). There a two-fold orbitally degenerate SC state is split by weak AF order that reduces the hexagonal symmetry to orthorhombic. This also leads to two critical field curves in the $B-T$ phase diagram. In PrOs$_4$Sb$_{12}$ no such symmetry breaking field exists and the split
transition has to belong to two different $T_h$ representations of the SC order parameter or combinations thereof as discussed below. The critical field curves associated with $T_{c1,2}$ have been investigated with magnetization (Tayama et al., 2003) and specific heat measurements (Measson et al., 2004) from which practically parallel curves are obtained. Only the upper critical field corresponding to $T_{c1}$ with $H_{c2}(0) \simeq 2.2$ is shown in fig. 63.

As usual in the early stage of investigation different experiments gave inconclusive results on the question of the nature of gap anisotropy. The low temperature specific heat exhibits a $C_s(T) \sim T^n$ power law in a rather reduced range between 0.65 K and 1.2 K which points to some kind of nodal state. In Sb-NQR experiments (Kotegawa et al., 2003) the nuclear spin lattice relaxation $1/T_1$ rate was determined. It has an itinerant quasiparticle contribution that contains information on the SC nodal state below $T_c$ and in addition a localised contribution from broadened CEF excitations which decreases exponentially for temperatures $T \ll \Delta$. There is no unique way to separate these contributions, this problem is similar to the two Knight shift contributions in the case of UPd$_2$Al$_3$ (sect. 4.2) with its isoelectronic $5f^2$ localised states. The NQR measurements did not show any evidence for a coherence peak below $T_c$ which points to an unconventional SC state, for lower temperatures an exponential decay of $T_1^{-1}$, in conflict with the existence of gap nodes was reported. However, this result depends critically on the subtraction procedure of the localised contribution.

6.4. **Thermal conductivity in the vortex phase and multiphase superconductivity in PrOs$_4$Sb$_{12}$**

The experiments on field-angle dependent thermal conductivity described in sect. 2 are a more powerful method to investigate the SC state. This method achieved the determination of critical field curves and the associated $B$–$T$ phase diagram in PrOs$_4$Sb$_{12}$ (Izawa et al., 2003) and at the same time the nodal structure of the gap function has at least been partly clarified. The same geometry as for borocarbides (sect. 5) with conical field rotation around the heat current
Fig. 65. (a) Field dependence of the four-fold ($C_{4\phi}$) and two-fold ($C_{2\phi}$) amplitudes of $\kappa_{zz}(\phi)$ oscillations in (b) as function of field strength at $T = 0.52$ K. Sharp transition at $H^* = 0.4H_{c2}$ from two-fold to four-fold oscillation is seen. (b) Four-fold oscillations from experiment for different polar field angle $\theta$ (Izawa et al., 2003). (c) Calculated angular variation of $\kappa_{zz}$ as function of azimuthal field angle $\phi$ exhibiting four-fold (A) and two-fold (B) oscillations ($\theta =$ polar field angle). The inset shows polar plots of A- and B-phase gap functions with four- and two-point nodes, respectively (Maki et al., 2003c).

Direction parallel to a cubic [001] axis was used and measurements in the whole temperature range from 0.3 K up to $T_c$ were performed. The azimuthal angle dependence of $\kappa_{zz}$ was found to be approximately of the empirical form

$$\kappa_{zz}(\theta, \phi, H, T) = \kappa_0 + C_{2\phi}(\theta, H, T) \cos 2\phi + C_{4\phi}(\theta, H, T) \cos 4\phi$$

(121)

containing both two-fold and four-fold rotations in $\phi$. The $\phi$ dependence for $H = 1.2T$ is shown in fig. 65 for a few polar angles $\theta$. It exhibits clearly a dominating four-fold oscillation. Since the amplitude rapidly decreases with $\theta$ similar as in the borocarbides this may be interpreted as evidence for point nodes in $\Delta(k)$ along the [100] and [010] cubic axes. Surprisingly, when the field is lowered to $H^* = 0.8T \ll H_{c2}$ a rather sudden change from a four-fold to a two-fold oscillation in $\phi$ as shown in fig. 65 is observed which is interpreted as at transition to a different SC state with only two point nodes along [010]. The second critical field $H^*$ can be followed to higher temperatures, although with rapidly decreasing oscillation amplitudes $C_{2\phi}$ and $C_{4\phi}$. Apparently the second critical field curve $H^*(T)$ ends at the lower critical temperature $T_{c2}$ observed already in the second specific heat jump at zero field.

This allows one to construct a $B$–$T$ phase diagram for PrOs$_4$Sb$_{12}$ shown in fig. 66. Most importantly one has two distinct regions with A- and B-phase superconductivity characterised by four and two point nodes along cubic axis [100] and [010]. In the applied geometry the field is swept around the [001] axis, and therefore possible nodes along this direction cannot be detected. Additional experiments with a sweep axis orthogonal to [001] would be necessary. Furthermore, three different domains (orientation of the node directions) of A- and B-phase are possible. Apparently for the sample investigated a specific domain has been selected, possibly by internal strain effects. From the $B$–$T$ phase diagram in fig. 66 we conclude that PrOs$_4$Sb$_{12}$ is the second pure multiphase HF superconductor found after UPt$_3$. There is, how-
ever, no crossing of critical field lines and no corresponding tetracritical point in PrOs$_4$Sb$_{12}$ as was observed in fig. 29 for UPt$_3$.

6.5. Gap models for SC A- and B-phases of PrOs$_4$Sb$_{12}$

The observation of a different nodal structure in the A- and B-phases raises the question of the symmetry of the gap function in PrOs$_4$Sb$_{12}$. Various proposals have been made (Maki et al., 2003c; Goryo, 2003; Miyake et al., 2003; Ichioka et al., 2003) based on an empirical approach and compatible with the observed point nodes. The gap function may be expanded in terms of basis functions $\psi_{\Gamma}^{i}(k)$ which transform like representations $\Gamma$ of the crystal symmetry group ($i = 1$–$d$ is the degeneracy index, the index $l$ denoting the degree of $\Gamma$ is suppressed).

So far there is no information from NMR Knight shift or $H_{c2}$-Pauli limiting effects whether PrOs$_4$Sb$_{12}$ has spin singlet or triplet pairing. In the singlet case the gap function should then be given by

$$\Delta(k) = \sum_{\Gamma,i} \eta_{i}^{\Gamma} \psi_{i}^{\Gamma}(k) \equiv \Delta f(k),$$

where the form factor $f(k)$ is normalized to one and $\Delta$ is the temperature dependent maximum gap value. In the spirit of the Landau theory only a single representation with the highest $T_c$ should be realized and for $T \geq T_c$ the free energy may then be expanded in terms of possible invariants of the order parameter components (Volovik and Gorkov, 1985) $\eta_{i}^{\Gamma}$ which are determined by Landau parameters $\alpha^{\Gamma}(T)$ and $\beta^{\Gamma}_{l}$. The node structure is then fixed by the specific symmetry class of $\Delta(k)$ defined by the set of $\eta_{i}^{\Gamma}$. However the pure second ($l = 2$) and fourth ($l = 4$) degree representations in tetragonal $T_h$ symmetry which are given in table 7 cannot realize a gap function with the observed nodal structure, and therefore, one has to consider the possibility of hybrid order parameter models. A striking example of such a hybrid gap function ($s + g$ wave model) has already been discussed in the previous section for the nonmagnetic borocarbide superconductor Y(Lu)Ni$_2$B$_2$C. The symmetry classification of
Table 7
Even parity basis functions for SC gap in tetrahedral (T₈) symmetry

<table>
<thead>
<tr>
<th>d</th>
<th>Γ</th>
<th>( \psi_i^{1}(\mathbf{k}) )</th>
<th>( \psi_i^{1/2}(\mathbf{k}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( A(\Gamma_1) )</td>
<td>( 1 )</td>
<td>( k_x^4 + k_y^4 + k_z^4 )</td>
</tr>
<tr>
<td>2</td>
<td>( E(\Gamma_2, \Gamma_3) )</td>
<td>( (2k_x^2 - k_y^2 - k_z^2, k_x^2 - k_y^2) )</td>
<td>( (2k_x^2 - k_y^2 - k_z^2, k_x^2 - k_y^2) )</td>
</tr>
<tr>
<td>3</td>
<td>( T(\Gamma_3) )</td>
<td>( (k_xk_y, k_xk_z, k_yk_z) )</td>
<td>( (k_xk_y, k_xk_z, k_yk_z) )</td>
</tr>
</tbody>
</table>

Symbols used: \( \psi_i^{1/2}(\mathbf{k}) \) and \( \psi_i^{1}(\mathbf{k}) \) \( (i = 1–d) \) are second \( (l = 2) \) and fourth degree \( (l = 4) \) basis functions, respectively, \( d \) denotes the degeneracy and \( \Gamma_2 \) and \( \Gamma_3 \) are complex conjugate combinations of \( E_1 \) and \( E_2 \) components.

Table 7 suggests the following simple proposal for a hybrid gap function \( \Delta(\mathbf{k}) = \Delta f(\mathbf{k}) \) for PrOs₄Sb₁₂ which has the observed nodes along the cubic axis, excluding the still hypothetical ones along [001]:

- **A-phase** \( \Delta(\mathbf{k}) = \Delta (1 - k_x^4 - k_y^4), \)
- **B-phase** \( \Delta(\mathbf{k}) = \Delta (1 - k_z^4). \)

According to table 7 A- and B-phase are superpositions of A and E representations with planar and axial symmetry, respectively. They are three-fold degenerate and particular domains with nodes in the \( k_xk_y \)-plane (A) or along \( k_z \) (B) have been chosen. This selection might be realized due to internal strains in the SC sample. Polar plots of A, B-phase gap functions are shown in the inset of fig. 65(c). The magnetothermal conductivity in the vortex state of phases A, B in the limit \( \Gamma < T \ll \Delta \) can be calculated as described in sect. 2 and the result for \( \kappa_{zz} \) is, up to logarithmic corrections

\[
\frac{\kappa_{zz}}{\kappa_n} \approx \frac{2}{\pi} \frac{v\sqrt{eH}}{\Delta} I_{A,B}(\theta, \phi)
\]

with the angular dependent functions \( I_i(\theta, \phi) \) \( (i = A, B) \) given by

\[
I_A(\theta, \phi) = \frac{1}{4} \left[ (1 - \sin^2 \theta \sin^2 \phi)^{1/2} + (1 - \sin^2 \theta \cos^2 \phi)^{1/2} \right],
\]
\[
I_B(\theta, \phi) = \frac{1}{4} \left( 1 - \sin^2 \theta \sin^2 \phi \right)^{1/2}.
\]

For in plane field \( \theta = \pi/2 \) one has \( I_A(\theta, \phi) = \frac{1}{4} (| \sin \phi | + | \cos \phi |) \) and \( I_B(\theta, \phi) = \frac{1}{4} | \cos \phi | \), their leading Fourier components vary like \( \cos 4\phi \) and \( \cos 2\phi \), respectively, in accordance with the empirical expressions in eq. (121). However, the above equations really predicts cusps at \( \phi = n(\pi/2) \) (fig. 65(c)) analogous to the situation in the borocarbides and not the smooth minima as seen in fig. 65(b). Therefore, the existence of cusps cannot be safely inferred from the experimental results in fig. 65, this would necessitate measurements at lower temperatures. Also experiments with variation of the polar angle \( \theta \) have to be performed to check the possible existence of node points along [001]. Finally \( \mu \)SR experiments show indications of the presence of magnetic moments in the SC phase which is interpreted, similarly as in \( U_{1-x}Th_xBe_{13} \) as evidence for a nonunitary triplet SC state (Aoki et al., 2003). This has also been claimed from a recent penetration depth study (Chia et al., 2003).
As a preliminary conclusion it seems clear that PrOs$_4$Sb$_{12}$ is a very unconventional multiphase HF superconductor of potentially the same interest as UPt$_3$. Recalling that heavy quasiparticles are presumably caused by coupling with virtual quadrupolar excitations from the nonmagnetic 5f ground state one is lead to speculate that SC in PrOs$_4$Sb$_{12}$ might also imply the presence of an unprecedented pairing mechanism based on the exchange of quadrupolar fluctuations. In addition to the spin-fluctuation and magnetic-exciton exchange mechanism this would be the third possibility for Cooper pair formation at work in heavy fermion compounds. However, at the moment the quadrupolar SC mechanism in PrOs$_4$Sb$_{12}$ is still a conjecture.

7. Summary and outlook

During the past decade a number of exciting and often unanticipated broken symmetry states and associated physical effects have been discovered in lanthanide and actinide intermetallic compounds. Prominent among them are superconductivity characterized by highly anisotropic unconventional order parameters and superconductivity coexisting with ferromagnetic order as well as the hidden order of unconventional density waves. A further example is the inhomogeneous superconducting phase appearing in an applied field which has been predicted theoretically a long time ago. A general hallmark of these systems is the coexistence and the competition of various different cooperative phenomena such as superconductivity and itinerant spin density wave magnetism.

The wealth of experimental data is only partly understood. To describe the low-temperature ordered phases, the determination of the type and symmetry of order parameters is of central importance. The latter restrict the possible excitations in the ordered phases and hence determine the low-temperature properties. Order parameters given in terms of expectation values of physical observables like spin- and charge-densities can be directly measured, e.g., by X-ray and neutron diffraction. The magnetic phases in lanthanide and actinide compounds are therefore rather well characterized. This, however, is not the case for hidden order like quadrupolar ordering or unconventional density waves.

Superconductivity corresponds to an off-diagonal long range order parameter which is not directly observable and it is not surprising that the type of pairing is still a matter of controversy in many heavy fermion compounds. Considerable progress has, however, been made recently in detecting unconventional order parameter symmetries. Angle-resolved studies of thermodynamic and transport properties in the vortex phase determine the position of the order parameter nodes relative to the crystal axes. These results help to strongly reduce the number of symmetry-allowed superconducting candidate states.

Important information about the material properties relevant for superconductivity is gained from the temperature and frequency dependence of thermodynamic and transport properties. Experiments indicate severe deviations from the universal behavior predicted by weak-coupling BCS theory. The current theoretical analysis of experimental data which relies on standard weak-coupling theory can yield only qualitative results. A quantitative treatment must account for strong-coupling effects. This, however, requires a microscopic picture of the normal state, i.e., the quasiparticles and their interactions.
A microscopic picture for the strongly renormalized quasiparticles has finally emerged for the actinide compounds. The hypothesis of the dual character of the 5f-electrons is translated into a calculational scheme which reproduces both the Fermi surfaces and the effective masses determined by dHvA experiments without adjustable parameter. The method yields also a model for the residual interaction leading to the various instabilities of the normal phase. The next step will be to develop an appropriate Eliashberg-type theory. The dual model approach should also provide insight into the mysterious hidden order phases of U-compounds.

In Ce-based compounds, the Cooper pairs are formed by heavy quasiparticles with predominantly 4f-character which arise through the Kondo effect in the periodic lattice. This picture has been confirmed in detail by dHvA and high-resolution photoemission studies. The strongly renormalised quasiparticles can be successfully reproduced by a semiphenomenological ansatz. Despite the efforts to implement modern many-body methods for strong correlations into realistic electronic structure calculations there is still no general concept for quantitative microscopic calculations. In particular, the subtle interplay between local and intersite effects continues to challenge theorists. The latter may lead to long-range order of local moments while the former favor the formation of a Fermi liquid state at low temperatures.

Many heavy fermion materials are on the verge of magnetic instability. By application of pressure and magnetic field, these materials can be tuned through a quantum critical point from a metallic antiferromagnet into a paramagnet. This may also trigger a transition to an unconventional SC state. Much effort has been devoted to the study of the behavior in the vicinity of the quantum critical point. Experimental data exhibit universality with unusual critical exponents. The theoretical picture, however, is highly controversial at present.

Superconductivity, magnetism and hidden order in lanthanide and actinide compounds pose an ongoing challenge. These compounds serve as model systems to study strong correlations in a broader context.

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References


Weickert, F. et al., 2004, unpublished.


Chapter 220

CIRCULARLY POLARIZED LUMINESCE
SPECTROSCOPY FROM LANTHANIDE SYSTEMS

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5. Magnetic Circularly Polarized Luminescence

List of symbols

<table>
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<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>A, B, and C</td>
<td>Faraday terms in MCPL</td>
</tr>
<tr>
<td>$b_0$</td>
<td>external magnetic field strength</td>
</tr>
<tr>
<td>$B/\tau$</td>
<td>measure of the reduction of $g^L_{\text{lum}}(\lambda)$ due to energy transfer</td>
</tr>
<tr>
<td>CD</td>
<td>acronym for Circular Dichroism</td>
</tr>
<tr>
<td>CPL</td>
<td>acronym for Circularly Polarized Luminescence</td>
</tr>
<tr>
<td>$D^g$</td>
<td>dipole strength</td>
</tr>
<tr>
<td>$e$</td>
<td>intermediate state</td>
</tr>
<tr>
<td>$e^{A(t)}$</td>
<td>probability that an initial excitation of an $A$ (or $\Delta$) complex remains on an $A$ (or $\Delta$) molecule at time $t$</td>
</tr>
<tr>
<td>$f_{\sigma}(\lambda)$</td>
<td>normalized lineshape function</td>
</tr>
<tr>
<td>$f_{\text{MCPL}}$ and $f'_{\text{MCPL}}$</td>
<td>MCPL lineshape and its first derivative</td>
</tr>
<tr>
<td>$g$</td>
<td>ground state</td>
</tr>
<tr>
<td>$g^\text{abs}$</td>
<td>absorption dissymmetry ratio</td>
</tr>
<tr>
<td>$g^\text{lum}$</td>
<td>luminescence dissymmetry ratio (or factor)</td>
</tr>
<tr>
<td>$I$</td>
<td>total emission intensity</td>
</tr>
<tr>
<td>$I_L$</td>
<td>left circularly polarized light intensity</td>
</tr>
<tr>
<td>$I_R$</td>
<td>right circularly polarized light intensity</td>
</tr>
<tr>
<td>$J$</td>
<td>total angular momentum quantum number</td>
</tr>
<tr>
<td>$k_0$</td>
<td>excited state decay rate</td>
</tr>
<tr>
<td>$k_{\text{rac}}$</td>
<td>racemization rate</td>
</tr>
<tr>
<td>$k^Q$ and $k^{AQ}$</td>
<td>diastereomeric bimolecular quenching rate constants</td>
</tr>
<tr>
<td>$L$</td>
<td>left circular polarization</td>
</tr>
<tr>
<td>$m^g$</td>
<td>imaginary magnetic dipole transition moment</td>
</tr>
<tr>
<td>$M^g$</td>
<td>magnetic-dipole transition moment vector</td>
</tr>
<tr>
<td>$n$</td>
<td>set of degenerate Zeeman levels</td>
</tr>
<tr>
<td>$N$ and $\not{N}$</td>
<td>total number of lanthanide complexes present</td>
</tr>
<tr>
<td>$N_L(\Delta)$</td>
<td>number of $\Delta$ enantiomers that absorb left circularly polarized light</td>
</tr>
<tr>
<td>$N_R(\Delta)$</td>
<td>number of $\Delta$ enantiomers that would absorb right circularly polarized light</td>
</tr>
<tr>
<td>$N_L^{\not{A}}(t)$</td>
<td>total number of emitting molecules</td>
</tr>
<tr>
<td>$N_l(\Omega, t)$</td>
<td>number of molecules in the emitting state $</td>
</tr>
<tr>
<td>$N_{\eta}(\omega_0)$</td>
<td>electric dipole absorption</td>
</tr>
<tr>
<td>$P^g$</td>
<td>electric-dipole transition moment vector</td>
</tr>
<tr>
<td>$R$</td>
<td>right circular polarization</td>
</tr>
<tr>
<td>$R^g$</td>
<td>rotatory (or rotational) strength</td>
</tr>
<tr>
<td>$t$</td>
<td>time</td>
</tr>
<tr>
<td>$W^g$</td>
<td>transition probability</td>
</tr>
<tr>
<td>$\Delta I$</td>
<td>differential emission intensity</td>
</tr>
<tr>
<td>$\Delta J$</td>
<td>change in angular momentum quantum number</td>
</tr>
<tr>
<td>$\Delta N_L$</td>
<td>differential population $(A - \Delta)$ of excited enantiomers that would result from left circularly polarized excitation</td>
</tr>
<tr>
<td>$\Delta W^g$</td>
<td>differential transition probability</td>
</tr>
</tbody>
</table>
1. Introduction

The very first report of the detection of circular polarization in the emission of a chiral molecular system was by Samoilov in 1948 who was studying crystals of sodium uranyl acetate, Na[UO$_2$(CH$_3$COO)$_3$] (Samoilov, 1948). This species crystallizes in the P2$_1$3 space group allowing for two enantiomeric forms, and the circular polarization is so large that it was easily detected with simple static optics. In the 1960s Professor Oosterhof and coworkers at the University of Leiden in The Netherlands published the first measurement from solution of the intrinsic circular polarization in the luminescence from a chiral organic molecule, namely, trans-b-hydrindanone, and from a resolved chiral transition metal complex, [Cr(ethylenediamine)$_3$]$_2$$^-$ (Emeis and Oosterhof, 1967, 1971). The extent of circular polarization from these systems is not nearly as large as the Na[UO$_2$(CH$_3$COO)$_3$] crystal, but advances in optics and electronics allowed for the detection of much smaller polarization values. In 1974, Luk and Richardson at the University of Virginia were the first to report the measurement of CPL from chiral complexes in solution containing a lanthanide ion (Luk and Richardson, 1974). It was quickly recognized that the luminescence from intraconfigurational f ↔ f transitions of the lanthanide (III) ions that were magnetic-dipole allowed often showed very large circular polarization. Since the mid 1970s applications involving the measurement of circular polarization from molecular systems to chiral luminescent lanthanide complexes have dominated the use of this spectroscopic technique.

In this article we will refer to the measurement of the net circular polarization in the luminescence as Circularly Polarized Luminescence and use the acronym CPL. This is now the most widely used term to describe this spectroscopic technique, but in the older literature one finds circularly polarized emission (CPE), emission circular intensity differentials (ECID), or the somewhat more specific terms of circularly polarized fluorescence (CPF) or circularly
polarized phosphorescence (CPP). All of these terms refer to the spontaneous emission of circularly polarized light by chiral, or so-called “optically active” systems. A chiral molecule is one in which the mirror image isomers are not superimposable. From a molecular symmetry point of view, molecules belonging to point groups that do not contain mirror planes or an inversion center are chiral. All interactions involving light with chiral molecules discriminate between the two possible circular polarizations (left = L and right = R). In the absence of a perturbing static electric or magnetic field, the light emitted by a molecular chromophore will be partially circularly polarized only if the emitting species is chiral.

There have been several reviews published on the general use of CPL to study chiral molecular systems. Steinberg wrote the first review on CPL spectroscopy emphasizing applications to biochemical systems (Steinberg, 1975), and Richardson and Riehl published a review of CPL in 1977 (Richardson and Riehl, 1977) and an updated review in 1986 (Riehl and Richardson, 1986). Brittain published a review of the use of CPL to study chiral lanthanide complexes in 1989 (Brittain, 1989). Several other recent articles describing various general aspects of CPL measurements and CPL theoretical principles are also available (Riehl and Richardson, 1993; Riehl, 1993, 2000; Maupin and Riehl, 2000). In this article we will review the various applications of CPL to the study of lanthanide complexes, as well as provide an up-to-date assessment of the state of theory and instrumentation. We will emphasize both the qualitative and quantitative molecular information that has so far been obtained from this technique, and discuss the future of CPL as a reliable probe of the molecular stereochemistry of lanthanide complexes.

2. Theoretical aspects

2.1. General theory

In CPL spectroscopy one is interested in measuring the difference in the emission intensity (∆I) of left circularly polarized light (I_L) versus right circularly polarized light (I_R). By convention this difference is defined as follows

$$\Delta I \equiv I_L - I_R.$$  

(1)

Just as in ordinary luminescence measurements, the determination of absolute emission intensities is quite difficult, so it is customary to report CPL measurements in terms of the ratio of the difference in intensity, divided by the average total luminescence intensity

$$g_{\text{lum}} = \frac{\Delta I}{\frac{1}{2}I} = \frac{I_L - I_R}{\frac{1}{2}(I_L + I_R)},$$  

(2)

g_{\text{lum}} is referred to as the luminescence dissymmetry ratio (or factor). The extra factor of $\frac{1}{2}$ in eq. (2) is included to make the definition of $g_{\text{lum}}$ consistent with the previous definition of the related quantity in circular dichroism, CD, namely, $g_{\text{abs}}$

$$g_{\text{abs}} = \frac{\Delta \varepsilon}{\varepsilon} = \frac{\varepsilon_L - \varepsilon_R}{\frac{1}{2}(\varepsilon_L + \varepsilon_R)},$$  

(3)
CIRCULARLY POLARIZED LUMINESCENCE SPECTROSCOPY

Fig. 1. Schematic energy level diagram for luminescence.

where in this equation $\varepsilon_L$ and $\varepsilon_R$ denote, respectively, the molar absorption coefficients for left and right circularly polarized light, and $\varepsilon$ has always been explicitly defined as an average quantity.

We refer to the simple energy level diagram given in fig. 1. The time dependence of the intensity of emitted light of polarization $\sigma$, at a particular wavelength, $\lambda$, for a specific transition $n \rightarrow g$ of a molecule with orientation $\Omega$ (in the laboratory frame) may be expressed in terms of the transition probability, $W_{gn}$, as follows

$$ I_\sigma(\lambda, t) = \left(\frac{\bar{h}c}{\lambda}\right)N_n(\Omega, t)W_{gn}(\Omega)f_\sigma(\lambda), $$

where $N_n(\Omega, t)$ denotes the number of molecules in the emitting state $|n\rangle$ at time $t$, and $f_\sigma(\lambda)$ is a normalized lineshape function. The number of molecules in the emitting state with orientation $\Omega$ depends on the orientation at the time of excitation ($t = 0$) with respect to the polarization $\pi$ and direction of the excitation beam

$$ N_n(\Omega, t) = N_e^\pi(\Omega : \Omega_0, t)\eta_n, $$

$\eta_n$ denotes the fraction of molecules that end up in state $|n\rangle$ that were initially prepared in the intermediate state $|e\rangle$ by the excitation beam. This quantity is assumed to be independent of orientation. We consider here only the two limiting cases for the orientational distribution; the so-called “frozen” limit appropriate for crystals, glasses, rigid sol-gels, and solutions of very high viscosity, or isotropic appropriate for most solution measurements where we assume that the orientational distribution of emitting species is random. No intermediate cases have been reported, since the long radiative lifetimes of the luminescent lanthanide ions usually ensures that, in solution, the emitting species have had sufficient time to scramble any initial photoslected orientational distribution.

2.2. Circularly polarized luminescence from rigid samples: theory

In the frozen limit we set $\Omega = \Omega_0$. The differential intensity of left minus right circularly polarized light may now be expressed as follows

$$ \Delta I(\lambda, t) = \left(\frac{hc}{\lambda}\right)N_n(\Omega_0, t)\Delta W_{gn}(\Omega_0)f_\sigma(\lambda), $$

where we have introduced the differential transition probability $\Delta W_{gn}$

$$ \Delta W_{gn}(\Omega_0) \equiv W_{gn}^L(\Omega_0) - W_{gn}^R(\Omega_0). $$
The probability of emitting a right or left circularly polarized photon may be related in the usual way to molecular transition matrix elements through Fermi’s Golden Rule. Under the assumption that the emitted light is being detected in the laboratory direction, and allowing for electric dipoles and magnetic dipoles in the expansion of the molecule-radiation interaction Hamiltonian we obtain the following expressions

\[ W^L_{gn}(\Omega_0) = K(\lambda^3)[|\mu_{1}^{gn}|^2 + |\mu_{2}^{gn}|^2 + |m_{1}^{gn}|^2 + |m_{2}^{gn}|^2 - 2i(\mu_{1}^{gn}m_{1}^{gn} + \mu_{2}^{gn}m_{2}^{gn})], \]

\[ W^R_{gn}(\Omega_0) = K(\lambda^3)[|\mu_{1}^{gn}|^2 + |\mu_{2}^{gn}|^2 + |m_{1}^{gn}|^2 + |m_{2}^{gn}|^2 + 2i(\mu_{1}^{gn}m_{1}^{gn} + \mu_{2}^{gn}m_{2}^{gn})], \]

where \( I \) and \( 2 \) refer to laboratory axes and the electric dipole transition moment, \( \mu^{gn} \), and the imaginary magnetic dipole transition moment \( m^{gn} \) are defined as follows

\[ \mu_{1}^{gn} \equiv \langle g|\mu_{1}|n\rangle, \]
\[ m_{1}^{gn} \equiv \langle g|m_{1}|n\rangle. \]  

The differential transition rate is, therefore,

\[ \Delta W_{gn}(\Omega_0) = K(\lambda^3)[2i(\mu_{1}^{gn}m_{1}^{gn} + \mu_{2}^{gn}m_{2}^{gn})]. \]  

The condition for this quantity to be non-zero is that the chromophore of interest must have a non-zero magnetic and electric transition dipole moment along the same molecular direction. In the absence of perturbing external fields, this is only true for molecules that are chiral. Expressions that include higher order multipole contributions to eqs. (8), (9), and (11) can be found in previous theoretical descriptions of CPL theory (Riehl and Richardson, 1976a, 1986).

The final connection between molecular properties and experimental observables requires knowledge of the orientational distribution of the emitting molecules with respect to the direction and polarization of the excitation light and the direction of detection, and the time dependence of this distribution. Substituting eq. (5) into eq. (6) we obtain the following general expression

\[ \Delta I^\pi(\lambda,t) = \eta_n(hc/\lambda)N^\pi_e(\Omega_0,t)\Delta W_{gn}(\Omega_0)f_{CPL}(\lambda). \]  

Note that we have a superscript, \( \pi \), to indicate the polarization of the excitation beam, and also added a subscript to the lineshape function. As indicated above all CPL measurements are made relative to the total luminescence intensity which we may express using similar formalism as

\[ I^\pi(\lambda,t) = \eta_n(hc/\lambda)N^\pi_e(\Omega_0,t)W_{gn}(\Omega_0)f_{TL}(\lambda) \]  

and

\[ W_{gn}(\Omega_0) = K(\lambda^3)[|\mu_{1}^{gn}|^2 + |\mu_{2}^{gn}|^2 + |m_{1}^{gn}|^2 + |m_{2}^{gn}|^2]. \]  

Equations (12) and (13) along with the transition probabilities given in eqs. (11) and (14) may be substituted into eq. (2) in order to relate the molecular transition moments (in the lab frame!) to the observable, \( g_{\text{Lum}} \). In sect. 2.2 we discuss the orientational averaging necessary to relate the molecular to laboratory coordinate system for randomly oriented samples. We will
not continue with any more specific theoretical development of CPL from “frozen” solutions or crystals in this section, but finish this discussion with some consequences of the formulas presented above for CPL measurements from lanthanide ions.

In order to measure CPL from crystals, the crystal system must be at least uniaxial with the optic axis oriented along the direction of emission detection, so that circularly polarized luminescence may be propagated through the crystal without scattering which leads to depolarization. Of course, the other requirement is that the emitting complex must be chiral. As we will discuss in sect. 2.5, it is also possible to study racemic systems containing an equal number of opposite mirror image isomers (enantiomers), if one uses a circularly polarized excitation beam to generate a non-racemic excited (emitting) state. Obviously, the circularly polarized excitation beam also must propagate along a crystal axis in which the two orthogonal directions are equivalent. These requirements have severely limited the use of CPL to study crystalline materials.

We also note the complicated dependence of luminescence intensity on molecular orientation and polarization of the excitation beam as given by eqs. (12) and (14). For chromophores with well-defined transition directions, one needs to know the orientation and direction of the excitation beam relative to the direction of emission detection in order to fully describe, and perhaps, exploit the polarization dependence of the emission. The circular and linear polarization properties of absorption and emission transitions of individual crystal field transitions of lanthanide (III) ions in crystals have been used to help assign the symmetry of the initial and final states (Hopkins et al., 1998; Berry et al., 1988). Meskers et al. (1993) have also measured the linear polarization in the luminescence from \([\text{Eu} (2,6\text{-pyridine-dicarboxylate})_3]^{3-}\) in a viscous glycerol/water mixture where it was assumed that no reorientation occurred during the excited state lifetime of Eu(III). This complex has almost exact D\(_3\) symmetry, so, for example, the luminescent crystal field components in the \(5\,D_0 \rightarrow 7\,F_1\) transition have the polarization properties listed in table 1. In this complex, the C\(_3\)-axis defines the molecular z-axis, so by using linearly polarized excitation at right angles to the direction of emission detection, a non-isotropic orientational distribution of excited complexes was generated. From the linear polarization of the luminescence, it was not only possible to identify the symmetry of the individual crystal field states, but it was also possible to estimate the percentage magnetic dipole and electric dipole character of the transitions. By knowing the direction of the \(\pi \rightarrow \pi^*\) absorption of the pyridine ring, the angle that the aromatic plane made with the z-axis was also determined. This experiment was possible because of the high symmetry of the complex,

<table>
<thead>
<tr>
<th>Transition</th>
<th>Experimental (p_{\text{num}}) (standard error &lt; 0.01)</th>
<th>Theoretical (p_{\text{num}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(5,D_0 \rightarrow 7,F_1)</td>
<td>(\lambda_{\text{exc}} = 465.8,\text{nm}) (\lambda_{\text{exc}} = 290,\text{nm})</td>
<td>(\mu_x, \mu_y^b) (\mu_z^b)</td>
</tr>
<tr>
<td>(A_1 \rightarrow A_2)</td>
<td>+0.26</td>
<td>+1/3</td>
</tr>
<tr>
<td>(A_1 \rightarrow E)</td>
<td>−0.10</td>
<td>−1/2</td>
</tr>
</tbody>
</table>

\(^a\)From reference (Meskers et al., 1993).
\(^b\)Transition moment in absorption.
and the simplicity (i.e. small $J$) of the Eu(III) states which results in this case in individual (i.e. non-overlapping) crystal field transitions.

The results presented in the previous paragraph demonstrate that it might be necessary in the study of CPL from lanthanides imbedded in frozen solutions or rigid matrices, to be concerned with the presence of a photosel ected orientational distribution of emitting species that is not random, even if the ground state orientational distribution is random. The key result here is that different components of the emission transition vectors may contribute unequally to the observable, $g_{\text{lum}}$. In principle, this would allow one to investigate the chirality of molecular transitions along specific molecular directions, but to date this has not been exploited. In this case, there also may be a dependence on the magnitude of CPL on laboratory excitation and emission geometry, and, perhaps more importantly, the luminescence may also be partially linearly polarized. As discussed in sect. 3.3, it is experimentally difficult to measure circular polarization in the presence of linear polarization in the emission. It is often possible, however, to eliminate the effects of linear polarization by careful experimental design.

2.3. Circularly Polarized Luminescence from solutions: theory

We restrict the discussion here to the measurement of CPL from randomly oriented molecular samples such as occur in liquid solutions. Of course, as described above, even if the ground state distribution is randomly oriented (isotropic), the emitting state may not be, due to the photoselection of the emitting sample by the excitation beam. We will denote an averaged orientational distribution by brackets $\langle \cdot \rangle$. Substitution of eqs. (5) and (11) into eq. (6), and allowing for an ensemble of orientations we obtain the following

$$
\Delta I(\lambda,t) = \left( \frac{2i \hbar c \eta_{n}}{\lambda} \right) K(\lambda^{3}) \langle N_{e}^{\pi}(\Omega : \Omega_{0},t) \left[ \mu_{1}^{gn} m_{1}^{gn} + \mu_{2}^{gn} m_{2}^{gn} \right] \rangle f_{\sigma}(\lambda). \quad (15)
$$

We may formally separate the time dependence of the orientational distribution, $\eta(\Omega, t)$, from the number of molecules excited at time $t = 0$ by an excitation pulse as follows

$$
N_{e}^{\pi}(\Omega_{0}) = N_{e}^{\pi}(\Omega_{0}) \eta(\Omega, t). \quad (16)
$$

Assuming electric dipole absorption, $N_{e}^{\pi}(\Omega_{0})$ may be calculated from knowledge of the initial distribution, and direction and polarization of the excitation beam

$$
N_{e}^{\pi}(\Omega_{0}) = \kappa^{2} |\mu_{\text{eg}}^{eq} \cdot \hat{n}|^{2} \quad (17)
$$

where $\mu_{\text{eg}}^{eq}$ is the absorption transition vector. Final expressions for the formal relationship between the time-dependent circularly polarized luminescence intensity or $g_{\text{lum}}$ require knowledge of experimental geometry, and $\eta(\Omega, t)$.

For purposes of illustration, we examine here the most common situation of random (isotropic) ground state distributions, and $90^\circ$ excitation/emission geometry with unpolarized excitation. If the emission is detected in the laboratory $3$ direction, then the excitation is polarized in the $13$ plane. Equation (17) may then be written as

$$
N_{e}^{\pi}(\Omega_{0}) = \kappa^{2} \left[ |\mu_{1}^{eq}|^{2} + |\mu_{3}^{eq}|^{2} \right] \quad (18)
$$
and the final formal expression is obtained by substituting this result into eq. (15).

\[
\Delta I(\lambda, t) = \left( \frac{2i\hbar c \eta_n \kappa^2}{\lambda} \right) K(\lambda^3) \left[ |\mu_{\text{eg},1}|^2 + |\mu_{\text{eg},3}|^2 \right]_0 \left[ \mu_{\text{gn},1} m_{\text{gn},1} + \mu_{\text{gn},2} m_{\text{gn},2} \right]_0 \eta(\Omega, t) f_0(\lambda).
\]

(19)

This equation is written, of course, in the laboratory coordinate frame, and the orientational average requires knowledge of the absorption and emission transition dipole directions in the molecular coordinate system. The average may be evaluated through the use of Euler matrix terms in the standard manner (Riehl and Richardson, 1976a). The reader is referred to previous publications in which various excitation emission geometries and molecular transition moments have been evaluated (Riehl and Richardson, 1976a, 1986).

Since the emission lifetimes of the luminescent lanthanide (III) ions are normally long (\(\mu\text{sec}\) to msec), it is appropriate to assume that in solution the orientational distribution of emitting species is completely random. In this case, the orientational average in eq. (19) reduces to the following “isotropic” limit.

\[
g_{\text{lum}}(\lambda) = 4i \frac{f_{\text{CPL}}(\lambda)}{f_{\text{TL}}(\lambda)} \left[ \frac{\vec{\mu}_{\text{gn}} \cdot \vec{m}_{\text{gn}}}{|\vec{\mu}_{\text{gn}}|^2} \right].
\]

(20)

We note again that the magnetic dipole operator is pure imaginary, and the multiplication by \(i\) results in a real value. In the denominator of eq. (20) we have also neglected the magnetic dipole transition moments in eq. (14) compared to the usually larger electric dipole moments. This equation is sometimes written as

\[
G_{\text{lum}} = 4 \frac{R_{\text{gn}}}{D_{\text{gn}}} = 4 \int_{\text{band}} f_{\text{CPL}}(\omega) \text{Re}(\vec{\mu}_{\text{gn}} \cdot \vec{m}_{\text{gn}}) \omega^{-4} d\omega \int_{\text{band}} f_{\text{TL}}(\omega) |\vec{\mu}_{\text{gn}}|^2 \omega^{-4} d\omega,
\]

(22)

where we have converted these equations to frequency (or energy), and we note that all of the multiplicative factors except for the frequency (or wavelength) dependence divide out. For the usually very sharp transitions seen for lanthanide (III) complexes, this integration over a band is seldom performed.

One final simplification is the assumption that the lineshapes for total luminescence and circularly polarized luminescence are identical. This is appropriate for the usually sharp isolated pure electronic transitions that are often the target of CPL measurements. In this case we rewrite eq. (21) as follows

\[
g_{\text{lum}}(\lambda) = 4 \left[ \frac{\text{Re}(\vec{\mu}_{\text{gn}} \cdot \vec{m}_{\text{gn}})}{|\vec{\mu}_{\text{gn}}|^2} \right] = 2 \frac{\Delta I(\lambda)}{I(\lambda)}
\]

(23)

and make the final connection between theory and experiment.
2.4. CPL intensity calculations, selection rules, and spectra-structure correlations

Although there have been a number of reports of the calculation of CPL (and CD) intensities for various lanthanide complexes using the formalism presented above, it is not yet the case that the sign and magnitude of the differential emission or absorption of a chiral complex can be calculated reliably from knowledge of structure only. There have been a number of publications in which a parametric model has been used to match experimentally determined energy levels, and dipole and rotatory strengths on lanthanide complexes with $D_3$ symmetry (May et al., 1987, 1992; Berry et al., 1988; Moran and Richardson, 1992; Hopkins et al., 1996). Overall these model calculations have been fairly successful in matching the experimental values; however, it is observed that the calculated rotatory strengths, in particular, are quite sensitive to the quality of the wavefunctions used in the simulation and modeling.

The form of eq. (23) leads to several general conclusions concerning CPL spectroscopy. Since allowed magnetic dipole transition moments are usually much smaller in magnitude than allowed electric dipole transition moments, it makes sense to study emissive transitions that are formally electric dipole forbidden, but magnetic dipole allowed. Intraconfigurational $f \leftrightarrow f$ transitions are, indeed, formally LaPorte (parity) forbidden, and transitions that satisfy the formal selection rules of $\Delta J = 0, \pm 1$ (except $0 \leftrightarrow 0$) are magnetic dipole allowed (Richardson, 1980). These transitions may pick up electric dipole intensity from admixture of the odd-parity $4f^{N-1}5d$ and $4f^{N-1}ng$ configurations, and odd-parity multipole-ligand interactions. Since the electric dipole intensity is in the denominator of eq. (23), a larger value will often be obtained for $g_{\text{lum}}$ if the transition is weak. This is essentially the paradox of CPL (and CD) spectroscopy, namely, that $g_{\text{lum}}$ may be quite large for magnetic dipole allowed transitions that are electric dipole forbidden, but these transitions have very low total intensity, and therefore are more difficult to measure. A list of selected magnetic-dipole allowed emissive transitions of the luminescent lanthanide (III) ions are given in table 2.

It is unfortunate that there is to date no generally applicable spectra-structure correlations for CPL measurements from lanthanide (III) complexes. However, the number of chiral lanthanide complexes with well-understood geometry and solution dynamics is increasing, al-

<table>
<thead>
<tr>
<th>Lanthanide (III) ion</th>
<th>Emission transition</th>
<th>Approximate wavelength</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sm(III)</td>
<td>$^4G_5/2 \rightarrow ^9H_7/2$</td>
<td>580 nm</td>
</tr>
<tr>
<td></td>
<td>$^4G_5/2 \rightarrow ^9H_5/2$</td>
<td>560 nm</td>
</tr>
<tr>
<td>Eu(III)</td>
<td>$^5D_0 \rightarrow ^7F_1$</td>
<td>590 nm</td>
</tr>
<tr>
<td>Gd(III)</td>
<td>$^8S_7/2 \rightarrow ^6P_{7/2}$</td>
<td>312 nm</td>
</tr>
<tr>
<td>Tb(III)</td>
<td>$^5D_4 \rightarrow ^7F_3$</td>
<td>625 nm</td>
</tr>
<tr>
<td></td>
<td>$^5D_4 \rightarrow ^7F_4$</td>
<td>580 nm</td>
</tr>
<tr>
<td></td>
<td>$^5D_4 \rightarrow ^7F_5$</td>
<td>540 nm</td>
</tr>
<tr>
<td>Dy(III)</td>
<td>$^4F_{9/2} \rightarrow ^9H_{11/2}$</td>
<td>650 nm</td>
</tr>
<tr>
<td>Yb(III)</td>
<td>$^2F_{5/2} \rightarrow ^2F_{7/2}$</td>
<td>980 nm</td>
</tr>
</tbody>
</table>
allowing for considerable progress in relating structural or solvent characteristics to CPL measurements. In particular, Bruce et al. have analyzed and correlated the magnitude of CPL from Eu(III) complexes with C_4 symmetry, and related the magnitude of CPL to the polarizability of axial ligands and to the internal square antiprism twist angle (Bruce et al., 2002).

It is, in fact, for the higher symmetry lanthanide complexes that one would expect to find some reliable correlation between absolute stereochemistry and the CPL spectrum. Hilmes and Riehl were able to measure the CPL and CD for a number of transitions of the nine coordinate tris-terdentate complex of Eu(III) with 2,6-pyridine-dicarboxylate (dipicolinic acid ≡ DPA) through perturbation of a racemic equilibrium by added chiral agents, or by the use of circularly polarized excitation (see sect. 2.5) (Hilmes et al., 1988; Hilmes and Riehl, 1986). This complex is known to possess D_3 symmetry in the solid, and presumably in aqueous solution. They compared their results to those of Morley et al. (1982c) and Sen et al. (1981) who measured, respectively, the CPL, and CD of a particular chiral crystal of Na_3Eu(oxydiacetate ≡ ODA)_3·2NaClO_4·6H_2O which was determined by X-ray crystallography to have a Δ-configuration around the central Eu(III) ion. [A Δ-configuration is defined such that if the complex is viewed along the principal C_3 axis, the coordinated ligands are clockwise oriented as they go away from the observer.] A summary of the spectroscopic results are given in table 3. Čoruh et al. (1988b) noted that the signs of all the magnetic dipole transitions for [Eu(DPA)_3]^{3−} were uniformly opposite to those of [Eu(ODA)_3]^{3−}, and the magnitudes were similar. They concluded that the complex that they were examining had the Λ-configuration. This conclusion has not as yet been verified with other Eu(III) complexes with local D_3 symmetry. As described in sect. 4.1.3.1, other much more empirical correlations of the sign of the observed CPL to various aspects of molecular structure have been attempted without much success.

Table 3
Absorption (g_{abs}) and luminescence (g_{lum}) dissymmetry ratios for selected transitions of a chiral crystal of Δ-Na_3[Eu(ODA)_3]·2NaClO_4·6H_2O^4

<table>
<thead>
<tr>
<th>Transition</th>
<th>g_{abs}</th>
<th>Emission</th>
</tr>
</thead>
<tbody>
<tr>
<td>5D_0 ← 7F_0</td>
<td>not observed</td>
<td>5D_0 → 7F_0</td>
</tr>
<tr>
<td>5D_1 ← 7F_0</td>
<td>−4.60 × 10^{-1}</td>
<td>5D_0 → 7F_1</td>
</tr>
<tr>
<td>5D_2 ← 7F_0 (a)</td>
<td>−6.60 × 10^{-2}</td>
<td>5D_0 → 7F_2 (b)</td>
</tr>
<tr>
<td>5D_2 ← 7F_0 (b)</td>
<td>−1.00 × 10^{-2}</td>
<td>(b) no CPL detected</td>
</tr>
<tr>
<td>5D_0 ← 7F_1</td>
<td>−1.70 × 10^{-1}</td>
<td>5D_0 → 7F_3 (a)</td>
</tr>
<tr>
<td>5D_1 ← 7F_1</td>
<td>large negative</td>
<td>5D_0 → 7F_3 (b)</td>
</tr>
<tr>
<td>5D_2 ← 7F_1 (a)</td>
<td>large negative</td>
<td>5D_0 → 7F_4 (a)</td>
</tr>
<tr>
<td>5D_2 ← 7F_1 (b)</td>
<td>large negative</td>
<td>5D_0 → 7F_4 (b)</td>
</tr>
</tbody>
</table>

^4From references (Morley et al., 1982c) and (Sen et al., 1981).
2.5. Circularly polarized luminescence from racemic mixtures: theory

One of the most useful applications of CPL spectroscopy has been in the study of racemic mixtures (Hilmes and Riehl, 1983). This experiment is possible, because, even though the ground state is racemic, i.e. equal concentrations of the two possible enantiomeric structures, the emitting state can sometimes be photoprepared in a significantly enantiomerically enriched state (non-racemic) by differential absorption of circularly polarized excitation. If the racemization (interconversion) rate of the excited state is less than the emission rate, then the emission will be circularly polarized. We consider a racemic sample containing equal concentrations of $\Lambda$ and $\Delta$ enantiomers. The preferential absorption of circularly polarized light is related to the CD through the absorption dissymmetry ratio $g_{\text{abs}}$ which was defined in eq. (3).

Rewriting this equation for the CD of the $\Delta$ enantiomer we obtain the following

$$g^\Delta_{\text{abs}} = \frac{\Delta\varepsilon}{\varepsilon} = \frac{\varepsilon_L(\Delta) - \varepsilon_R(\Delta)}{\frac{1}{2}(\varepsilon_L(\Delta) + \varepsilon_R(\Delta))}.$$  \hspace{1cm} (24)

The number of $\Delta$ enantiomers that absorb left circularly polarized light, $N_L(\Delta)$ is proportional to $\varepsilon_L(\Delta)$, and similarly for $\varepsilon_R(\Delta)$. Thus, we may make the following substitution

$$g^\Delta_{\text{abs}} = \frac{N_L(\Delta) - N_R(\Delta)}{\frac{1}{2}(N_L(\Delta) + N_R(\Delta))}$$  \hspace{1cm} (25)

since all of the proportionality constants cancel. The number of $\Delta$ enantiomers that would absorb right circularly polarized light is exactly equal to the number of $\Lambda$ enantiomers that would absorb left circularly polarized light, i.e. $N_R(\Delta) = N_L(\Lambda)$. This substitution yields the following

$$g^\Delta_{\text{abs}} = \frac{2\Delta N_L}{N_L}.$$  \hspace{1cm} (26)

$\Delta N_L$ is the differential population ($\Lambda - \Delta$) of excited enantiomers that would result from left circularly polarized excitation.

In general, an exact description of CPL from racemic mixtures requires consideration of the competition between racemization and emission (Hilmes and Riehl, 1983). Under the assumption that racemization is much slower than emission, we may write the following expression for CPL from a racemic mixture under left circularly polarized excitation as follows

$$g^L_{\text{lum}}(\lambda) = 4i f_{\text{CPL}}(\lambda) \Delta N_L \left[ \frac{\mu(\Lambda) \cdot \mu(\Lambda)'}{|\mu(\Lambda)|^2} \right].$$  \hspace{1cm} (27)

Substituting from above we obtain the following expression

$$g^L_{\text{lum}}(\lambda) = \frac{1}{2}g^\Delta_{\text{abs}}(\lambda') g^\Lambda_{\text{lum}}(\lambda'),$$  \hspace{1cm} (28)

where we have explicitly labeled the excitation wavelength as $\lambda'$. Examination of eq. (28) shows that the measurement of CPL from racemic mixtures depends on the product of the CD and CPL. Since the measurement of $g_{\text{lum}}$ is generally limited to magnitudes greater than $10^{-4}$, the magnitude of the intrinsic dissymmetry ratios must usually be greater than approximately
10^{-2} for this unique measurement to be feasible. Since f \leftrightarrow f transitions that obey magnetic
dipole selection rules often are associated with large dissymmetry ratios, both the excitation
and emissive transition should involve these transitions if possible.

2.6. Theoretical aspects of time-resolved circularly polarized luminescence

The measurement of the time dependence of $g_{lum}$ may be used to probe various chiral aspects
of excited state energetics, molecular dynamics, and reaction kinetics. Although there are
some time-dependent circular polarization effects due to molecular reorientations that parallel
time-dependent linear polarization measurements, the most interesting studies are those that
involve the time-dependence of intrinsic molecular chirality. For a sample containing one
chiral luminescent lanthanide chromophore, it might be the case that there are processes that
affect chirality occurring on the same time scale as emission that could be probed by time-
dependent CPL. To date, however, there have been no reports of such studies, and all of the
time-dependent CPL measurements have involved racemic mixtures.

2.6.1. Time-resolved CPL from racemic mixtures: chemical racemization

We consider again the case that a non-racemic excited state of a racemic mixture is generated
by use of a circularly polarized excitation beam. Neglecting orientational effects, and assum-
ing a concentration sufficiently dilute ($< 0.01$ M) that excited state energy transfer between
eanantiomers does not occur (Sokolnicki et al., 2002), we may derive the following expression
for the time-dependence of the excited state population following an excitation pulse of left
circularly polarized light

$$
\Delta N^L(t) = \Delta N^L(0) \exp(-2k_{rac}t - k_0 t),
$$

(29)

where $k_{rac}$ is the racemization rate and $k_0$ is the excited state decay rate. The total number of
emitting molecules excited by left circularly polarized excitation decays as

$$
N^L(t) = N^L(0) \exp(-k_0 t).
$$

(30)

Allowing for the time-dependence of the excited state population, we may substitute eqs. (29)
and (30) into eq. (27), and express the time-dependence of the luminescence dissymmetry as
follows

$$
g_{lum}^L(t) = g_{lum}^L(0) \exp(-2k_{rac}t).
$$

(31)

This technique has been used in a studies in which the decay of $g_{lum}^L(\lambda)$ has been analyzed to
extract racemization rate constants (Metcalf et al., 1990c; Glover-Fischer et al., 1998; Metcalf
et al., 1991).

It is also possible to extract dynamic information concerning racemization rates through
measurement of steady-state CPL (Belair et al., 2000; Mondry et al., 1994). Again neglect-
ing time-dependent orientational effects and excited state energy transfer, one may integrate
eqs. (29) and (30) over long times and obtain the following result

$$
g_{lum}^L(\lambda) = \frac{1}{2} g_{abs}(\lambda') g_{lum}^L(\lambda) \left( \frac{k_0}{2k_{rac} + k_0} \right)
$$

(32)
This equation has the expected limiting behavior. If $k_{\text{rac}} \ll k_0$ then the result of eq. (28) is obtained, and if $k_{\text{rac}} \gg k_0$ then the magnitude of $g^L_{\text{lum}}(\lambda)$ becomes negligibly small.

2.6.2. Time-resolved CPL from racemic mixtures: excited state energy transfer

A complete description of CPL from racemic mixtures must include the possibility of excited state energy transfer. Transfer of energy between identical enantiomers, of course, has no effect on the excited state differential population, transfer between opposite enantiomers will decrease the excited state population and, therefore, $g^L_{\text{lum}}(\lambda)$. There have been several reports of CPL from lanthanide complexes dissolved in rigid sol-gels, that have been interpreted in terms of possible effects of excited state energy transfer on measured CPL (Huskowska et al., 2000; Sokolnicki et al., 2002, manuscript in preparation). These sol-gels are made by slow evaporation of dilute samples, but the final concentrations of lanthanide complexes are quite high, so that intermolecular energy transfer effects may be important. Assuming that structural racemization does not occur, Sokolnicki et al. (2002) showed that the differential excited state concentration may be expressed as

$$\Delta N_n(t) = \Delta N_n(0) e^{-k_0 t} e^{A(t)}, \quad (33)$$

where

$$A(t) = -\int_0^t N k_{et}(t) \, dt. \quad (34)$$

$N$ is the total number of lanthanide complexes present, and $e^{A(t)}$ is the probability that an initial excitation of an $\Lambda$ (or $\Delta$) complex remains on an $\Lambda$ (or $\Delta$) molecule at time $t$. In the derivation of this result, it was also assumed that the energy transfer rate between identical enantiomers was the same as between opposite enantiomers. Determination of the form of $e^{A(t)}$ depends on the particular energy transfer mechanism, and various assumptions concerning intermolecular orientation and distances.

So far no one has interpreted any time-resolved results as being due to effects of diminishing of CPL due to energy transfer. The results that have appeared involved “steady-state” measurements. Substitution of eqs. (33) and (30) into eq. (27), and integrating over long times as above yields the following result

$$g^L_{\text{lum}}(\lambda) = \frac{1}{2} g^A_{\text{abs}}(\lambda') g^A_{\text{lum}}(\lambda) \int_0^\infty e^{-k_{et} t} e^{A(t)} \, dt = \frac{1}{2} g^A_{\text{abs}}(\lambda') g^A_{\text{lum}}(\lambda) B/\tau. \quad (35)$$

The term $B/\tau$, which is defined by eq. (35), is a measure of the reduction of $g^L_{\text{lum}}(\lambda)$ due to energy transfer. This term is approximately zero if racemization due to energy transfer is fast compared to the emission lifetime, and equal to 1 if no racemization occurs due to excited state energy transfer.

2.6.3. Time-resolved CPL from racemic mixtures: differential excited state quenching

We consider here the schematic energy level diagram given in fig. 2. In this experiment an unpolarized or linearly polarized beam excites a racemic mixture to an emitting state. Since the
excitation beam is not circularly polarized the initial concentration of the two excited enantiomers is equal, i.e. \([\Lambda^\ast]_0 = [\Delta^\ast]_0\). The solution under study also contains a chiral quencher molecule, \(Q\), and the interaction of the quencher with the excited enantiomers is such that one of the enantiomers is quenched at a faster rate than the other. This, of course, results in an excited state that becomes increasingly chiral as time passes from an initial excitation pulse.

It is easy to show that the time-dependence of the excited state population of the two enantiomers may be expressed as (Metcalf et al., 1990a)

\[
[\Lambda^\ast](t) = [\Lambda^\ast]_0 e^{-(k_0 + k_q^{\Delta Q}(Q))t}, \tag{36}
\]

\[
[\Delta^\ast](t) = [\Delta^\ast]_0 e^{-(k_0 + k_q^{\Lambda Q}(Q))t}, \tag{37}
\]

where \(k_q^{\Delta Q}\) and \(k_q^{\Lambda Q}\) the diastereomeric bimolecular quenching rate constants. In this experiment we have two emitting species, thus we can write the following equation describing the time-dependence of the enantiomeric excess

\[
\eta^\ast(t) = \frac{[\Lambda^\ast](t) - [\Lambda^\ast](t)}{[\Lambda^\ast](t) + [\Delta^\ast](t)} \tag{38}
\]

and substituting from eqs. (36) and (37) we obtain

\[
\eta^\ast(t) = \frac{[\Lambda^\ast]_0 e^{-(k_0 + k_q^{\Delta Q}(Q))t} - [\Delta^\ast]_0 e^{-(k_0 + k_q^{\Lambda Q}(Q))t}}{[\Lambda^\ast]_0 e^{-(k_0 + k_q^{\Delta Q}(Q))t} + [\Delta^\ast]_0 e^{-(k_0 + k_q^{\Lambda Q}(Q))t}} \tag{39}
\]

since \([\Delta^\ast]_0 = [\Lambda^\ast]_0\), after some cancellation we get

\[
\eta^\ast(t) = \frac{e^{-k_q^{\Delta Q}t} - e^{-k_q^{\Lambda Q}t}}{e^{-k_q^{\Delta Q}t} + e^{-k_q^{\Lambda Q}t}} = \tanh(\frac{1}{2}kd[Q]t), \tag{40}
\]

where

\[
k_d = k_q^{\Delta Q} - k_q^{\Lambda Q}. \tag{41}
\]
In this case the time-dependence of \( g_{\text{lum}} \) is given through the time-dependence of the enantiomeric excess

\[
g_{\text{lum}}(\lambda, t) = g_{\text{lum}}^A(\lambda) \eta^*(t) = g_{\text{lum}}^A(\lambda) \tanh \left( \frac{1}{2} k_d |Q| t \right).
\]  

Therefore, one may fit the time-dependence of \( g_{\text{lum}} \) to a hyperbolic tangent function to determine \( k_d \). Note from the denominator of eq. (39), that, for this system, the total luminescence decay is described as a biexponential, and it is sometimes easier to determine the individual diastereomeric decay constants from a biexponential analysis.

Just as above, it is possible to determine the “steady-state” result for this differential quenching by integrating over long times. The final result is

\[
g_{\text{lum}}(\lambda) = g_{\text{lum}}^A \left( \frac{k_q^\Lambda Q - k_q^\Delta Q |Q|}{2k_0 (k_q^\Lambda Q + k_q^\Delta Q |Q|)} \right)
\]

which again has the correct limiting result; if there is no difference in quenching rates, there will be no CPL observed.

2.7. Circularly polarized luminescence from perturbed racemic equilibria

The addition of chiral complexes to a solution containing a racemic mixture may lead to a perturbation of the ground state equilibrium without changing the local structure of the complexes involved. This perturbation, which is sometimes referred to as the “Pfeiffer effect” leads to an enantiomeric excess in the ground state, \( \eta_g \), defined in analogy to eq. (38) as follows

\[
\eta_g = \frac{[\Lambda] - [\Delta]}{[\Lambda] + [\Delta]},
\]

where the square brackets in this equation denote ground state concentrations. It is easy to see that the measured absorption or luminescence dissymmetry factors for a perturbed equilibrium may be related to the values that one would obtain if the solution contained a pure enantiomer by the following equations

\[
g_{\text{abs}}(\lambda') = \eta_g g_{\text{abs}}^A(\lambda'), \quad (45)
\]

\[
g_{\text{lum}}(\lambda) = \eta_n g_{\text{lum}}^A(\lambda), \quad (46)
\]

where we have introduced the symbol, \( \eta_n \), to indicate the enantiomeric excess in the emitting state, and we explicitly denoted the absorption wavelength as \( \lambda' \), and the emission wavelength as \( \lambda \). In the absence of enantioselective excited state processes, one expects that the enantiomeric excess in the emitting and ground states are equal

\[
\eta \equiv \eta_n = \eta_g. \quad (47)
\]
Using this substitution in eqs. (45) and (46) and rewriting eq. (28) we have the following set of three equations with three unknowns

\[ g_{\text{abs}}(\lambda') = \eta g_{\text{abs}}^A(\lambda'), \]
\[ g_{\text{lum}}(\lambda) = \eta g_{\text{lum}}^A(\lambda), \]
\[ g_{\text{lum}}^L(\lambda) = \frac{1}{2} g_{\text{abs}}^A(\lambda') g_{\text{lum}}^A(\lambda). \]

(48)

If one can measure the CD of a perturbed racemic solution at the same wavelength \( \lambda' \) that one uses for circularly polarized excitation of a racemic mixture, and also measures the CPL of the perturbed racemic equilibrium at wavelength, \( \lambda \), then one can not only determine the enantiomeric excess, but also the dissymmetry ratios of the pure enantiomeric complexes even if they can not be resolved. Application of this procedure is described in sect. 4.1.3.1.

3. Experimental measurement

3.1. CPL instrumentation

3.1.1. Steady-state CPL instrumentation

Unlike CD spectrometers, there are no commercial instruments available for the detection of CPL and, therefore, the research groups involved in this measurement over the last thirty have designed and built their own instruments (Brittain, 1991, 1989, 1997a, 1998, 2000; Riehl and Richardson, 1993, 1986; Schippers et al., 1982). All of these various custom-built instruments use the same basic optical design as described below, in which the circular polarizations are detected by conversion to linear polarization by a modulated quarter wave retarder/advancer. The earlier instruments used a lock-in amplifier to determine the extent of circular polarization in the emission, the more modern instruments use various methods of gated photon counting. Photon counting techniques have been shown to be much more stable, and not susceptible to the electronic problems (e.g., ground loops) associated with the small net signal usually seen in CPL spectroscopy.

To illustrate the basic design of a CPL spectrometer we present a short description of the CPL instrument used in our laboratory at the University of Minnesota Duluth. A schematic diagram for this instrument is illustrated in fig. 3. This instrument is designed to measure “steady-state” CPL. As can be seen in this figure, the source of excitation in this instrument is either a continuous wave (CW) dye laser (Coherent 599) pumped by a CW argon-ion laser (Coherent Innova 70-4), the argon-ion laser without the dye laser, or a CW 450 watt xenon arc lamp. In our experimental set-up, the laser beam is situated below the sample and emission light path, and is reflected through the sample quartz fluorescence cuvette with polished surfaces. In addition, the polarization of the laser beam is aligned along the direction of emission (laboratory 3 axis) so that the luminescence detected at 90° will not be linearly (1 or 2) polarized. The presence of linear polarization in the emission beam has been known to cause artifacts in the detection of circular polarization (see sect. 3.3). This precise experimental alignment is not important when the emission is isotropic, but it is extremely important for
slowly rotating or oriented systems in which the emission could be linearly polarized (Field et al., 2003; Riehl and Richardson, 1986). It should be mentioned that the incident excitation beam may be linearly polarized, circularly polarized, or unpolarized, and that unpolarized or circularly polarized excitation at 90° may lead to linear polarization in the emission even if the ground state distribution of emitting species is isotropic.

The design of CPL instrumentation is based on an ordinary fluorescence instrument with the addition of a circular analyzer between the emitting sample and the emission monochromator. The circular analyzer is composed of a photo-elastic (or elasto-optic) modulator (PEM) followed by a high-quality linear polarizer. The PEM is composed of an isotropic clear optical material that becomes anisotropic on application of a periodic stress. PEMs that operate around 50 kHz are commercially available (Hinds Int., Eugene, OR) and in this application act as a dynamic quarter-wave polarization modulator. During one half-cycle of the modulation left circularly polarized light is converted to linearly polarized light which then passes through the linear polarizer, and then, during the other half-cycle, right circularly polarized light is passed on to the monochromator. Thus, the monochromator sees light with only one polarization throughout the modulator cycle, which is a necessity due to the polarization sensitivity of monochromators. In polarization-sensitive detection experiments it is obviously necessary to minimize sources of depolarization. Thus, for example, one normally places no optical elements between the sample compartment and the PEM. These precautions are es-

![Schematic diagram for instrumentation used to measure circularly polarized luminescence.](image)
especially important in CPL measurements owing to the fact that the difference in intensities between the left and right circularly polarized emitted light are 10 to 100 times less than that detected in linearly polarized luminescence measurements.

In order for the PEM to act as a quarter-wave device over the entire UV-visible spectral region, it must be driven at the appropriate amplitude corresponding to the detection wavelength by employing an inexpensive digital-to-analog (D/A) interface, and setting the appropriate voltage for the PEM controller. Another possibility may be by coupling the wavelength drive of the emission monochromator to the PEM controller. After passing through the PEM and linear polarizer, the emitted light travels through an appropriate filter to eliminate scattered excitation and other stray light, and a suitable lens is used to focus the beam onto the entrance slits of the emission monochromator. Detection is accomplished by a thermoelectrically cooled photomultiplier tube (PMT) operating in photon-counting mode. In the case of near-IR detection, a liquid N₂ cooled PMT is used. As shown in fig. 3, the output of the PMT can either be analyzed for a DC voltage which is a direct measurement of relative total luminescence in arbitrary units, or directed to an amplifier/discriminator which outputs photopulses to a custom-built differential photon counter (DPC). This latter was designed and constructed at the University of Leiden, The Netherlands, under the supervision of H.P.J.M. Dekkers (Schippers, 1982). In this case, the leading edge of the 50 kHz reference signal from the PEM controller is used to define a time window centered on the peak of the modulation reference signal. During the “open” time of this variable time window, photopulses are directed into two separate counters. One of the counters counts every pulse that passes through the time-window, and the other (up/down) counter adds the photo-pulses that enter the gate when left circularly polarized light is passing through the PEM and subtracts the pulses when right circularly polarized light intensity is being measured. Thus one counter contains a number proportional to \( I \), and the other counter contains a number proportional to \( \Delta I \). These two quantities can be used to obtain \( g_{\text{lum}} \) (see eq. (2)). Perhaps the most difficult aspect of this gated-counter technique is the requirement that the time windows for left and right circular polarization detection must be positioned properly, have a high temporal resolution, and be exactly equal in width (Schippers, 1982). The computer used in this experimental setup monitors the DC voltage and differential photon count, steps the excitation and emission monochromators, and stores and displays \( g_{\text{lum}} \) values and total emission signals as a function of wavelength. These functions are accomplished using commercially available digital and analog input/output boards.

3.1.2. Time-resolved CPL instrumentation

The measurement of the time dependence of the emitted circular polarization is complicated by the inherent time variation of the PEM modulator, and the important requirement that the time windows corresponding to the detection of left and right circular polarization following a pulsed excitation be sampled equally. Blok et al. reported the modification of a commercial fluorescence spectrometer to measure time-dependent CPL (Blok et al., 1990). Metcalf et al. (1989) have briefly described their photon counting based time-resolved CPL instrumentation, and Rexwinkel et al. (1993c) have published a detailed account of their instrumentation in
which time resolution approaching nanoseconds could be obtained. An instrument using an analog detection system has also been described (Schauerte et al., 1995).

There have been two approaches to the sampling issue, one method is to ensure that the excitation pulses are positioned appropriately by coupling of the timing of the excitation pulse with the modulator cycle (Metcalf et al., 1989), the other is to have the excitation pulses occur randomly throughout the modulation cycle (Rexwinkel et al., 1993c; Blok et al., 1990). The photon counting detection system of Rexwinkel et al. (1993c) is composed of 2048 decay channels that are adjustable to time windows of 50 ns to 1 s. Just as described above, two counters associated with the total counts and differential counts are used, and the content of the counters is transferred into computer memory through high-speed direct memory access. The optical components of these instruments are essentially the same as the “steady-state” instrument described above.

3.2. Experimental and statistical limitations of CPL measurements

The use of photon-counting techniques in combination with sinusoidal variation of stress-induced polarization modulation imposes quantifiable limitations on the accuracy and precision of CPL measurements. We consider first the optical characteristics of the PEM. The phase difference ($\phi$) between the two orthogonal crystal axes of the PEM is related to the sinusoidally varying periodic stress ($\sin \alpha t$) through the following Bessel function

\[ \phi = \sin(\sin \alpha t) . \]  

The stress modulation and polarization modulation are plotted in fig. 4. We also plot in this figure a square wave signal corresponding to a time-window (gate) equal to 50% of the half-period that would correspond to detection of left and right circularly polarized emitted photons. As one can see, only at the peaks of the modulation cycles is the PEM acting as an exactly quarter-wave advancing (+90°) or retarding (−90° or +270°) optical element. Clearly, the smaller the time-window, the more closely photons passing through the PEM are being properly selected, and the more accurate is the measurement. With a time-window corresponding to 50% of the modulation cycle, one is not counting 50% of the available photons, and reducing the time-window even smaller for more accurate measurements has obvious consequences, and makes the time for the measurement even longer.

Assuming perfect operating conditions, one can show that for a 50% window that the intrinsic error associated with the PEM in the measurement of $g_{\text{lin}}$ is less than 3% (Kemp, 1969).

Fig. 4. Variation of applied stress and induced polarization modulation as a function of time. The square wave signal corresponds to a time-window equal to 50% of the half-period.
For highly luminescent samples, the error may be reduced by decreasing the time-window (the error is approximately 1% for a 20% window) (Schippers and Dekkers, 1981), or one might be justified in simply “correcting” the value to account for this known effect. Obviously, there are other sources of error in polarization measurements of this type. The PEM itself is not a perfect optical element, and other elements including linear polarizers, sample containers, filters, mirrors, etc. all lead to sources of error. One also needs to consider the statistical nature of these measurements as described below, and, as a result, $g_{\text{lum}}$ values obtained with 50% time-windows are usually presented as obtained without correction.

One of the principal advantages of the use of photon-counting methods over analog methods for detection and analysis of CPL is the statistical nature of these measurements. In fig. 5 we plot the distribution of $g_{\text{lum}}$ results obtained for a sample measurement. Schippers has shown that the observed spread in $g_{\text{lum}}$ values is what one expects from Poisson statistics (Schippers, 1982). The standard deviation, $\sigma_d$, of a particular measurement of $g_{\text{lum}}$ in which a total of $N$ photons were counted is as follows

$$\sigma_d = \sqrt{\frac{2}{N}}.$$  \hspace{1cm} (50)

The data presented in fig. 5 is for 10,000 measurements of $g_{\text{lum}}$ for $N = 1 \times 10^5$, thus we calculate a standard deviation of $4.5 \times 10^{-3}$ which should be compared to the experimental $\sigma_d$ of $6.1 \times 10^{-3}$. Obviously, if the instrument is working properly it is easy to determine the standard deviation for a particular measurement, or if a particular precision is required, the number of photons that must be counted may be predetermined.

Although selected transitions involving lanthanide (III) ions in chiral rigid complexes often exhibit large $g_{\text{lum}}$ values, it is often the case that one is interested in studying racemic or
other types of mixtures, in which the circularly polarized component of the emitted light represents only a small fraction of the total emitted light intensity. The CPL spectrometers which have been constructed are able to determine $g_{\text{lum}}$ with a sensitivity of approximately $10^{-4}$–$10^{-5}$, depending on the total light intensity. From the discussion above, one can easily see that the time required for a CPL measurement is dependent on the intensity of luminescence of the system studied, and the “chirality” of the transition being analyzed. For example, using eq. (45), in order to measure $g_{\text{lum}}$ with a standard deviation of $1 \times 10^{-4}$, a total of $2 \times 10^8$ photon pulses must be counted. As a consequence, accurate $g_{\text{lum}}$ measurements can be achieved in a short time for highly luminescent systems for transitions associated with large $g_{\text{lum}}$ values, while in the case of weakly luminescent samples with small $g_{\text{lum}}$ values, a much longer collection time is required for the same percent error. These arguments, of course, pertain only to measurement at one wavelength. For measurement of a CPL spectrum, one must decide whether to collect data for approximately the same amount of time at each wavelength to present spectra in which the relative error at each wavelength point is the same, or to collect the same number of photon pulses at each wavelength so that the absolute error at each point is the same. This choice is usually accomplished in the controlling software.

3.3. Artifacts in CPL measurements

It has long been recognized that the main source of artifacts in CPL measurements is the passing of linearly polarized light through the very slightly birefringent PEM (Riehl and Richardson, 1986; Dekkers et al., 1985). Even though this birefringence is usually small ($< 5$%), it may lead to a signal of comparable magnitude to many true CPL signals, since linear polarization is usually a much larger effect. Luminescence that is 10% linearly polarized may be converted by this effect to circular polarization of say 0.5%, which is often larger than the true signal.

Some success has been obtained in eliminating linear polarization effects by rotating the PEM and linear polarizer such that the birefringent axis of the PEM is parallel to the plane of polarization of the emitted light; however, it is our experience that one can only be confident of the accuracy of a CPL measurement by ensuring that the emitted light contains no linearly polarized component in the plane perpendicular to the emission direction. As described in sect. 3.1, one way of accomplishing this is to use a 90° excitation/emission geometry with a linearly polarized beam oriented parallel to the direction of emission detection. It is the case, however, that to date, no one has observed linear polarization in the luminescence from lanthanide (III) ions in ordinary solutions. This is due to the nature of the electronic transitions involved, and the fact that the lifetimes of the luminescent lanthanide ions are long, allowing for any initial photoselected orientational distribution to randomize in the time between excitation and emission.

3.4. CPL calibration and standards

In the analog detection of CPL, the differential emission intensity, $\Delta I$, is assumed to be proportional to the output of the lock-in amplifier, and the total emission intensity, $I$, is proportional to a DC output voltage. These are generally independent measurements, so that a
Table 4

<table>
<thead>
<tr>
<th>Emission transition</th>
<th>Approximate wavelength</th>
<th>$g_{\text{lum}}$ value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^5\text{D}_0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table><p>ightarrow ^7\text{F}_1$ | 588.24 nm | $-0.25$ |
| $^5\text{D}_0ightarrow ^7\text{F}_1$ | 595.24 nm | $-0.78$ |
| $^5\text{D}_0ightarrow ^7\text{F}_2$ | 613.50 nm | $+0.072$ |</p>

*a* From reference (Schippers, 1982).

determination of $g_{\text{lum}}$ requires the use of a calibration standard. One of the main advantages of the photon counting method is that $g_{\text{lum}}$ is determined directly, so that, in principal, no independent calibration is necessary.

It is obviously important that one be sure that the magnitude and sign of the CPL signal are being measured accurately. Although there have been efforts at standardization and calibration using variable quarter-wave plates (Steinberg and Gafni, 1972), or passing unpolarized light through solutions of known CD (Barnett et al., 1979), the most common approach is to use a solution containing a chiral species of known CPL. Brittain was the first to suggest the use of the commercially available NMR chiral shift reagent tris(3-trifluoroacetyl-d-camphorato)europium (III), Eu(facam)$_3$, as a CPL standard (Brittain, 1980). This complex is available in high purity, is readily soluble in DMSO, and may be excited either by a UV source around 350 nm or Ar-ion laser excitation at 345 nm. This complex has also been used by Schippers (1982), and the results reported in this reference are reproduced in table 4. As can be seen, three different Eu(III) transitions are observed with variable signs and magnitudes. It should be noted that Maupin has noted that the chirality of lanthanide facam complexes in DMSO are quite sensitive to the presence of water, so that care should be taken to ensure dry complex and dry solvent when using this species as a CPL standard (Maupin, 1999).

4. Survey of recent experimental results (CPL)

Since the last complete review of CPL spectroscopy (Riehl and Richardson, 1986) there have been approximately 150 articles published involving this technique with the vast majority being concerned with lanthanide (III) ions. In this section we will review the various types of applications of CPL to the study of the molecular and electronic structure of chiral systems containing lanthanide ions. We will attempt to be as complete as possible concerning work appearing since this last full review, but we will also include some of the earlier work in order to put some of the more recent results in context.

4.1. Lanthanide complexes with achiral ligands

In order for a luminescent lanthanide (III) ion to emit circularly polarized light, it must be in a chiral environment. In sect. 4.2 we will describe CPL measurements in which the chirality is due to the chirality induced in lanthanide (III) coordination due to the fact that the ligands
surrounding the lanthanide ion are themselves chiral. In this section we describe studies involving lanthanide complexes with ligands that are not chiral (achiral). If one mixes together non-chiral ligands with lanthanide ions which, of course, are not chiral, then the solution or solid that is formed can have no net chirality. As we will see below, however, the complexes that are formed may be of such a symmetry that mixtures are formed of opposite chirality. If a pair of molecular species are formed in equal concentration that are mirror images of each, the solution is said to be racemic, i.e. it contains a racemic mixture of enantiomers. This system has no optical rotation, no circular dichroism, and no CPL, unless one is able to generate an enantio-enriched population through some chemical or optical technique.

The simplest way to prepare the enantiomers for chiroptical study would be to physically separate the species through recrystallization or other classical chemical separation techniques. This is a well known process for racemic transition metal complexes. Enantiomeric complexes can be prepared, separated, and solutions of individual enantiomers studied by the various chiroptical techniques. This is much more problematic for lanthanide complexes, since the ligands are generally very labile. The first CPL results for a chiral complex of a luminescent lanthanide ion containing achiral ligands that has been resolved and studied in solution has just been reported (Cantuel et al., 2004). The much more common situation is the generation of enantioenriched excited states from racemic mixtures either by employing circularly polarized excitation, enantioselective quenching, or perturbation of a ground state equilibrium through the addition of chiral agents. These experiments are described in the next sections.

4.1.1. CPL from racemic mixtures following circularly polarized excitation

4.1.1.1. Steady-state measurements. As presented in sect. 2.6.1, CPL spectroscopy has been shown to be a particularly useful technique for studying chiral lanthanide complexes that occur in solution as racemic mixtures, but do not completely interconvert during the emitting state lifetime. The use of a left circularly polarized excitation beam results in a preferential excitation of one of the two possible enantiomers over the other. Excitation with right circularly polarized light gives the exactly opposite effect. This experiment relies upon differential absorption to generate a non-racemic excited state, so that if possible one should excite the species of interest at a wavelength corresponding to a magnetic-dipole allowed transition. A list of selected magnetic-dipole allowed absorptions of the luminescent lanthanides are given in table 5. In table 6 we list the racemic lanthanide complexes for which CPL has been reported either through circularly polarized excitation, or by perturbation of the ground state equilibrium.

Perhaps, the best example of this kind of experimental study are the numerous reports involving the 9-coordinate complex of lanthanide (III) ions with 2,6-pyridine-dicarboxylate = dipicolinic acid = DPA. These complexes are known to possess almost exact D3 symmetry, and as such occur in solution as racemic mixtures of complexes with A and Δ helicity. It has been demonstrated that the CPL lineshape and the magnitude of the luminescence dissymmetry ratios, $g_{\text{lum}}(\lambda)$, of a 3.5 : 1 mixture of dipicolinic acid (DPA) to Tb(III) in aqueous solution excited with circularly polarized light were independent of pH in the range 7.0–8.8, additional ligand, or upon a 4-fold dilution (Hilmes et al., 1985;
Table 5

<table>
<thead>
<tr>
<th>Lanthanide (III) ion</th>
<th>Absorption transition</th>
<th>Approximate wavelength</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nd(III)</td>
<td>(^{4}G_{7/2} \leftarrow ^{4}I_{9/2})</td>
<td>526 nm</td>
</tr>
<tr>
<td></td>
<td>(^{4}G_{9/2} \leftarrow ^{4}I_{9/2})</td>
<td>514 nm</td>
</tr>
<tr>
<td></td>
<td>(^{4}G_{11/2} \leftarrow ^{4}I_{9/2})</td>
<td>465 nm</td>
</tr>
<tr>
<td>Sm(III)</td>
<td>(^{4}G_{5/2} \leftarrow ^{6}H_{5/2})</td>
<td>560 nm</td>
</tr>
<tr>
<td>Eu(III)</td>
<td>(^{5}D_{1} \leftarrow ^{7}F_{0})</td>
<td>526 nm</td>
</tr>
<tr>
<td></td>
<td>(^{5}D_{1} \leftarrow ^{7}F_{1})</td>
<td>535 nm</td>
</tr>
<tr>
<td></td>
<td>(^{5}D_{1} \leftarrow ^{7}F_{2})</td>
<td>557 nm</td>
</tr>
<tr>
<td></td>
<td>(^{5}D_{2} \leftarrow ^{7}F_{2})</td>
<td>488 nm</td>
</tr>
<tr>
<td></td>
<td>(^{5}D_{2} \leftarrow ^{7}F_{1})</td>
<td>472 nm</td>
</tr>
<tr>
<td>Dy(III)</td>
<td>(^{4}I_{15/2} \leftarrow ^{4}I_{15/2})</td>
<td>457 nm</td>
</tr>
<tr>
<td>Yb(III)</td>
<td>(^{2}F_{5/2} \leftarrow ^{2}F_{7/2})</td>
<td>980 nm</td>
</tr>
</tbody>
</table>

This study, which was the first example of CPL from a racemic 9-coordinate tris lanthanide complex, namely \([\text{Tb(DPA)}_{3}]^{3-}\), showed that an optically active species was indeed formed in aqueous solution, and was not labile on the time scale of Tb(III) emission from the \(^{5}D_{4}\) excited state (~4 msec). This result may be explained by the high stability of the complexes formed between Tb(III) ion and DPA ligands. The observation of a CPL signal from complexes of DPA with Eu(III), Dy(III), or Sm(III) has also been reported, while for the Pr(III) complex, the lack of sufficient CD at the excitation wavelength is probably responsible for the absence of any measurable CPL signal.

CPL following circularly polarized excitation has also been observed for lanthanide (III) complexes with chelidamic acid (CDA) (Metcalf et al., 1990b), and oxydiacetic acid (ODA) (Hilmes and Riehl, 1986) when the metal:ligand ratio was greater than 1:3, but not for complexes with iminodiacetic acid (IDA) (Hilmes and Riehl, 1986). By analogy with \([\text{Eu(DPA)}_{3}]^{3-}\), the chiroptical measurements from \([\text{Eu(CDA)}_{3}]^{6-}\) complexes demonstrated that this species also exists as a racemic mixture of structural enantiomers, \(\Delta\)-[Eu(CDA)]\(^{6-}\) and \(\Lambda\)-[Eu(CDA)]\(^{6-}\) in basic aqueous solution that do not racemize on the emission lifetime. The lack of measurable CPL in the case of the complexes with IDA may be explained by facial coordination of the ligands to the metal ion in the tri-capped trigonal prism geometry, resulting in achiral \(C_{3v}\) symmetry, instead of the meridional geometry observed for DPA and ODA that gives \(D_{3}\) symmetry. Although CPL has been observed for \([\text{Dy(ODA)}_{3}]^{3-}\), no CPL was detected in aqueous solution for ODA complexes with Eu(III), Tb(III), or Sm(III) even though these species are known to possess \(D_{3}\) symmetry in the solid state (Albertsson and Elding, 1977). These types of experiments are successful only if the differential excited state population is maintained during the excited state lifetime. Indeed, for the ODA complexes, Dy(III) has the shortest lived emitting state (approximately 0.07 ms in D\(_2\)O), while for the Sm(III) and Eu(III) compounds, the lifetime is longer by a factor of 2 and 14, respectively. Thus these results have been interpreted in terms of complete racemization of all of the complexes except \([\text{Dy(ODA)}_{3}]^{3-}\). It must be noted that it might be the case that the solution structure of the
Table 6
List of the racemic lanthanide (III) complexes for which CPL measurements have been reported, following circularly polarized excitation and/or perturbation of the ground state equilibrium by an added chiral agent (Pfeiffer effect)

<table>
<thead>
<tr>
<th>Racemic lanthanide complex</th>
<th>Sample description</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>DPA = dipicolinic acid</td>
<td>Rigid sol-gels</td>
<td>(Hilmes and Riehl, 1986) (Metcalf et al., 1990b) (Riehl, 1996)</td>
</tr>
<tr>
<td>([\text{Sm(DPA)}_3]^3-)</td>
<td>(\text{D}_2\text{O}) circularly polarized excitation</td>
<td>(Hilmes and Riehl, 1986)</td>
</tr>
<tr>
<td>([\text{Yb(DPA)}_3]^3-)</td>
<td>(\text{D}_2\text{O}) Pfeiffer effect</td>
<td>(Çoruh et al., 1988b) (Huskowska and Riehl, 1995) (Muller and Riehl, 2004)</td>
</tr>
<tr>
<td>([\text{Tb(DPA)}_3]^3-)</td>
<td>(\text{H}_2\text{O}, \text{D}_2\text{O}) circularly polarized excitation</td>
<td>(Hilmes and Riehl, 1986) (Hilmes et al., 1985) (Wu et al., 1989)</td>
</tr>
<tr>
<td>([\text{Dy(DPA)}_3]^3-)</td>
<td>(\text{H}_2\text{O}) Pfeiffer effect</td>
<td>(Hilmes and Riehl, 1986) (Hilmes et al., 1985) (Wu et al., 1989)</td>
</tr>
</tbody>
</table>

*continued on next page*
### Table 6, continued

<table>
<thead>
<tr>
<th>Racemic lanthanide complex</th>
<th>Sample description</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>[Eu(PEDPA)$_3$]$^{3-}$</td>
<td>PEDPA = 4-phenyl-ethyl-lyl-dipicolinic acid</td>
<td>H$_2$O, circularly polarized excitation</td>
</tr>
<tr>
<td>[Eu(CDA)$_3$]$^{6-}$</td>
<td>CDA = chelidamic acid</td>
<td>D$_2$O, circularly polarized excitation</td>
</tr>
<tr>
<td>[Dy(ODA)$_3$]$^{3-}$</td>
<td>ODA = oxydiacetic acid</td>
<td>D$_2$O, circularly polarized excitation</td>
</tr>
<tr>
<td>[Eu(ODA)$_3$]$^{3-}$</td>
<td>rigid sol-gels</td>
<td>circularly polarized excitation</td>
</tr>
<tr>
<td>[Tb(ODA)$_3$]$^{3-}$</td>
<td>H$_2$O, D$_2$O</td>
<td>Pfeiffer effect</td>
</tr>
<tr>
<td>[Dy(L$_1$)Cl$_2$]$^{-}$</td>
<td>L$_1$ = 6,6'-bis[bis(2-pyridylmethyl)amino-methyl]-2,2'-bipyridine</td>
<td>H$_2$O, circularly polarized excitation</td>
</tr>
<tr>
<td>[Eu(bpyO$_2$)$_3$]$^{3-}$</td>
<td>bpyO$_2$ = 2,2'-bipyridine-1,1'-dioxide</td>
<td>anhydrous MeCN, circularly polarized excitation</td>
</tr>
<tr>
<td>[Eu(ceDOTA)$^{-}$</td>
<td>ceDOTA = RRRR/SSSS-tetra(alpha-carboxyethyl)DOTA</td>
<td>D$_2$O, circularly polarized excitation</td>
</tr>
<tr>
<td>[Dy(N$_4$P$_4$Bz$_4$)$^{-}$</td>
<td>[Eu(N$_4$P$_4$Bz$_4$)$^{-}$, [Tb(N$_4$P$_4$Bz$_4$)$^{-}$</td>
<td>H$_2$O, circularly polarized excitation</td>
</tr>
<tr>
<td>[Eu(N$_4$P$_4$Me$_4$)$^{-}$, [Tb(N$_4$P$_4$Me$_4$)$^{-}$</td>
<td>[Eu(N$_4$P$_3$Me$_3$RM$_2$)$^{-}$, [Tb(N$_4$P$_3$Me$_3$RM$_2$)$^{-}$</td>
<td>[1,4,7,10-tetraazacyclododecane-1,4,7,10-tetrayltetraakis(methylene)]tetraakis[a-b]-phosphinic acid</td>
</tr>
<tr>
<td>[Eu(TTHA)$^{3-}$</td>
<td>TTHA = triethylenetetraminehexaaacetic acid</td>
<td>D$_2$O, circularly polarized excitation</td>
</tr>
</tbody>
</table>
Another recent study involving the use of circularly polarized excitation is for the compound $[\text{Eu}(L^1)\text{Cl}_2]\text{Cl}$ with the ligand 6,6′-bis[bis(2-pyridylmethyl)amino-methyl]-2,2′-bipyridine (Dossing et al., 2002; Gawryszewska et al., manuscript in preparation). Luminescence measurements have shown the existence of two forms in aqueous and MeOH solutions, where one of them ($[\text{Eu}(L^1)\text{Cl}_2]^{2+}$) appears to involve exchange of a Cl$^{-}$ ion by a solvent molecule in the form $[\text{Eu}(L^1)\text{Cl}_2]^{+}$. To confirm the presence of a racemic mixture, use of the Eu(III) complex was not possible, because the species presumably racemizes during the Eu(III) emitting state lifetime. By looking at the complex with the Dy(III) ion, which has a shorter lifetime than the related Eu(III) compound, it was possible to record a CPL spectrum (fig. 6). Variation of the excitation polarization from right to left circular polarization produced an oppositely signed CPL spectrum of about equal magnitude as expected.

This technique has also been used to study a number of racemic lanthanide complexes with approximate C$_4$ or D$_2$ symmetries in solution. The ligand 2,2′-bipyridine-1,1′-dioxide (bpyO$_2$) (Scheme 2) forms 8-coordinate tetrakis lanthanide complexes in anhydrous acetonitrile (Huskowska and Riehl, 2000; Huskowska et al., 2002). Based on high-resolution luminescence studies and X-ray diffraction analysis, the solution structure of $[\text{Eu}(\text{bpyO}_2)_4](\text{ClO}_4)_3$ is probably distorted from the D$_4$ symmetry observed for the La(III) analog in the solid state and, thus, has an overall geometry of D$_2$. This symmetry also results in a system composed of enantiomers, entirely consistent with the CPL results given in fig. 7.

Lanthanide complexes with macrocyclic ligands such as those based on 1,4,7,10-tetraazacyclododecane or polyaminocarboxylates have been studied extensively by CPL spectroscopy. These complexes often possess large stability constants in aqueous solution and, moreover, present interesting chiroptical properties. There have been a number of reports of CPL
Scheme 2.

Fig. 6. Circularly polarized luminescence (upper curve) and total luminescence (lower curve) spectra for the $^6H_{13/2} \leftrightarrow ^4F_9/2$ transition of a 0.001 M aqueous solution of [Dy(L1)Cl2]$^+$ at pH = 7, following left and right circularly polarized 476-nm excitation (Gawryszewska et al., manuscript in preparation).

Fig. 7. Circularly polarized luminescence (upper curve) and total luminescence (lower curve) spectra for the $^7F_1 \leftrightarrow ^5D_0$ (left) and $^7F_2 \leftrightarrow ^5D_0$ (right) transitions of [Eu(bpyO2)$_4$]$^{3+}$ in MeCN at 295 K, following left circularly polarized 553.7-nm excitation (Huskowska and Riehl, 2000).
Following circularly polarized excitation of racemic complexes involving DOTA-type ligands (DOTA = 1,4,7,10-tetraazacyclododecane-1,4,7,10-tetraaceticacid), although no CPL has been detected from complexes of lanthanides with DOTA itself in aqueous solution presumably due to rapid racemization. As depicted in Fig. 8, upon complexation with a lanthanide ion, square anti-prism (SAP) or twisted square antiprism (TSAP) structures are formed with a vacant coordination site in the “cap” position which is typically occupied by a solvent molecule. In these structures two distinct type of chiral stereochemistry are present. In analogy with D₃ or C₃ symmetry species, the sense of rotation of the pendant “arms” is denoted Λ or Δ depending upon if the arms rotate clockwise (Δ) or counterclockwise (Λ) as one proceeds down the direction of the C₄ axis. There is also chirality (or helicity) associated with
the non-planar 12 member ring. If one looks along the line connecting the coordinated nitrogens, the carbon atoms either have a right-handed (+ NCCN torsion angle) ($\delta$) or left handed ($-\text{NCCN torsion angle}$) ($\lambda$) twist. Since in this ligand there are four connecting segments, the full description of the molecular stereochemistry must include the overall helicity of the arms, and the helicity of the four segments. Since for this ligand all the segments are connected, however, one finds only $\delta\delta\delta\delta$ or $\lambda\lambda\lambda\lambda$ conformations. The ring conformations are further coupled to the arm configuration, so that the $\Delta(\delta\delta\delta\delta)$ and $\Lambda(\lambda\lambda\lambda\lambda)$ enantiomers of $[\text{Eu(DOTA)}]$ have a TSAP structure, whereas the $\Delta(\lambda\lambda\lambda\lambda)$ and $\Lambda(\delta\delta\delta\delta)$ enantiomers have a SAP geometry (Dickins et al., 1998a).

As shown in fig. 8, complexes with these DOTA-type ligands racemize by inverting both the helicity of the pendant arms as well as the helicity of the ring conformation. We also show in this figure the equilibrium between the SAP and TSAP structures. It has been shown that when the pendant arms attached to the ring nitrogens are not $\alpha$-substituted, arm rotation can lead to interconversion of TSAP (twisted square anti-prism) and SAP (square anti-prism) isomers, as well as a change in absolute configuration ($\Delta \leftrightarrow \Lambda$). However, for the complex formed between Eu(III) and $RRRR/SSSS$-tetra($\alpha$-carboxyethyl)DOTA ([Eu(ceDOTA)]$^-$), and for other structurally related complexes, the only process that effectively interconverts these two isomers at ambient temperature is cyclen ring inversion as illustrated schematically in fig. 9 (Muller et al., 2002b). Verification that the solution prepared from this ligand was indeed a racemic mixture, was made by measuring the circularly polarized luminescence following circularly polarized excitation. An exactly opposite value was obtained when the excitation beam was converted from left to right circular polarization. This confirms that the structure in solution is chiral and does not racemize completely on the luminescence time-scale. The same technique has been used to examine a series of phosphinate macrocyclic complexes of Eu(III), Tb(III), and Dy(III) with octadentate ligands derived from the 1,4,7,10-tetraazacyclododecane ring: $N_4P_4Bz_4$, $N_4P_4Me_4$ and $N_4P_3Me_3RMe_2$ (Scheme 3) (Huskowska et al., 1997; Luck et al., 2001).

It has generally been observed that the overall chirality of the electronic transitions from these DOTA-type lanthanide complexes appears to be quite sensitive to the square antiprism

![Fig. 9. Pictorial representation of the possible ring inversion that interconverts the two isomers of the complex [Eu(ceDOTA)]$^-$](image-url)
twist angle associated with the coordinated atoms, and relatively insensitive to the helicity of the ring (Bruce et al., 2002, 2001). Some generalizations concerning the averaged solution structure of these complexes may be possible from the magnitude of the observed effect. Using a simple model, it has been predicted that a twist angle of 22.5° would yield the largest $g_{\text{lum}}$ (Bruce et al., 2002). Examination of the sharp crystal field splitting and large $g_{\text{lum}}$ for the Tb(III) complexes of the three DOTA-phosphinates described above, suggested that the overall average chiral structure of these three complexes were very similar. The CPL spectra of the related compounds with the Eu(III) ion had quite small values for $g_{\text{lum}}$ (fig. 10), even though the excitation wavelength corresponded to a magnetic dipole allowed transition, whereas the excitation of the Tb(III) complexes did not (Huskowska et al., 1997). These important variations suggested that there were differences between the emitting state structures of the Eu(III) and Tb(III) complexes. For comparison, the opposite trend was observed for the $g_{\text{lum}}$ values when studying the same transitions on the Tb(III) and Eu(III) complexes with the DPA ligand (Hilmes and Riehl, 1986; Hilmes et al., 1985), consistent with the selection rules for the excitation transition used.
Circularly polarized luminescence spectroscopy has also been a useful technique for studying solution complexes of lanthanides with achiral ligands of low symmetry. For example, CPL was observed from solutions of Eu(III) with triethylenetetraminehexaacetic acid (TTHA) following circularly polarized excitation (Mondry et al., 1994). The spectroscopic studies of 1:1 solutions of Eu(III) with TTHA, corroborated by the X-ray diffraction analysis of the Gd(III) complex and temperature and pressure dependence of the $^7F_0 \rightarrow ^5D_0$ excitation spectrum, clearly showed the presence of two distinct species corresponding to the dimer and monomer complexes (Maupin et al., 2001). Presumably, each of these species exists in solution as a racemic mixture. In the dimer species, the inner coordination sphere contains three nitrogen and six oxygen donors from TTHA (Scheme 4), while in the monomer there are four nitrogen and five oxygen donors.

**Scheme 4.**

**4.1.1.2. Racemization kinetics from CPL measurements.** Experiments concerned with the enantiomer interconversion processes were carried out on the [Eu(TTHA)]$^{2-}$ compound in neutral aqueous solution (Mondry et al., 1994). It has been demonstrated that the CPL spectrum of this system shows only a very slight temperature dependence from 283 to 353 K,
indicating that the complex does not undergo significant rearrangement that would lead to racemization in this temperature range. In these experiments it is assumed that the excited state lifetime is independent of temperature, and any decrease in $g_{\text{lum}}$ is due to the increased racemization at elevated temperatures. The interpretation of these experiments is somewhat complicated by the fact that there is an equilibrium between two species that also changes with temperature (Mondry et al., 1994). By comparison, $g_{\text{lum}}$ values measured for $[\text{Eu(DPA)}_3]^{3-}$ in aqueous solution decreased by a factor of 4 or 5 when the temperature was increased from 293 to 353 K (Metcalf et al., 1990c). The estimated value of the activation energy for enantiomeric conversion for $[\text{Eu(DPA)}_3]^{3-}$ was $E_a = 48.9$ kJ/mol using a simple Arrhenius analysis for $k_{\text{rac}}$ determined in the steady state mode (eq. (31)) (Belair et al., 2000), or 50.7 kJ/mol derived from $k_{\text{rac}}$ determined by time-resolved CPL studies (Glover-Fischer et al., 1998). This is about two times lower than the one corresponding value obtained for the complex with TTHA, $E_a = 96.1$ kJ/mol in D$_2$O (Mondry et al., 1994).

It should be mentioned that these types of measurements are only possible if the ligand-solvent or bound ligand-free ligand exchange processes are slow compared to optical enantiomer interconversions, and the complex remains intact on the time scale of these possible stereochemical transformations. The excited-state racemization kinetic may also be influenced by the nature of the solvent, as observed for $[\text{Eu(DPA)}_3]^{3-}$ (Metcalf et al., 1990c) and $[\text{Eu(CDA)}_3]^{6-}$ (Metcalf et al., 1990b). The racemization rates are respectively 1.44 and 4.70 times faster in H$_2$O than D$_2$O at 293 K, indicating relatively important complex-solvent interactions attributable to solvent mass and hydrogen-bonding capabilities. This solvent dependence is greater for $[\text{Eu(CDA)}_3]^{6-}$ than for $[\text{Eu(DPA)}_3]^{3-}$ reflecting stronger complex-solvent interactions via the 4-oxo substituent on each CDA ligand. Further studies have been carried out in ethylene glycol-H$_2$O mixture and in non-aqueous solvents such as methanol, ethanol, and acetonitrile (Glover-Fischer et al., 1998). The slower racemization rate and higher activation energy observed for $[\text{Eu(DPA)}_3]^{3-}$ in ethylene glycol-H$_2$O solution than in H$_2$O or D$_2$O was attributed to the higher viscosity of the ethylene glycol-H$_2$O solvent mixture, while the experiments performed in non-aqueous solvent solutions show no evidence of enantiomer interconversion processes over time periods as long as 6 ms, at temperature up to 313–333 K.

4.1.1.3. CPL measurements in rigid sol-gels. In order to measure CPL from racemic mixtures without having to be concerned with competition from chemical racemization, a number of studies have been reported in which the racemic lanthanide complexes have been imbedded in rigid sol-gels (Sokolnicki et al., 2002; Huskowska et al., 2000). In this work it has been shown that at low concentrations in the sol-gel, the CPL results obtained for $[\text{Eu(DPA)}_3]^{3-}$ are approximately the same as obtained in aqueous solution at room temperature, consistent with the observation that under these conditions racemization may be neglected. A somewhat surprising result was that at higher concentrations a reproducible decrease in $g_{\text{lum}}$ was observed. This concentration dependent effect was ascribed to racemization due to excited state energy transfer. A theoretical description of this phenomena is presented in sect. 2.6.2. A resonance energy transfer occurring between opposite enantiomer will result in a decrease in $g_{\text{lum}}$, while a transfer between identical enantiomers will not have any effect on the CPL signal. Results have also been reported for the concentration dependence of $g_{\text{lum}}$ for $[\text{Eu(bpyO}_2)_4]^{3+}$,
and a energy transfer model based on a diagrammatic Green’s function approach (Gochanour and Fayer, 1981) has been used to estimate Förster critical transfer distances, \( R_0 \), from the concentration dependence (Sokolnicki et al., 2002). A large value of the critical transfer distance \( (R_0 = 69 \text{ Å}) \), was determined for \([\text{Eu(bpyO}_2\text{)}_4]^{3+}\), and a somewhat more reasonable value of 24 Å was determined for \([\text{Eu(DPA)}_3]^{3–}\). These results were obtained under the assumptions that the complexes remain intact in the sol-gel, and are evenly distributed through the matrix. There is evidence for some structural modifications, and for some clustering of complexes in sol-gel cavities that would impact the results reported above.

In order to verify the suggestion that no CPL was detected for \([\text{Eu(ODA)}_3]^{3–}\) in solution because of racemization during the emission lifetime, this complex was also prepared and studied in a sol-gel. Small, oppositely signed \( g_{\text{lum}} \) results were obtained (-0.0063 and +0.0043 at 592.6 nm) following left and right circularly polarized excitation (Huskowska et al., 2000), but a concentration dependent study was not performed. It should be noted that preparation of strain-free sol-gels of sufficient quality for these types of polarization measurements are difficult, and one can not completely eliminate effects due to sample alignment, orientation, or depolarization of the circularly polarized excitation and emission.

4.1.2. CPL from racemic mixtures: differential excited-state quenching

As described in sect. 2.6.3, there have been a number of recent studies concerned with interconverting racemic mixtures in which a non-racemic excited state has been generated by differential quenching of one enantiomer over the other by an added chiral quencher. This has been commonly referred to enantioselective quenching, although the discriminatory interactions leading to the difference in excited state populations are properly described as diastereomeric interactions. A summary of the enantioselective quenching systems is given in table 7. In these experiments, optically active quenchers such as transition-metal complexes (Metcalf et al., 1990b; Morita et al., 2000; Maupin et al., 1996, 1998a; Glover-Fischer et al., 1995; Bolender et al., 1994, 1993; Metcalf et al., 1993; Rexwinkel et al., 1992a, 1992b, 1993a, 1993b; Glover et al., 1992; Çoruh et al., 1991; Richardson et al., 1991; Metcalf et al., 1990a, 1989; Wu et al., 1991), transition metal-nucleotide complexes (Stockman et al., 1996; Metcalf et al., 1992), metalloproteins (Meskers and Dekkers, 2001; Meskers et al., 1998a, 1996, 1998b; Meskers and Dekkers, 1998a), vitamin B12 derivatives (Meskers and Dekkers, 1998b, 1999a, 1999b, 2001), organic dye molecules (Meskers and Dekkers, 1997) or dicopper trefoil knots (Meskers et al., 2000) have been used. Almost all of the studies have used the tris complexes of lanthanide(III) with DPA and CDA ligands. These complexes are known to be quite emissive and have luminescent transitions with large \( g_{\text{lum}} \) values, and, therefore, emit light with a high degree of circular polarization. This makes it possible to detect even small enantiomeric excesses in the excited state.

The first quencher used for this purpose was the resolved tris(1,10-phenanthroline)ruthenium(II)complex, \([\text{Ru(phen)}_3]^{2+}\) (Metcalf et al., 1990a, 1989). The enantioselective excited-state quenching of \([\text{Tb(DPA)}_3]^{3–}\), \([\text{Eu(DPA)}_3]^{3–}\), and \([\text{Dy(DPA)}_3]^{3–}\) by resolved-[Ru(phen)]_3^{2+} has provided important information regarding the structure and excited state energetics of the racemic donor and chiral acceptor molecules (e.g., size, shape, electrostatic...
Table 7
List of the enantioselective quenching systems used

<table>
<thead>
<tr>
<th>Enantioselective quenching system</th>
<th>Racemic system</th>
<th>Solvent</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. $\Lambda$-($+$)-Co(en)$_3^{3+}$</td>
<td>$[\text{Eu(CDA)}]_3$</td>
<td>H$_2$O, D$_2$O</td>
<td>(Metcalf et al., 1990b)</td>
</tr>
<tr>
<td>en = ethylenediamine = 1,2-diaminoethane</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b. $\Lambda$-($-$)-Co(en)$_3^{3+}$</td>
<td>$[\text{Eu(CDA)}]_3$</td>
<td>H$_2$O</td>
<td>(Metcalf et al., 1993)</td>
</tr>
<tr>
<td>c. $\Delta$-Co(en)$_2$($R,R$-chxn)$_3^{3+}$</td>
<td>$[\text{Tb(CDA)}]_3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R,R$-chxn = trans-1$R,2R$-diaminocyclohexane</td>
<td>$[\text{Eu(DPA)}]_3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d. $\Delta$-Co(en)$_2$($R,R$-chxn)$_3^{3+}$</td>
<td>$[\text{Tb(CDA)}]_3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>e. $\Delta$-Co($R,R$-chxn)$_3^{3+}$</td>
<td>$[\text{Eu(DPA)}]_3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>f. $\Lambda$-Co(en)$_2$(bpy)$_3^{3+}$</td>
<td>$[\text{Tb(DPA)}]_3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>bpy = 2,2$'$-bipyridine</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>g. $\Lambda$-Co(en)$_2$(phen)$_3^{3+}$</td>
<td>$[\text{Eu(DPA)}]_3$</td>
<td>H$_2$O, D$_2$O</td>
<td>(Metcalf et al., 1990a)</td>
</tr>
<tr>
<td>phen = 1,10-phenanthroline</td>
<td>$[\text{Tb(DPA)}]_3$</td>
<td></td>
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</tr>
<tr>
<td>h. $\Lambda$-Co(phen)$_3^{3+}$</td>
<td>$[\text{Eu(DPA)}]_3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>i. $\Delta$-Co(phen)$_3^{3+}$</td>
<td>$[\text{Eu(DPA)}]_3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>j. $\Delta$($\lambda\lambda\lambda$)-Co($R,R$-chxn)$_3^{3+}$</td>
<td>$[\text{Eu(DPA)}]_3$</td>
<td>H$_2$O, D$_2$O, Glycol</td>
<td>(Glover-Fischer et al., 1995)</td>
</tr>
<tr>
<td>k. $\Delta$($\delta\delta\delta$)-Co($R,R$-chxn)$_3^{3+}$</td>
<td>$[\text{Eu(DPA)}]_3$</td>
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<td></td>
</tr>
<tr>
<td>l. $\Delta$($\lambda\lambda\lambda$)-Co($R,R$-chxn)$_3^{3+}$</td>
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<td>m. $\Delta$($\delta\delta\delta$)-Co($R,R$-chxn)$_3^{3+}$</td>
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<tr>
<td>n. $\Delta$($\lambda\lambda\lambda$)-Co($R,R$-chxn)$_3^{3+}$</td>
<td>$[\text{Eu(DPA)}]_3$</td>
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<tr>
<td>mc = mono-capped</td>
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<tr>
<td>o. $\Delta$($\lambda\lambda\lambda$)-Co($R,R$-chxn)$_3^{3+}$</td>
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<tr>
<td>bc = bi-capped</td>
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<tr>
<td>Enantioselective quenching system</td>
<td>Racemic system</td>
<td>Solvent</td>
<td>Reference</td>
</tr>
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<tr>
<td>p. $A(\pm)-Ru(phen)_3^{2+}$</td>
<td>([\text{Tb(DPA)}_3]^{3-}[q])</td>
<td>H$_2$O, MeOH</td>
<td>(Maupin et al., 1998a)</td>
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<td>q. $A(-)-Ru(phen)_3^{2+}$</td>
<td>([\text{Eu(DPA)}_3]^{3-}[p, q, r])</td>
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<td>(Maupin et al., 1996)</td>
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<tr>
<td>r. $A(\pm)-Ru(bpy)_3^{2+}$</td>
<td>([\text{Tb(DPA)}_3]^{3-}[p, q, r])</td>
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<td>(Metcalf et al., 1990a)</td>
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<td>([\text{Tb(DPA)}_3]^{3-}[p, q) H$_2$O, D$_2$O, MeOH</td>
<td>(Rexwinkel et al., 1993a)</td>
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<tr>
<td></td>
<td>([\text{Tb(DPA)}_3]^{3-}[p, q) H$_2$O, D$_2$O, MeOH</td>
<td>(Rexwinkel et al., 1992a, 1992b, 1993b)</td>
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<td>t. $\Delta(a)$- and $\Delta(e)$-Cr(H$_2$O)$_4$(ATP)</td>
<td>([\text{Eu(DPA)}_3]^{3-}[t, u, v])</td>
<td>H$_2$O</td>
<td>(Çoruh et al., 1991)</td>
</tr>
<tr>
<td>u. $\Delta(a)$-Cr(H$_2$O)$_4$(ATP)</td>
<td>([\text{Tb(DPA)}_3]^{3-}[t, v])</td>
<td>H$_2$O/MeOH</td>
<td>(Metcalf et al., 1989)</td>
</tr>
<tr>
<td>v. $\Delta(e)$-Cr(H$_2$O)$_4$(ATP)</td>
<td>([\text{Eu(DPA)}_3]^{3-}[p, q) H$_2$O</td>
<td>(Meskers and Dekkers, 1997)</td>
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<td>$\Delta$- and $A$-endo-Rh(H$_2$O)$_3$(ATP)</td>
<td>([\text{Tb(DPA)}_3]^{3-})</td>
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<td>(Wu et al., 1991)</td>
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<td>H$_2$O</td>
<td>(Stockman et al., 1996)</td>
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<tr>
<td>$\Delta$- and $A-(a/e)$-Rh(H$_2$O)$_4$(ATP)</td>
<td>([\text{Eu(DPA)}_3]^{3-})</td>
<td>H$_2$O</td>
<td>(Stockman et al., 1996)</td>
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*$\Delta$ = pseudo-axial orientation
* $\Delta$ = pseudo-equatorial orientation

* ATP = adenosine triphosphate

* continued on next page
Table 7, continued

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<tr>
<th>Enantiomeric quenching system</th>
<th>Racemic system</th>
<th>Solvent</th>
<th>Reference</th>
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<tr>
<td>Co(NH$_3$)$_4$(ntp)</td>
<td>[Eu(DPA)$_3$]$^{3-}$</td>
<td>D$_2$O</td>
<td>(Metcalf et al., 1992)</td>
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<tr>
<td>ntp = triphosphate nucleotide</td>
<td>[Tb(DPA)$_3$]$^{3-}$</td>
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<tr>
<td>Co(NH$_3$)$_4$(ndp)</td>
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<td>ndp = diphosphate nucleotide</td>
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<tr>
<td>Metalloproteins:</td>
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<td></td>
</tr>
<tr>
<td>cytochrome c, cytochrome c-550, azurin, amicyanin, plastocyanin</td>
<td>[Eu(DPA)$_3$]$^{3-}$</td>
<td>H$_2$O</td>
<td>(Meskers and Dekkers, 2001)</td>
</tr>
<tr>
<td></td>
<td>[Tb(DPA)$_3$]$^{3-}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M44K azurin, wt azurin, amicyanin, plastocyanin</td>
<td></td>
<td></td>
<td>(Meskers et al., 1998a)</td>
</tr>
<tr>
<td>cytochrome c, wt cytochrome c-550</td>
<td></td>
<td></td>
<td>(Meskers et al., 1998b)</td>
</tr>
<tr>
<td>Vitamin B$_{12}$ and derivatives</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>cyanocobalamin = vitamin B$<em>{12}$ = CNCbl, aquacobalamin = vitamin B$</em>{12a}$ = H$_2$OCbl$^+$, hydroxocobalamin = HOCbl, dicyanocobinamide = (CN)$_2$Cbi, heptamethyl ester of dicyanocobyrinic acid = (CN)$_2$Cby(OMe)$_7$</td>
<td>[Eu(DPA)$_3$]$^{3-}$</td>
<td>H$_2$O</td>
<td>(Meskers and Dekkers, 2001)</td>
</tr>
<tr>
<td></td>
<td>[Tb(DPA)$_3$]$^{3-}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>cyanocobalamin = vitamin B$<em>{12}$ = CNCbl, aquacobalamin = vitamin B$</em>{12a}$ = H$_2$OCbl$^+$, dicyanocobinamide = (CN)$_2$Cbi</td>
<td></td>
<td></td>
<td>(Meskers and Dekkers, 1999a)</td>
</tr>
<tr>
<td></td>
<td>[La(DPA)$_3$]$^{3-}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[Tb(DPA)$_3$]$^{3-}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[Nd(DPA)$_3$]$^{3-}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[Gd(DPA)$_3$]$^{3-}$</td>
<td></td>
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</table>
Table 7, continued

<table>
<thead>
<tr>
<th>Enantioselective quenching system</th>
<th>Racemic system</th>
<th>Solvent</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>cyanocobalamin = vitamin B$<em>{12}$ = CNCbl, aquacobalamin = vitamin B$</em>{12a}$ = H$_2$OCbl$^+$</td>
<td>[Tb(DPA)$_3$]$^{3-}$</td>
<td></td>
<td>(Meskers and Dekkers, 1998b)</td>
</tr>
<tr>
<td>Rhodamine 6G, Rhodamine B, Uro, MeUro (+) and (−)-K-2Cu-2PF$_6$</td>
<td>[Tb(DPA)$_3$]$^{3-}$</td>
<td>MeOH</td>
<td>(Meskers and Dekkers, 1997)</td>
</tr>
<tr>
<td></td>
<td>[Eu(DPA)$_3$]$^{3-}$</td>
<td>H$_2$O/MeCN</td>
<td>(Meskers et al., 2000)</td>
</tr>
<tr>
<td>Λ(-)(+)-Co(en)$_3$$^{3+}$</td>
<td>[Tb(d-EDDS)]$^{-}$</td>
<td>H$_2$O</td>
<td>(Morita et al., 2000)</td>
</tr>
<tr>
<td>l-EDDS = ($S,S'$)-ethylenediamine-N,N$'$-disuccinic acid</td>
<td>[Tb(l-EDDS)]$^{-}$</td>
<td>measured separately</td>
<td></td>
</tr>
<tr>
<td>d-EDDS = ($R,R'$)-ethylenediamine-N,N$'$-disuccinic</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Note:** The racemic system refers to the mixture of enantiomers, while the enantioselective system specifically targets one enantiomer.
charge distributions, stereochemical dynamics, or electronic state structures) as well as the stability of the diastereomeric encounter complexes that are formed, and other structural and energetic aspects of the energy transfer process. Studies have been carried out on solution samples in which the kinetics of the quenching reactions have been examined under variable temperature, pressure, ionic strength, solvent type, and solution viscosity conditions (Metcalf et al., 1990c; Glover-Fischer et al., 1998; Maupin et al., 1998a, 1996; Glover-Fischer et al., 1995; Rexwinkel et al., 1992a, 1992b, 1993a, 1993b; Çoruh et al., 1991). All those experiments have been very helpful in developing a better understanding of the characteristics of the observed chiral recognition/discrimination.

One of the more interesting results from these studies is the solvent dependence seen in the excited-state quenching of lanthanide (III) DPA complexes by [Ru(phen)3]2+. Since in this case both donor and acceptor have D3 symmetry, we can properly label the mutual interactions as being heterochiral (ΔΛ or ΛΔ) or homochiral (ΔΔ or ΛΛ). For this system it has been shown that heterochiral quenching is more efficient in methanol solution, whereas in water solution, homochiral quenching is the faster process (Rexwinkel et al., 1992b, 1993a). Analysis of the temperature dependence of the quenching kinetics in the two solvents has demonstrated that the competition between enthalpy and entropy effects is the driving force of the chiral discriminatory interactions. From all of these data, it was concluded that differential entropy effects control the overall quenching kinetics in methanol and favor heterochiral versus homochiral quenching. Differential enthalpy controls the quenching kinetics in aqueous solutions and favors heterochiral quenching. It is perhaps important to note that changing of the sign of g_lum from positive to negative by switching achiral solvents does not violate any symmetry considerations, since as stated above, the fundamental interactions are diastereomeric in nature and not enantiomeric. Comparative studies of enantioselective quenching in water and methanol solutions of the [Tb(DPA)3]3−/[Ru(phen)3]2+ system under variable pressure conditions have also been performed (Maupin et al., 1998a, 1996). It was observed that the rate of this enantioselective quenching process increased with pressure when the reaction was carried out in water, but decreased with pressure in methanol. The origin of the observed differences between the quenching in methanol versus aqueous solution has been attributed to different structural aspects of the solvated diastereomeric encounter pairs formed in these two solvents. A negative volume of activation in water is consistent with a close approach of donor:acceptor species without much solvent rearrangements. A positive volume of activation in methanol has been interpreted as being due to methanol molecules having to leave the interstitial spaces surrounding the [Ru(phen)3]2+ before the donor can attain the close distance necessary for energy transfer to occur.

In addition to resolved [Ru(phen)3]2+ complex, other enantiomerically pure transition-metal complexes have been examined as quenchers for either [Ln(DPA)3]3− or [Ln(CDA)3]6− (Ln = Eu(III) or Tb(III)) in aqueous solution. These include resolved optical isomers of [Co(en)3]3+, [Co(en)2(R,R-chxn)]3+, [Co(en)(R,R-chxn)2]3+, or [Co(R,R-chxn)3]3+ (where en = ethylenediamine and R,R-chxn = trans-1R,2R-diaminocyclohexane) (Glover-Fischer et al., 1995; Metcalf et al., 1993). These studies have shown that the structural changes as well as the additional number of chiral centers in the coordinated ligands, brought by the substitution of R,R-chxn for en ligands had considerable effect on the degree and sense of chi-
Table 8

<table>
<thead>
<tr>
<th>Racemic lanthanide complexes</th>
<th>Resolved quenchers</th>
<th>( k_q^- / k_q^+ )</th>
</tr>
</thead>
<tbody>
<tr>
<td>([\text{Eu(DPA)}_3]^{3-})</td>
<td>(\Lambda - [\text{Co}(en)_3]^{3+}) en = ethylenediamine</td>
<td>0.78</td>
</tr>
<tr>
<td>DPA = dipicolinic acid</td>
<td>(\Lambda - [\text{Co}(en)_2(R,R\text{-chxn})]^{3+}) (R,R\text{-chxn} = \text{trans-}1R,2R)-diaminocyclohexane</td>
<td>1.67</td>
</tr>
<tr>
<td></td>
<td>(\Lambda - [\text{Co}(en)(R,R\text{-chxn})_2]^{3+})</td>
<td>3.02</td>
</tr>
<tr>
<td></td>
<td>(\Lambda - [\text{Co}(R,R\text{-chxn})_3]^{3+})</td>
<td>4.01</td>
</tr>
<tr>
<td>([\text{Eu(CDA)}_3]^{6-})</td>
<td>(\Lambda - [\text{Co}(en)_3]^{3+})</td>
<td>0.75</td>
</tr>
<tr>
<td>CDA = chelidamic acid</td>
<td>(\Lambda - [\text{Co}(en)_2(R,R\text{-chxn})]^{3+})</td>
<td>1.33</td>
</tr>
<tr>
<td></td>
<td>(\Lambda - [\text{Co}(en)(R,R\text{-chxn})_2]^{3+})</td>
<td>2.01</td>
</tr>
<tr>
<td></td>
<td>(\Lambda - [\text{Co}(R,R\text{-chxn})_3]^{3+})</td>
<td>2.77</td>
</tr>
<tr>
<td>([\text{Tb(DPA)}_3]^{3-})</td>
<td>(\Lambda - [\text{Co}(en)_3]^{3+})</td>
<td>0.87</td>
</tr>
<tr>
<td></td>
<td>(\Lambda - [\text{Co}(en)_2(R,R\text{-chxn})]^{3+})</td>
<td>1.58</td>
</tr>
<tr>
<td></td>
<td>(\Lambda - [\text{Co}(en)(R,R\text{-chxn})_2]^{3+})</td>
<td>2.57</td>
</tr>
<tr>
<td></td>
<td>(\Lambda - [\text{Co}(R,R\text{-chxn})_3]^{3+})</td>
<td>3.40</td>
</tr>
<tr>
<td>([\text{Tb(CDA)}_3]^{6-})</td>
<td>(\Lambda - [\text{Co}(en)_3]^{3+})</td>
<td>0.71</td>
</tr>
<tr>
<td></td>
<td>(\Lambda - [\text{Co}(en)_2(R,R\text{-chxn})]^{3+})</td>
<td>1.42</td>
</tr>
<tr>
<td></td>
<td>(\Lambda - [\text{Co}(en)(R,R\text{-chxn})_2]^{3+})</td>
<td>2.00</td>
</tr>
<tr>
<td></td>
<td>(\Lambda - [\text{Co}(R,R\text{-chxn})_3]^{3+})</td>
<td>3.21</td>
</tr>
</tbody>
</table>

*From references (Glover-Fischer et al., 1995) and (Metcalf et al., 1993).\n
*\(k_q^-\) and \(k_q^+\) denote the homochiral and heterochiral quenching constants, respectively.

CIRCULARLY POLARIZED LUMINESCENCE SPECTROSCOPY

Among the search of new chiral quenchers for the enantioselective quenching of \([\text{Ln(DPA)}_3]^{3-}\), the use of vitamin B\(_{12}\) derivatives (Meskers and Dekkers, 1999a, 1999b) has shown that the intermolecular forces contribute to the stability of the encounter complexes and, also, that the H-bonding interactions between the carboxylate groups of the DPA ligand and the amide hydrogens of the corrinoid play an important role, as suggested by NMR and CD studies. On the other hand, the quenching of \([\text{Ln(DPA)}_3]^{3-}\) by biomolecules such as vitamin B\(_{12}\) derivatives or metalloproteins (c-type cytochrome (Meskers and Dekkers, 1998a; Meskers et al., 1998b, 1996)) can be used to probe structural aspects of complexation reactions of corrinoids with proteins or, in the near future, to study enantioselectivity in the luminescence of metal containing membrane proteins (Meskers and Dekkers, 2001).
4.1.3. CPL from racemic mixtures: perturbation of the equilibrium

In the two previous subsections, we have described experiments involving CPL from racemic solutions in which either a non-racemic emitting state is prepared by circularly polarized excitation or a time-dependent optically enriched excited state is generated through enantioselective quenching by an added chiral species. A third possibility is the preparation of a non-racemic emitting state by disturbance of the ground state racemic equilibrium by an added chiral substance, a phenomenon which has been called the Pfeiffer effect. The first examples of the measurement of the CPL from perturbed racemic solution involving a lanthanide complex have been reviewed by Brittain (1989). Brittain and coworkers showed that a variety of chiral species such as tartrate substrates, amino acids, or sugar derivatives, may cause a perturbation of the D₃ equilibrium between Λ- and Λ-[Tb(DPA)₃]³⁻. Additional experiments (Hilmes and Riehl, 1986; Huskowska and Riehl, 1995; Wu et al., 1989; Çoruh et al., 1988b; Hilmes et al., 1988) have shown that the nature of this equilibrium shift was an outer-sphere interaction between the racemic lanthanide complex and the added chiral agent. It is important in this type of experiment to compare the CPL spectra from the Pfeiffer-perturbed system with the “pure” enantiomer generated in excess by a circularly polarized excitation beam. If the total emission and CPL lineshape are identical as shown in fig. 11, then one can assume that the structure of the emitting species is the same in the racemic mixture and the Pfeiffer-active systems. These types of measurements have been extended to Eu(III), Dy(III), and Yb(III) complexes of DPA (Çoruh et al., 1988b; Huskowska and Riehl, 1995; Hilmes et al., 1988; Maupin et al., 2000).

The experiments described above have led to the conclusion that the chiral-induced equilibrium shift could be induced in the lanthanide (III) complex by a combination of electrostatic and hydrophobic interactions. Hydrogen bonding effects appear to be less important as suggested by experiments carried out under variable pH, temperature, concentration, solvent type, and solution dielectric constant conditions. By analogy to the associated/dissociated equilibrium shift models of Schipper (1978) in which the source of the equilibrium perturbation is attributed to a free energy of mixing, Brittain (1981) and Wu et al. (1989) attempted to ascertain the complex mechanism responsible for this type of Pfeiffer effect. However, their conclusions were opposite as they concluded that the [Tb(DPA)₃]³⁻ complex interacted with the

![Fig. 11.](image_url)
chiral probe, L-histidine, in the associated and in the dissociated models, respectively. An explanation of this difference of behavior might be due to the experimental conditions concerning the dependence of $g_{\text{lum}}$ on the ratio of chiral substrate ($N_p$) to complex ($N$). Indeed, it has recently been shown by CD or CPL spectroscopy that these chiral discrimination interactions involving the $[\text{Tb(ODA)}_3]^{3-}/\text{L-proline}$ (Parac-Vogt et al., 2002) or $[\text{Tb(DPA)}_3]^{3-}/\text{L-proline}$ benzyl ester (Muller and Riehl, 2004) system exhibit the behavior predicted by the associated model. Compared to the $[\text{Tb(DPA)}_3]^{3-}/\text{L-histidine}$ system where the ratio of $N_p/N$ did not exceed 10, the new experiments were carried out on solutions with ratios up to 40.

As presented in sect. 2.7, if one is able to measure CPL from a racemic mixture at wavelength $\lambda$ using circularly polarized excitation at wavelength $\lambda'$, the CD of a perturbed system also at $\lambda'$, and the CPL from the same perturbed system at $\lambda$, then one should be able to determine the enantiomeric excess, and the $g_{\text{lum}}$ and $g_{\text{abs}}$ values for transitions of the pure enantiomers, even though they may be not be separable by ordinary chemical means. This has been accomplished for $[\text{Dy(DPA)}_3]^{3-}$ (Hilmes et al., 1988). In this work it was possible to measure the CPL of a racemic solution of this complex using a circularly polarized 457.9 nm Ar-ion laser line, and measure the CD at 457.9 nm of a solution of this complex which contained 0.075 M L-histidine. The CPL of this latter solution was also measured. These three experimental measurements are reproduced in fig. 12. The most difficult measurement of the three is the measurement of the CD, since the intraconfigurational $f \rightarrow f$ absorptions are so weak. The results displayed were obtained using a pathlength of 5 cm. From these three measurements, it was determined that the addition of 0.075 M L-histidine resulted in an enantiomeric excess of 3.7%. The magnitude of the dissymmetry ratios for the enantiomeric complexes were also determined, however, this analysis alone does not yield the specific sign of $g_{\text{lum}}$ or $g_{\text{abs}}$ for a specific enantiomer. These measurements were also reported for the same complex in the

![Fig. 12. Circular dichroism (upper curve) and absorption (lower curve) spectra (left), for the $^4I_{15/2} \leftrightarrow ^6H_{15/2}$ transition of an aqueous solution of 5:1 L-histidine:[Dy(DPA)$_3$]$^{3-}$  at pH = 3. Circularly polarized luminescence (upper curve) and total luminescence (lower curve) spectra for the $^6H_{11/2} \leftrightarrow ^4F_{9/2}$ transition of [Dy(DPA)$_3$]$^{3-}$ at pH = 3 (middle) and of a 5:1 L-histidine : [Dy(DPA)$_3$]$^{3-}$ aqueous solution (right), following circularly polarized 457.9-nm excitation and excitation at 488 nm, respectively (Hilmes et al., 1988).](image)
presence of 4.5M (+)-dimethyl-L-tartrate (Çoruh et al., 1988b). It can be seen that the CPL spectra for the racemic and perturbed equilibria are very similar. In this case the enantiomeric excess was estimated to be 37%. If one makes the reasonable assumption that the addition of the same amount of (+)-dimethyl-L-tartrate to other tris lanthanide (III) complexes with DPA results in the same enantiomeric excess, dissymmetry ratios for enantiomers for these other complexes may also be determined from eqs. (45) and (46).

4.1.3.1. Perturbation of the racemic equilibrium as a structural probe. Since the luminescence dissymmetry factors for the magnetic dipole allowed transitions of a number of lanthanide complexes are often quite large and easily measurable, it has been suggested that the perturbation of a ground state racemic equilibrium might be useful as a reliable structural indicator for chiral species. This practical use of CPL spectroscopy, of course, hinges on the ability to relate the created enantiomeric excess to some structural characteristic of the perturber molecule. This effort is further complicated by the inherent difficulty in the nomenclature of chiral molecules as expressed by Professor S.F. Mason “The principal nomenclature systems [for chiral molecules] are artificial in the sense that two analogous enantiomers with the same configurational label do not necessarily have the same absolute stereochemical form, nor need be related chemically, by interconversions, or physically, by the sign of their optical rotation properties” (Mason, 1982).

An obvious target for this kind of analysis is the perturbation of the racemic equilibria by L-amino acids, since the L, D nomenclature used for these molecules is based on the common structure indicated in Scheme 5.

![Scheme 5](image)

L-amino acid

Under physiological pH conditions (6–7) these are perhaps more correctly written as their zwitterionic form. There has been a number of studies in which various amino acids have been added to racemic solutions of [Tb(DPA)3]3− or [Eu(DPA)3]3− (Yan et al., 1982; Brittain, 1981; Huskowska and Riehl, 1995). Although the early work indicated that L-amino acids all gave the same sign for the CPL from selected transitions of the DPA complexes, more recent work has shown that such a simple structural picture is unfortunately not correct (Muller and Riehl). It does appear from this work that simple substitutions of similar groups lead to similar results. For example, in table 9 we show results for the luminescence dissymmetry values obtained from an aqueous solution of [Tb(DPA)3]3− in which the amino acids L-histidine, L-histidine methyl ester, and L-histidine benzyl ester have been added. One can conclude from this limited set of data that minor modifications of the alkyl group do not change the overall chirality of the perturbation.
Just as in the development of other spectroscopic-based methods at determining chiral structure, the exploitation of the perturbation of racemic equilibria is based on very small differential effects. In the specific experiments involving neutral amino acids, the presumably outer sphere association is very weak, and the difference between the diastereomeric association constants is very small. There have been several other attempts at using the perturbation of the racemic equilibrium of DPA complexes as diagnostic of chiral structure. For example, Brittain has attempted to correlate the sign of the CPL to some element of the chiral structure of monosaccharide aldose sugars (Brittain, 1984), but in a more quantitative study using modern instrumentation it has been shown that, in fact, no simple correlation is evident in the CPL results from racemic [Tb(DPA)$_3$]$^{3-}$ into which simple or complex sugars have been added (Huskowska and Riehl, 1995).

Although there has been some progress in understanding the nature of the equilibrium shift induced by added chiral agents to racemic equilibria (Huskowska and Riehl, 1995; Schipper, 1978; Wu et al., 1989), no one has yet “designed” a system based on specific consideration of the possible intermolecular interactions in which differences in diastereomeric perturbations would be expected to be maximized. Various speculations concerning the importance of hydrogen bonding, coulombic forces, π-stacking, steric factors, hydrophobic effects, etc. have been considered, but only in attempts to understand empirical observations. To be useful in determination of unknown chirality, and to develop a useful tool for determining molecular chirality, a more rational process of designing racemic complexes, and possibly chemical derivitization of target chiral molecules will probably be necessary.

4.2. Lanthanide complexes with chiral ligands

If the ligands surrounding a luminescent lanthanide ion are chiral due to, for example, an asymmetric carbon atom, then the luminescence from the lanthanide ion must be chiral. The “source” of the circular polarization of a particular transition may be due to vicinal effects associated with the chiral center(s) or due to a net chiral configuration about the lanthanide ion due to the influence of the chiral center(s). It is impossible to completely separate these two contributions, but it is generally believed that due to the short range nature of f-orbitals, that the effect of chiral centers in non-coordinating atoms is negligible compared to the chiral arrangement of ligands surrounding the luminescent center. Much of the experimental work involving lanthanide complexes with chiral ligands is, therefore, mostly a study of the chiroptical properties of the chiral configuration caused by the fact that the ligands are chiral.
### Table 10
List of the enantiopure lanthanide complexes for which CPL measurements have been reported

<table>
<thead>
<tr>
<th>Enantiopure lanthanide complex</th>
<th>Sample description</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>[Eu(DOTMPhA)]$^{3+}$</td>
<td>H$_2$O, D$_2$O</td>
<td>(Dickins et al., 1999)</td>
</tr>
<tr>
<td>[Dy(DOTMPhA)]$^{3+}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[Yb(DOTMPhA)]$^{3+}$</td>
<td>MeOH/D$_2$O</td>
<td>(Dickins et al., 1999)</td>
</tr>
<tr>
<td>[Th(DOTMPhA)]$^{3+}$</td>
<td>H$_2$O, D$_2$O, MeCN, MeOH</td>
<td>(Dickins et al., 1999)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(Morita et al., 2000)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(Dickins et al., 1997a)</td>
</tr>
<tr>
<td>[Yb(DOTMBrPhA)]$^{3+}$</td>
<td>MeOH/D$_2$O, D$_2$O</td>
<td>(Dickins et al., 1999)</td>
</tr>
<tr>
<td>[Nd(DOTMBrPhA)]$^{3+}$</td>
<td>D$_2$O</td>
<td>(Rau and Morita, 2001)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[Eu(DOTMCNPhA)]$^{3+}$</td>
<td>H$_2$O, D$_2$O</td>
<td>(Dickins et al., 1999)</td>
</tr>
<tr>
<td>[Tb(DOTMCNPhA)]$^{3+}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[Eu(DOTMCO$_2$MePhA)]$^{3+}$</td>
<td>D$_2$O</td>
<td>(Dickins et al., 1999)</td>
</tr>
<tr>
<td>[Tb(DOTMCO$_2$MePhA)]$^{3+}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{18%}$ Er$_{3+}$-doped GMO (β'-phase)</td>
<td>Crystal</td>
<td>(Herren and Morita, 1996)</td>
</tr>
<tr>
<td>GMO = ferroelectric Gd$_2$(MoO$_4$)$_3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{18%}$ Er$_{3+}$-doped GMO (β'-phase)</td>
<td>Crystal</td>
<td>(Morita et al., 2003)</td>
</tr>
<tr>
<td>$^{18%}$ Er$_{3+}$-doped GMO (β'-phase)</td>
<td>Crystal</td>
<td>(Morita et al., 2002a)</td>
</tr>
</tbody>
</table>

- DOTMPhA = 1,4,7,10-tetakis-[(S or R)-1-(1-phenylethylcarbamoyl)methyl]-1,4,7,10-tetraazacyclododecane
- DOTMBrPhA = 1,4,7,10-tetakis-[(S or R)-1-(1-4-bromophenylethylcarbamoyl)methyl]-1,4,7,10-tetraazacyclododecane
- DOTMCNPhA = 1,4,7,10-tetakis-[(S or R)-1-(1-4-cyanophenylethylcarbamoyl)methyl]-1,4,7,10-tetraazacyclododecane
- DOTMCO$_2$MePhA = 1,4,7,10-tetakis-[(S or R)-1-(1-4-phenylethylcarbamoyl)methyl]-1,4,7,10-tetraazacyclododecane
- DOTMPHa-based palladium porphyrin derivative

- CD$_3$OD/D$_2$O, EtOH (Beeby et al., 2000)
- DO3MPhA-based palladium porphyrin derivative

- MeOH/D$_2$O, D$_2$O (Maupin et al., 2000)
- Na$_3$[Tb(ODA)$_3$] ODA = oxydiacetic acid
- Na$_3$[Gd$_{1-x}$Tb$_x$(ODA)$_3$]·2NaBF$_4$·6H$_2$O ($x = 0.2$) (Maupin et al., 2000)
In table 10 we list the enantiopure lanthanide complexes for which CPL measurements have been reported.

In the previously described studies on racemic mixtures, one had to circularly polarize an excitation beam at a wavelength corresponding to an $f \rightarrow f$ absorption in order to generate a significantly enantio-enriched excited state concentration, in order to see CPL. For enantiopure chiral complexes, one needs only to generate species in their emitting state, so it is very often the case that indirect excitation through ligand absorption and radiationless energy transfer to excited lanthanide (III) states is much more effective at generating luminescence. This is due, of course to the weakness of direct $f \rightarrow f$ absorption transitions. As stated above, for a pure chiral complex, $g_{\text{lum}}$ will not depend upon the polarization of the exciting light. It might be possible to modulate the intensity of the luminescence due to preferential absorption of linear or circular polarized light, particularly for oriented systems, but to date no experiments of this type on pure chiral species have appeared in the literature. If a species is composed of an inherently chiral absorption chromophore, $A_R$, and an inherently chiral luminescent chromophore, $L_R$, then one might expect to see a difference in intensity between $A_R : L_R$ versus $A_S : L_S$ due to differences in energy transfer efficiency. This has been incorrectly referred to in the literature as “chiral energy transfer”, but is actually due to differences in diastereomeric interactions which can often be explained in terms of different donor: acceptor distances in the diastereomeric complexes.

In general, all of the fundamental principles concerning spectral overlap, absorption strength, etc., that are critical to understanding the ligand to metal energy transfer process in the design of luminescent lanthanide complexes, also apply to the luminescence from chiral complexes.

4.2.1. Chiral DOTA-based ligands

In sect. 4.1 we described a series of experiments concerning the measurement of CPL from racemic mixtures of complexes containing DOTA type ligands. As described previously, there is chirality associated with the pendant arms, and chirality associated with the 12 member ring (Scheme 6). In these structures, the introduction of a chiral center in the pendant arm $\alpha$ or $\beta$ to the ring nitrogen may influence the sense of rotation (clockwise or counterclockwise) of the pendant arms, and result in a $C_4$ chirality that can be partially or completely locked.

It has been demonstrated that the $C_4$-symmetric chiral tetraamide ligand, 1,4,7,10-tetrais-[((S)-1-(1-phenylethyl)carbamoylmethyl]-1,4,7,10-tetraazacyclodecane ($Ar = \text{Ph}$, DOTMPhA), and its $para$-substituted derivatives (DOTMBBrPhA, DOTMCNPhA, and DOTMCO$_2$MePhA where $Ar = p$-Br-Ph, $p$-CN-Ph, and $p$-CO$_2$Me-Ph, respectively) give enantiomeric lanthanide (III) complexes that do not readily undergo interconversion of the $\Delta/\Lambda$ isomers in the temperature range 220–320 K and, also, exist as one predominant isomer in aqueous solution (Dickins et al., 1999, 1997a). The crystal structures of the Eu(III) complexes with both enantiomers of DOTMPhA lead to formation of one single isomer in which the chirality is controlled by the absolute configuration at the remote chiral carbon center ($S$ and $R$ give $\Delta(\lambda\lambda\lambda\lambda)$ and $\Lambda(\delta\delta\delta\delta)$, respectively) (Dickins et al., 1999, 1997a). These structural conclusions have been based on X-ray crystallographic analysis and NMR studies of these...
compounds. Confirmation and additional details concerning the chiroptical and structural properties in solutions containing these complexes have been provided by CPL spectroscopy (Maupin et al., 2000, 1998b; Dickins et al., 1999, 1997a). It was shown, for example, that the CPL spectra obtained were independent of the polarization of the excitation beam (left-, right-, or plane-polarized light). This is consistent with the presence of only one species in solution as indicated by NMR and X-ray crystallographic studies. As expected, complexes prepared with ligands of opposite chirality yield mirror image CPL spectra.

As noted above, the energetic and spectral properties that are important in the design of lanthanide-based luminescent species are also important in the development of chiral luminescent complexes. It has been shown, for example, that Tb(III) complexes with DOTMPH A or DOTMBrPhA are more emissive, due to the absence of efficient quenching processes, than the related complexes with Dy(III), Eu(III), and Yb(III) (Dickins et al., 1999, 1997a). However, all of these species exhibit strong circularly polarized luminescence following either direct excitation of the lanthanide (III) ion or indirect sensitization through the ligand absorption bands in aqueous solution. The functions of (R)-[Ln(DOTMPH A)]$^{3+}$ or (R)-[Ln(DOTMBrPhA)]$^{3+}$ at, e.g., 548, 657, 995, or 590 nm are $-0.25$, $+0.35$, $+0.18$, or $-0.12$ in the spectral range of the $5D_4 \rightarrow 7F_5$ (Tb(III)), $4F_{9/2} \rightarrow 6H_{11/2}$ (Dy(III)), $2F_{5/2} \rightarrow 2F_{7/2}$ (Yb(III)), or $5D_0 \rightarrow 7F_1$ (Eu(III)) transitions, which are particularly well-suited for CPL measurements since they satisfy magnetic-dipole selection rules, $\Delta J = 0, \pm 1$. It should be mentioned that the sign of the observed CPL was the same for all of the Eu(III) and Tb(III) analogues complexes in (S)- or (R)-[Ln(DOTMPH A)]$^{3+}$ in which different aryl substituents such as naphtyl (Dickins et al., 1998a, 1998b, 1997b), or phenyl and its para-substituted derivatives (Dickins et al., 1999, 1997a) on the pendant arms have been studied. Such behavior is consistent with the adoption of an identical structure for each complex that has the same absolute configuration in the helicity of the pendant arm and the macrocyclic ring, and, therefore, indicates that the sign and magnitude of the CPL is controlled primarily by
this aspect of chiral structure. As stated above, this is controlled by the absolute configuration at the remote chiral carbon center: $S$ and $R$ give $\Delta(\lambda\lambda\lambda\lambda)$ and $\Lambda(\delta\delta\delta\delta)$, respectively.

An interesting and potentially useful result on these DOTA-ligand based systems is the observation that the sign and magnitude of the CPL is essentially “fixed” when only three of the pendant arms are chiral (Maupin et al., 2000, 1998b). Similar CPL spectra have been recorded for the enantiomeric pair of (RRRR)-$\Lambda$ or (SSSS)-$\Delta$-[Ln(phen)$_4$]$^{3+}$ (where (phen)$_4$ = DOTMPhA) and (RRR)-$\Lambda$ or (SSS)-$\Delta$-[Ln(Rphen)$_3$(phenan)]$^{3+}$ (Ln(III) = Eu, Tb, and Yb) in which one of the chiral amine groups has been substituted by an achiral phenanthridine group (see Scheme 7). Total luminescence and circularly polarized luminescence spectra of the Yb(III) complexes are plotted in fig. 13 (Maupin et al., 2000).

Although the experiments described above have been conducted on enantiopure tetraamide complexes, existing as one isomeric species in solution, it is possible to gain structural in-
formation from related compounds which exist as two non-interconverting diastereomers. Examples of such ligands are the chiral monoamidetris(phosphinates), DO3MPPhA and DO3MPNpA, in which one of the phosphinate groups has been replaced with a chiral amide substituent (α-phenylethyl and α-1-naphthylethyl) (Aime et al., 1998). Thus, the introduction of a chiral center possessing a $S$ configuration at the carbon atom into the amide group leads to the formation of two diastereomeric complexes, ($S$-$SSS$)/($S$-$RRR$), in ratios of 2 : 1 and 4 : 1, respectively. Observation of similar CPL spectra for Eu(III) and Tb(III) complexes of these two ligands indicated that the helicity and probably the macrocyclic ring conformation are determined by the configuration of the chiral carbon center in the amide group. This conclusion is based on the fact that the two diastereomers have identical ring conformations and pendant arm layout and, therefore, the observed CPL is due to the contribution from both isomers. If the two diastereomers had enantiomeric ring conformations and pendant arm layout, the CPL from the minor isomer would have been canceled by the CPL from the major. As a consequence, the magnitude of $g_{\text{lim}}$ should have been higher for [Tb(DO3MPNpA)] compared to [Tb(DO3MPPhA)] with respect to the ratio difference between these complexes (4 : 1 and 2 : 1). The $g_{\text{lim}}$ values amounted to +0.17 and +0.18 at 543 and 540 nm ($^5\text{D}_4 \rightarrow ^7\text{F}_5$ transition) for [Tb(DO3MPPhA)] and [Tb(DO3MPNpA)], respectively (Aime et al., 1998). Therefore, the two diastereomers in each system only differ in the configuration at each of the three phosphorus centers: ($RRR$) and ($SSS$). On the other hand, the ratio of the two diastereomers is modulated by the size of the remote chiral group as pointed out by NMR studies for DO3MPPhA and DO3MPNpA (2 : 1 for α-phenylethyl and 4 : 1 for α-naphthylethyl) (Scheme 8) (Aime et al., 1998).

As discussed briefly in sect. 2.4, these C$_4$ symmetric species have been a prime target for the development of useful correlations between CPL spectrum and chemical structure (Bruce et al., 2002). The sign and magnitude of CPL for this class of complexes have been correlated to the degree of helical twist of the complex, the nature of the ligand field, and the polarizability of the axial donor group. The local helicity at the lanthanide (III) center has been related to the angle, $\phi_{gn}$, between the electric-dipole ($|P_{gn}|$) and magnetic-dipole ($|M_{ng}|$) transition moment vectors and, therefore, the rotatory strength, $R_{gn}$, associated with a given transition between two states $g$ and $n$ as described below.

$$R_{gn} = \left| P_{gn} \right| \left| M_{ng} \right| \cos \phi_{gn}. \quad (51)$$

The rotatory strength has been shown to have a $\sin 2\theta \cos 2\theta$ (or $\sin 4\theta$) dependence, where $\theta$ is the twist angle between the N$_4$ and O$_4$ planes. It is, therefore, predicted to vanish at 0°, 45°, or 90° and to be maximum at ±22.5° for square-antiprismatic complexes (fig. 14). This latter value is closer to twist angles determined by crystallographic analysis for twisted square antiprismatic isomers ($\sim 28^\circ$) than those associated with regular square antiprismatic species ($\sim 40^\circ$) (Parker et al., 2002). Another factor which may perturb the size of the twist angle is the degree of conformational rigidity in the complex. Indeed, the CPL will reflect the time-averaged local helicity around the lanthanide (III) ion. For example, the magnitude of $g_{\text{lim}}$ has been shown to increase with an increase in the degree of conformational rigidity of the complex (Bruce et al., 2002). The tetramide complex, ($SSSS$)-[Eu(DOTMPPhA)(solvent)]$^{3+}$, which does not readily undergo interconversion of the $\Delta / \Lambda$ isomers in the temperature range
220–320 K exhibited the highest value, while the triamide and the isomeric 1,4 and 1,7 diamide compounds showed lower values (table 11). It has been demonstrated by NMR studies that in these latter systems there were exchange processes involving the interconversion of isomeric species. Finally, the authors have concluded from their NMR, CD, and CPL experiments that it is for the more polarizable axial donors that the affinity of that axial donors for the lanthanide (III) ion and $g_{\text{lum}}$ values are greater and smaller, respectively (Dickins et al., 2003; Bruce et al., 2002, 2001; Di Bari et al., 2000). This study has been conducted on a series of eight- and nine-coordinate axially symmetric complexes based on DOTA derivatives bearing phosphonate, phosphinate, or carboxamide groups. In particular, the detailed analysis of the Eu(III) total luminescence and CPL sign and magnitude changes for $^{(SSSS)}$-[Eu(DOTMPhA)(solvent)]$^{3+}$ showed that the more polarizable axial donors (e.g., DMSO > EtOH > H$_2$O) gave the higher intensity ratios of $^5D_0 \rightarrow ^7F_2$ to $^5D_0 \rightarrow ^7F_1$ transi-
Fig. 14. Correlation of the effect of twist angle in axially symmetric eight- and nine-coordinate antiprismatic lanthanide complexes on their CPL or CD intensity (Bruce et al., 2002).

Table 11
Luminescence dissymmetry ratio values (g_lum) in the spectral range of the 5D_0 → 7F_1, 7F_2, and 7F_4 transitions for di-, tri, and tetra-amide Eu(III) triflate complexes in acetonitrile solution at 295 K.

<table>
<thead>
<tr>
<th>Lanthanide complex</th>
<th>g_lum/λ (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5D_0 → 7F_1</td>
</tr>
<tr>
<td>cis-(RR)-[Eu(DO2Ph)(H_2O)_2]^{3+}</td>
<td>+0.013 (594)</td>
</tr>
<tr>
<td>trans-(RR)-[Eu(DO2Ph)(H_2O)_2]^{3+}</td>
<td>+0.02 (594)</td>
</tr>
<tr>
<td>(SSS)-(DOTMPhA)(MeCN)]^{3+}</td>
<td>−0.06 (594)</td>
</tr>
<tr>
<td>(SSS)-(DOTMPhA)(H_2O)]^{3+}</td>
<td>−0.09 (594)</td>
</tr>
<tr>
<td>(SSS)-(DOTMPhA)(H_2O)]^{3+}</td>
<td>−0.05 (593)</td>
</tr>
</tbody>
</table>

a From reference (Bruce et al., 2002).
b See text for nomenclature.
c Recorded in H_2O.

tions (2.8 > 1.0 > 0.6) and the smaller g_lum values (0 < +0.03 < +0.08 at 614 nm) (Bruce et al., 2002).

DOTA-based ligands have also been used to prepare chiral complexes of Yb(III) and Nd(III) for the measurement of CPL in the near-IR. CPL from the 2F_5/2 → 2F_7/2 transition of Yb(III) at about 980 nm, and from the 4F_3/2 → 4I_9/2 and 4F_3/2 → 4I_11/2 transitions of Nd(III) centered at 880 and 1060 nm, respectively, have been reported (Maupin et al., 2000; Beeby et al., 2000; Maupin et al., 1998b). CPL from Yb(III) complexes with the ligands DOTMPhA and DOTMBrPhA have been studied in some detail. Although the conclusions from these CPL studies were consistent with results obtained previously from related complexes with Eu(III) and Tb(III) ions, the measurement of CPL in the near-IR may be particularly suited for the development of chiral luminescent probes of biological molecules. Indeed, helical tetraamide complexes of Yb(III) as well as of Eu(III) and Tb(III), bearing an
Fig. 15. Circularly polarized luminescence (upper curve) and total luminescence (lower curve) spectra for the $^{4}I_{9/2} \rightarrow ^{4}F_{3/2}$ (left) and $^{4}I_{11/2} \rightarrow ^{4}F_{5/2}$ (right) transitions of $(RRR)[Nd(DO3MPhA)(PdPorph)]^{3+}$ in EtOH at 295 K, following excitation at 415 nm (Maupin et al., 2000).

CIRCULARLY POLARIZED LUMINESCENCE SPECTROSCOPY

$N$-methylphenanthridinium group ([Ln(DO3PhphenMe)]$^{4+}$) have been shown to selectively bind to DNA (Bobba et al., 2001; Govenlock et al., 1999).

The CPL spectra plotted in fig. 15 are the first examples of small, but measurable, CPL based on the luminescence in the near-IR region from a chiral Nd(III) complex incorporating a Pd-centered porphyrin moiety, $(RRR)[Nd(DO3MPhA)(PdPorph)]^{3+}$ (Maupin et al., 2000). In particular, it has been shown that the Pd-porphyrin chromophore sensitizes the near-IR emission from Nd(III) and Yb(III) which are enhanced either in the presence of a nucleic acid and in the absence of oxygen (Beeby et al., 2000). Another new development of using CPL is based on the structural changes resulting from the binding of anions (e.g., carbonate, phosphate, acetate, malonate) to the lanthanide center from coordinately unsaturated chiral lanthanide (III) complexes such as [Eu(DO3Ph)]$^{3+}$ and [Eu(DO3PhphenMe)]$^{4+}$ (Bruce et al., 2000; Parker, 2000; Dickins et al., 1998b). Upon coordination of the added anion to the lanthanide(III) ion, a new anion-bound complex was formed in which the helical twist about the center was modified and thus led to a different CPL spectrum. As a consequence, CPL spectroscopic measurements can be used as a probe of luminescent lanthanide complexes as sensory systems for anion binding in aqueous solution (Scheme 9).

4.2.2. Chiral complexes with three-fold symmetry

It is well known that triple-helical nine-coordinate complexes using achiral ligands are inherently chiral but they usually appear as racemic mixtures both in the solid state and in solution (Bünzli and Piguet, 2002). Since the high lability of the lanthanide (III) ions favors easy racemization of the triple helical complexes, it is possible to produce edifices in which the chirality is brought about by the ligands, a method that can only be used if the diastereomers have a large enough energy difference. During the last two years, the CPL technique has been used to probe the chiroptical properties of such systems with chiral tridentate aromatic ligands derived from 2,6-pyridine-dicarboxylic acid, bis(benzimidazole)pyridine, and terpyridine derivatives. The purpose of these studies was to attempt to quantify the importance of the helical wrapping of the ligand strand contribution and, therefore, the influence of this latter on the diastereomeric induction. Although the X-ray crystallographic analysis has shown that there is little or no helical arrangement of the ligand strands upon complexation of
2,6-bis[(1-S-neopentylbenzimidazol-2-yl)]-pyridine to lanthanide(III) in 1 : 3 Eu : L\textsuperscript{2} complex (Muller et al., 2001a) the study of this system as well as of 1 : 1 and 1 : 2 complexes gave a better understanding about the influence of the helical wrapping on the chiroptical properties (Muller et al., 2003b). It should be mentioned that the steric hindrance generated by the presence of the bulky chiral neopentyl group precludes the wrapping of one of the three ligands in a helical way, as was usually observed for related 1 : 3 compounds (Muller et al., 2001a) while the ligand adopts a meridional planar coordination to the lanthanide (III) ion in the 1 : 1 complex (Scheme 10) (Muller et al., 2003b).

CPL comparative studies have been performed on anhydrous MeCN solutions of [Eu(NO\textsubscript{3})\textsubscript{3} (L\textsuperscript{2})(MeCN)], [Eu(ClO\textsubscript{4})\textsubscript{2}(L\textsuperscript{2})\textsubscript{2}\textsuperscript{+}], and one with a Eu : L\textsuperscript{2} ratio of 1 : 34 in which 20% and 80% of the Eu(III) ion are in the form of the 1 : 3 and 1 : 2 complexes, respectively (Muller et al., 2003b). The measured CPL of [Eu(NO\textsubscript{3})\textsubscript{3}(L\textsuperscript{2})(MeCN)] was due to the remote asymmetric centers’ influence on chiral coordination, while for the 1 : 2 and 1 : 3 complexes, the structural contribution from the chiral helicate intervened also, as corroborated by the specific rotary dispersion values. Indeed, a little structural contribution effect (4–5 deg dm\textsuperscript{2} mol\textsuperscript{−1}) has been evidenced for 1 : 2 complexes (33.9–35.2 deg dm\textsuperscript{2} mol\textsuperscript{−1}) with respect to that generated by the sum of the two ligand molecules (15.1 deg dm\textsuperscript{2} mol\textsuperscript{−1} for the free ligand) (Muller et al., 2001a). Finally, the CPL data are consistent with the presence of a small diastereomeric excess in solution as indicating by the dependence of these latter on the polarization of the excitation light. Further experiments such as luminescence titration or temperature dependence of the \( ^{5}\text{D}_0 \leftrightarrow ^{7}\text{F}_0 \) excitation spectrum have demonstrated that the 1 : 3 and 1 : 2 species coexist in solution for a Eu : L\textsuperscript{2} ratio of 1 : 34, in line with the observation of two CPL effects on the spectrum of this solution (Muller et al., 2003b). A similar conclusion has been found for a 10\textsuperscript{−3} M-solution of [Eu(ClO\textsubscript{4})\textsubscript{2}(L\textsuperscript{2})\textsubscript{2}\textsuperscript{−}] in which 3% of the Eu(III) ion is in the form of the 1 : 1 species, as well as for related 1 : 3 complexes with the ligands 2,6-bis[(1-methylbenzimidazol-2-yl)]-pyridine-4-carboxylate (L\textsuperscript{2}) (Muller et al., 2002c) or
3-[2,6-bis(diethylcarbamoyl)pyridine-4-yl]-N-tert-butoxycarbonyl)alanine ester (L⁴) (Muller et al., 2001b). However, the CPL spectra of [Eu(L)₃]³⁺ with both enantiomers, L(+) and L(−), of the terdentate chiral C₂-symmetric ligand ‘dipineno-[5,6:5′,6′]-fused-2,2’-6’,2”-terpyridine were independent of the polarization of the excitation beam, which confirms the presence of only one diastereomer in solution as pointed out by NMR (Muller et al., 2002a, 2003a). Moreover, Eu(III) and Tb(III) complexes with both enantiomers, L(+) and L(−), showed mirror image CPL spectra, respectively (fig. 16). The resultant emitted light is polarized in a direction determined by the helicity of the metal ion, which in turn is controlled by the absolute configuration at the remote carbon centers in the pinene moieties. This latter case is the first example of diastereoselective formation of lanthanide (III) triple helical complexes with a terdentate chiral C₂-symmetric ligand.

4.2.3. Chiral complexes of low or no symmetry

CPL studies have also been used to investigate the solution structures of Eu(III) 1:1 complexes of chiral 18-crown-6 ligands in which the chirality is derived from stereospecific placement of methyl groups onto the crown ring (Metcalf et al., 1988, 1986). In particular, Eu(III) nitrate and chloride complexes of (2R,3R)-2,3- or (2S,6S)-2,6-dimethyl-1,4,7,10,13,16-hexaoxacyclooctadecane (L⁵) and (2R,3R,11R,12R)-2,3,11,12- or (2S,6S, 11S,15S)-2,6,11,15-tetramethyl-1,4,7,10,13,16-hexaoxacyclooctadecane (L⁶), [Eu(L)(X)₂]⁺ (X = NO₃⁻ and Cl⁻), have been studied in trifluoroethanol (TFE) solution (Metcalf et al., 1988) (Scheme 11). Although the opposite CPL sign observed for the R- and S-type com-
plexes resulted from the opposite absolute configuration of the chiral centers, its magnitude for the \( R \)-type systems was twice that of the corresponding \( S \)-type complexes indicated that the chirality also arose from conformational and configurational contributions. This magnitude difference has been explained by the greater importance of the steric interactions of the methyl substituents in the \( R \)-type crowns (methyl groups two atoms apart) which resulted in a larger twist of the crown ether ligands than in the \( S \)-type systems (methyl groups four atoms apart). Comparison of the CPL spectra of the Eu(III) nitrate and chloride complexes have shown that the complex chirality might be influenced by the nature of the anion. The
optical activity of the nitrate complexes also contained a configurational contribution due to the chiral arrangement of the opposing bidentate coordinated nitrates, while the analogous Eu(III) chloride complexes did not contain such contribution. This arrangement, formed by the opposing nitrate chelate rings, has been described as a dissymmetric two-bladed propeller in which the nitrate twist angle was influenced by the steric interactions of the methyl groups onto the crown ring. As a consequence, the nitrate configurational contribution has been found more significant (larger nitrate twist angles) in the $D_2$-symmetric tetramethyl crown systems than in the $C_2$-symmetric dimethyl crown compounds, because of the more even spacing of the methyls around the macrocyclic ring (greater steric interactions). A similar pattern has been found for the metal transition complexes with these ligands as pointed out by the CD study (Dyer et al., 1986).

CPL investigations on Eu(III) and Tb(III) nitrate complexes with the chiral $D_2$-symmetric ligand $4R,9R,19R,24R$-$3,10,18,25,31,32$-hexaaazapentacyclo$[25.3.1.1.0.0]$-dotriaconta-1-(31),2,10,12,14,16(32),17,25,27,29-decaene ($RR$-pydach) (Scheme 12) and its $SS$-pydach enantiomer have shown that the strong measured CPL, $g_{\text{lum}} = -0.19$ at 596 nm ($^4D_0 \rightarrow ^7F_1$ transition), was only due to the twisted conformation (Tsubomura et al., 1992). In the case of the complexes, $[\text{Ln}(RR\text{-pydach})]^3^+$ and $[\text{Ln}(SS\text{-pydach})]^3^+$, there was no evidence of a contribution from coordinated nitrate anions on the complex chirality as previously observed. The complex structures which have been characterized by NMR and luminescence spectroscopy have suggested that the nitrate anions were not coordinated to the lanthanide (III) ion in these species and, also, that approximately three water molecules were bound to the metal center.

Chiroptical structures of lanthanide (III) complexes with chiral amino-polycarboxylate ligands such as $(S,S)$- and $(R,R)$-ethylenediamine-$N,N$'disuccinic acid (edds) and L-alaninediacetate (alada) (Scheme 13) have been characterized by CPL (Murata et al., 1996). In particular, CPL spectra of nine-coordinate lanthanide(III) complexes of the hexadentate $(S,S)$-edds in aqueous solution have revealed strong and weak but measurable signal for Eu(III) or Tb(III) and Dy(III) or Sm(III) ions, respectively (Morita et al., 2000; Murata et al., 1988). The optical activity of these systems has been explained by conformational effects induced by the arrangement of the ligand coordinated to the metal ion, and by the asymmetric atoms in the ligand. As a comparison, the CPL lineshape and its mag-
nitude were different in the lanthanide(III)-L-alada system (Murata et al., 1996) which can be understood by a structural consideration. In an alada complex, the lanthanide (III) ion is coordinated to two tetradeutate ligands and one or two water molecules, whereas in an edds complex, the metal ion is bound to one hexadentate ligand and three water molecules, leading to different symmetry fields. The presence of water molecules in the inner coordination sphere of these compounds has been assumed by analogy to the structure of Tb(III) complexes with ethylenediamine-tetraacetate (edta) and nitrilotriacetate (nta) ligands (Horrocks and Sudnick, 1979).

4.2.3.1. Mixed ligand systems. CPL measurements have also been performed on so-called mixed-ligand lanthanide systems in which a luminescent lanthanide ion is presumably coordinated to more than one type of ligand with one or more of the ligands being optically active. For example, the formation of Tb(PDTA) mixed-ligand complexes (PDTA = R-, S-, and R,S-1,2-propanediaminetetraacetic acid) with selected α-hydroxycarboxylic acids such as the R-, S- and R,S-isomers of both mandelic (MAN) and 3-phenyllactic (PLA) (Scheme 14) acids have been investigated (Brittain, 1997b). The results from these studies were interpreted by the authors in terms of the influence of differential steric interactions, originating from perturbations in ligand conformations, accompanying the formation of the [Tb(PDTA)(MAN)] complex and, therefore, influencing the lineshape of its resulting CPL signal. No such effects were observed in the case of [Tb(PDTA)(PLA)], but the measured CPL signal in this case was found to be the sum of the PDTA and PLA contributions.
In a previous review article, CPL measurements from mixed-ligand lanthanide complexes appearing in the literature prior to 1985 have been tabulated. In table 12 we list the publications involving CPL from these types of systems that have appeared since this last review.

In the interpretation of CPL results from mixed-ligand lanthanide complexes, one must be concerned with the possibility that the luminescence originates from a complicated distribution of species, and not simply a complex composed of ligands in the exact stoichiometry that the solution was prepared. It is always the case that the measurement of CPL for solutions in which the distribution of chiral species is undetermined is liable to misinterpretation. To illustrate this point, we consider the results obtained for an aqueous solution of Eu(III) in which two equivalents of DPA and one equivalent of L-malic acid (L-mal) (Scheme 15) have been added (Riehl, 1994; Hilmes and Riehl, 1987; Riehl, 1992).

In figs. 17 and 18 we plot, the total luminescence and CPL for a solution of Eu(III), DPA and L-mal prepared with a ratio of 1 : 2 : 1.82. We have taken care here not to plot these two spectra on the same figure, because it has been shown that the total luminescence comes almost exclusively from the very stable racemic-[Eu(DPA)_3]^{3-} complex, but this racemic mixture exhibits no CPL. The weak CPL given in fig. 18, however, originates from some

\[ \text{Scheme 15.} \]

<table>
<thead>
<tr>
<th>Complex</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tb(III)(PDTA)(MAN)</td>
<td>(Brittain, 1997b)</td>
</tr>
<tr>
<td>Tb(III)(PDTA)(PLA)</td>
<td></td>
</tr>
<tr>
<td>PDTA = 1,2-propanediaminetetraacetate</td>
<td></td>
</tr>
<tr>
<td>MAN = mandelic acid</td>
<td></td>
</tr>
<tr>
<td>PLA = phenyllactic acid</td>
<td></td>
</tr>
<tr>
<td>Tb(III)(R-PDTA)(AC)</td>
<td>(Spaulding et al., 1986)</td>
</tr>
<tr>
<td>Tb(III)(R,R-CDTA)(AC)</td>
<td></td>
</tr>
<tr>
<td>R-PDTA = (R)-1,2-propanediaminetetraacetate</td>
<td></td>
</tr>
<tr>
<td>R,R-CDTA = (R,R)-trans-1,2-cyclohexanediamine-</td>
<td></td>
</tr>
<tr>
<td>tetraacetic acid</td>
<td></td>
</tr>
<tr>
<td>AC = various achiral carboxylic acids</td>
<td></td>
</tr>
<tr>
<td>Tb(III)(EDDS)(AC)</td>
<td>(Spaulding and Brittain, 1985)</td>
</tr>
<tr>
<td>EDDS = (S,S)-ethylenediamine-(N,N')-disuccinic acid</td>
<td></td>
</tr>
<tr>
<td>AC = various achiral carboxylic acids</td>
<td></td>
</tr>
<tr>
<td>Eu(III)(R-methyl-p-tolyl-sulfoxide)β-diketone</td>
<td>(Brittain and Johnson, 1985)</td>
</tr>
<tr>
<td>(1 : 1 and 1 : 2 adducts of various β-diketones were investigated)</td>
<td></td>
</tr>
</tbody>
</table>
unknown weakly luminescent distribution of complexes containing L-mal, H₂O, DPA and Eu(III). Clearly, it would be a mistake to divide the value for ∆I from fig. 18 by I from fig. 17 to determine a g_{lum} value for some “mixed-ligand” complex of fixed stoichiometry.

4.2.3.2. Other chiral mixtures. It is obviously difficult to obtain useful detailed structural information from CPL measurements from lanthanide complexes with a mixture of ligands, some of which may be chiral, or in situations in which one chiral ligand may form multiple solution species. In general, unless there is additional structural information available, the measurement of CPL from these systems can only indicate that the average solution structure is chiral, and intact to some extent on the emission timescale. A tabulation of CPL results on various lanthanide (III) systems composed of mixtures of ligands that appeared prior to 1985 has been given in a previous review article (Riehl and Richardson, 1986). Two more recent studies will be briefly described here. A CPL study of Eu(III) complexes of chiral Schiff base ligands derived from the condensation of substituted salicyaldehydes with amino alcohols in a ratio of 1 : 1 has shown that oligomers are formed in solution under the conditions used, and the results were, therefore, not interpretable in terms of a simple solution structure (Calienni and Brittain, 1986). A similar situation was encountered in a CPL study of Tb(III) complexes with tetridentate S-carboxymethoxysuccinic acid (CMOS) (Scheme 16) (Brittain et al., 1991). In this work, the concentration dependence of the observed CPL has been related to the polynuclear nature of the complexes formed in solution, although, again, no specific structural information could be obtained.

For the first ten to fifteen years of CPL spectroscopy, a fairly large number of papers were published in which the main purpose of the work was to determine whether or not the solution species formed exhibited circularly polarized luminescence, and was therefore, chiral...
and stable on the emission lifetime. Variation of ligand ratios, ligand structure, solvent, etc. were used in a general way to probe the sources of chirality and some elements of solution structure. This pioneering work exploited the lability and variable coordination properties of lanthanide complexes. Within the last fifteen years, however, the field of CPL spectroscopy from lanthanide complexes that warrants publication, has been focused almost exclusively on complexes of well-defined structure or solution speciation.

4.3. CPL from Lanthanide (III) ions in biological systems

Although the measurement of CPL from lanthanide ions associated with biomolecular systems has great potential as a local probe of chirality around metal binding sites, the number of applications of CPL to such chiral systems reported to date has been quite limited. CPL from Tb(III) and Eu(III) have been used to probe conformational changes in calcium-binding (Çoruh and Riehl, 1991, 1992; Çoruh et al., 1988a; Riehl and Çoruh, 1991) and iron-binding proteins (Abdollahi et al., 1996). Whereas, CD measurements on proteins are widely used to obtain macromolecular features of secondary and tertiary structure (e.g., % alpha-helicity), CPL measurements from lanthanides may yield very local information concerning the chiral environment of the metal ion. In these works, Tb(III) and Eu(III) are bound to the Ca or Fe binding sites of the proteins calmodulin (CaM), parvalbumin (Parv), trypsin (Tryp) or transferrin (Tf), lactoferrin (Lf), and ovotransferrin (ovoTf), respectively. In particular, the differences in lineshape and magnitude of the CPL spectra from Tb(III) bound to Tf or Lf and ovoTf have been explained by structural changes of the binding sites of these systems (Abdollahi et al., 1996). In the case of the calcium-binding proteins, it has been shown that the CPL spectra for EF-hand proteins such as CaM and Parv are very similar and, therefore, the coordination environments in these proteins are very closely related, as already pointed out by structural studies. On the other hand, the sign and splitting pattern for other protein classes such as intestinal calcium-binding proteins (e.g., Tryp) are different. In this work, the sequence of binding and conformational changes induced by the addition of metal ion to the CaM in buffered aqueous media could be followed by variation in the magnitude of the CPL signal (Çoruh and Riehl, 1991, 1992; Çoruh et al., 1988a). These results need to be interpreted in the context of differential binding affinities, as compared to Ca(II). The generally accepted order of binding for Tb(III) or Eu(III) is I and II, and then III and IV as corroborated by competitive and sequential binding experiments in which $g_{\text{Lum}}$ was monitored (Çoruh and Riehl, 1991, 1992). Ca(II) binds in the order III and IV, then I and II. The
variation of $g_{\text{lum}}$ with added lanthanide ion has been used to confirm previous CD results, namely, that CaM has essentially reached its final conformation after the binding of two metal ions and, also, that the environment of the second two sites is not as chiral as the first two. It should be mentioned that all bound lanthanide ions contribute to the CPL signal and that the results must be interpreted in terms of the average coordination environment of all four sites.

As implied in the previous discussions, a single CPL spectrum is often representative of several independent contributions either from different emitting species or from multiple conformations of these latter that exist simultaneously in the solution. As a consequence, these overlapping contributions do not allow the obtaining of individual information from the different emitting species present in the solution. One way to resolve this lack of information, in situations where the number of distinct species is small, might be to use a technique based on the determination of time-resolved CPL as already employed for enantioselective quenching of lanthanide luminescence, as a probe of energy-transfer dynamics in rare-earth doped crystals (Eu(III) ions in YAG, LaF$_3$, and LaAlO$_3$ crystals) (Reid, 1990), or in the study of biomolecules such as reduced nicotinamide adenine dinucleotide (NADH) bound to horse liver alcohol dehydrogenase (LADH) (Schauerte et al., 1995, 1992). It has been demonstrated that each emitting species has its own characteristic decay time associated with its electronically excited state and, therefore, can be used to obtain information on the individual CPL contribution of each luminophore. For example, a time-resolved CPL study of a mixture of Tb(III)/transferring and Tb(III)/conalbumin have demonstrated that it was possible to distinguish the two luminophores contributing for the overall CPL signal that differ by only 7% in lifetime and have a difference in $g_{\text{lum}}$ factors of only 0.05 (Schauerte et al., 1996).

5. Magnetic Circularly Polarized Luminescence

Whereas only species that are “chiral” exhibit circularly polarized luminescence in the absence of an external magnetic field, all luminescence will be circularly polarized when detected axial (i.e. parallel) to an external magnetic field. This technique is commonly referred to as Magnetic Circularly Polarized Luminescence (MCPL), or alternatively as Magnetic Circularly Polarized Emission (MCPE), or Magnetic Induced Circular Emission (MICE). A general theoretical description of MCPL has been presented by Riehl and Richardson (1976b), and other theoretical descriptions of MCPL are available (Wagnière, 1984; Hipps, 1977, 1978; Schatz et al., 1978). A number of the early experimental applications of this technique have been discussed in previous review articles (Richardson and Riehl, 1977; Riehl and Richardson, 1986). The reader is referred to these articles for a more complete description of this spectroscopic technique. MCPL instrumentation is virtually identical to the instrumentation presented in sect. 3.1 with the addition of external magnetic field oriented in the direction of emission detection. Instruments specifically designed for MCPL measurements have also been described (Hipps and Crosby, 1977; Schultheiss and Gliemann, 1986).
5.1. Theoretical aspects

As shown in sect. 2.1, in natural CPL spectroscopy, the final formal relationships connecting experimental measurements and molecular transition matrix elements are developed so that comparison can be made with natural CD spectroscopy. The theoretical description of MCPL spectroscopy has been based on previous development of the much more widely used Magnetic Circular Dichroism (MCD) technique (Stephens, 1970; Dooley and Dawson, 1984). MCD is the measurement of the differential absorption of circularly polarized light in the direction of (axial to) the imposed external magnetic field. An external magnetic field will lead to splitting of Zeeman level degeneracies, and field strength-dependent mixing of ground and excited electronic states. The splitting of initial state Zeeman levels will also cause the spectra to be temperature dependent due to Boltzmann distribution among these closely space levels. Because of these complicated effects, the analysis of experimental MCD and MCPL spectra has traditionally been accomplished by resolving the resultant spectrum into the so-called Faraday $A$, $B$ and $C$ terms. $A$ terms are associated with the appearance of a first derivative lineshape due to the fact that a degenerate state has been split so that one component absorbs or emits predominantly left circular polarization and the other component absorbs or emits predominantly right circular polarization. $C$ terms are associated with the temperature dependent part of the spectrum, and the rest of the spectrum is defined as a $B$ term.

In analogy with eq. (12) we may relate the differential emission intensity to the Faraday $A$, $B$ and $C$ terms for MCPL from a transition $\mathcal{N} \rightarrow \mathfrak{g}$ in which the magnetic field induced Zeeman splitting is much less than the line width, as follows

$$
\Delta I(\lambda) = \left( \frac{\hbar c \eta_{\mathcal{N}}}{2\lambda} \right) \left\{ A(\mathcal{N} \rightarrow \mathfrak{g}) f'_{\text{MCPL}}(\lambda)/\hbar + \left[ B(\mathcal{N} \rightarrow \mathfrak{g}) + C(\mathcal{N} \rightarrow \mathfrak{g})/kT \right] f_{\text{MCPL}}(\lambda) \right\} b_0. \tag{52}
$$

$\mathcal{N}$ and $\mathfrak{g}$ in this equation refer to the set of degenerate Zeeman levels, $f_{\text{MCPL}}$ and $f'_{\text{MCPL}}$ refer, respectively, to the MCPL lineshape and its first derivative, and $b_0$ denotes the external magnetic field strength. Just as in the case for natural CPL, more specific expressions relating molecular transition moments in the laboratory or molecular coordinate system may be derived given an experimental geometry. Some common results may be found in the review by Richardson and Riehl (1977).

5.2. MCPL experimental results from luminescent lanthanide systems

In table 13 we list the applications of MCPL to lanthanide systems that have been reported. Applications of this technique in solids have focused on its use as a probe of Zeeman level degeneracy as a probe of site symmetry (Schwartz et al., 1977), or in studies interested in examining the field-induced mixing of electronic states as shown in recent applications of MCPL measurements from Tb(III) and Ho(III) doped into garnets (Valiev et al., 2002a, 2002b).

In an early study, Brittain and Richardson have examined the effect of an axial magnetic field on the energy levels of Eu(III) (Richardson and Brittain, 1981), and used the results...
<table>
<thead>
<tr>
<th>Lanthanide complex or compound</th>
<th>Sample description</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>EuCl$_3$</td>
<td>0.10 M in DMF and MeOH</td>
<td>(Foster et al., 1983)</td>
</tr>
<tr>
<td>Eu(NO$_3$)$_3$</td>
<td>Dissolved in various protic and aprotic solvents</td>
<td>(Stephens et al., 1986)</td>
</tr>
<tr>
<td>[Eu(DPA)$_3$]$^{3-}$</td>
<td>DPA = dipicolinic acid</td>
<td>0.10 M in H$_2$O</td>
</tr>
<tr>
<td>[Eu(ODA)$_3$]$^{3-}$</td>
<td>ODA = oxydiacetic acid</td>
<td>0.10 M in H$_2$O</td>
</tr>
<tr>
<td>[Eu(IDA)$_3$]$^{3-}$</td>
<td>IDA = iminodiacetic acid</td>
<td>0.10 M in H$_2$O</td>
</tr>
<tr>
<td>[Eu(MIDA)$_3$]$^{3-}$</td>
<td>MIDA = methyl-iminodiacetic acid</td>
<td>0.10 M in H$_2$O</td>
</tr>
<tr>
<td>[Eu(EDTA)]$^{1-}$</td>
<td>25 mM in D$_2$O</td>
<td>(Schultheiss and Gliemann, 1986)</td>
</tr>
<tr>
<td>[Eu(citrate)$_3$]$^{3-}$</td>
<td>50 mM in D$_2$O</td>
<td>(Schultheiss and Gliemann, 1986)</td>
</tr>
<tr>
<td>[Eu(2,2'-bipyridine-1,1'dioxide)$_4$]$^{3+}$</td>
<td>0.25 mM in nitromethane</td>
<td>(Morita et al., 1992)</td>
</tr>
<tr>
<td>[Eu(fod)$_3$]</td>
<td>2.0 mM in H$_2$O</td>
<td>(Morita et al., 1992)</td>
</tr>
<tr>
<td>Fod = 6,6,7,8,8-heptafluoro-2,2-dimethyl-3,5-octanedionate</td>
<td>0.67 mM in DMF</td>
<td>(Richardson and Brittain, 1981)</td>
</tr>
<tr>
<td>[Eu(thd)$_3$]</td>
<td>thd = 2,2,6,6-tetramethyl-3,5-heptanedionate</td>
<td>0.67 mM in DMF</td>
</tr>
<tr>
<td>[Eu(dbm)$_3$]</td>
<td>dbm = dibenzoylmethanate</td>
<td>0.67 mM in DMF</td>
</tr>
<tr>
<td>[Eu(Bzac)$_3$]</td>
<td>Bzac = 1-benzoylacetonate</td>
<td>0.67 mM in DMF</td>
</tr>
<tr>
<td>[Eu(2,2'-bipyridine)$_2$]$^{3+}$</td>
<td>2.0 mM in DMSO</td>
<td>(Morita et al., 1992)</td>
</tr>
<tr>
<td>[Eu(facam)$_3$]</td>
<td>facam = 3-(trifluoromethyl-hydroxymethylene)-d-camphorato</td>
<td>Dissolved in poly(methylmethacrylate) at 77 K</td>
</tr>
<tr>
<td>[Eu(β-diketonates a-d)$_3$]</td>
<td>a. 1,1,1,5,5,5-hexafluoro-2,4-pentanedionato</td>
<td>0.01 M in DMF</td>
</tr>
<tr>
<td>b. 1,1,1-trifluoro-2,4-pentanedionato</td>
<td>0.01 M in DMF</td>
<td>(Foster et al., 1983)</td>
</tr>
<tr>
<td>c. 1,3-bis(3-pyridyl)-1,3-propanedionato</td>
<td>0.01 M in DMF</td>
<td>(Foster et al., 1983)</td>
</tr>
<tr>
<td>d. 4,4,4-trifluoro-1-(2-furyl)-1,3-butanedionato</td>
<td>0.01 M in DMF</td>
<td>(Foster et al., 1983)</td>
</tr>
<tr>
<td>Cs$_2$[NaTbCl$_6$]</td>
<td>Crystal</td>
<td>(Schwartz et al., 1977)</td>
</tr>
<tr>
<td>Cs$_2$[NaEuCl$_6$]</td>
<td>Crystal</td>
<td>(Morley et al., 1982a)</td>
</tr>
<tr>
<td>Cs$_2$[NaPrCl$_6$]</td>
<td>Crystal</td>
<td>(Morley et al., 1982b)</td>
</tr>
<tr>
<td>Y$_3$Al$<em>5$O$</em>{12}$:Tb$^{3+}$</td>
<td>5% doped crystal</td>
<td>(Valiev et al., 2002a)</td>
</tr>
<tr>
<td>Y$_3$Al$<em>5$O$</em>{12}$:Ho$^{3+}$</td>
<td>5% doped crystal</td>
<td>(Valiev et al., 2002b)</td>
</tr>
</tbody>
</table>
to gain information concerning whether or not the Eu(III) complex possessed axial symmetry (i.e. a $C_n$ axis where $n > 2$). Similar studies have been undertaken on 9-coordinate tris terdentate complexes in aqueous (Foster and Richardson, 1983) and non-aqueous solutions (Foster et al., 1983), a tetrakis bidentate complex (Morita et al., 1992), and other less symmetric systems as given in table 13.

6. Summary

Although still limited to a very few laboratories around the world, the measurement of the usually small net circular polarization in the luminescence from chiral species containing lanthanide ions can now be accomplished with a high degree of sensitivity and reliability. Recent advances in instrumentation, such as the ability to measure the time-dependence of $g_{\text{lum}}$, should allow for additional applications of this technique in the microsecond to millisecond time frame. This time domain is an interesting one since structural changes associated with many important biological processes are known to occur on this time scale. Instrumental improvements, such as the use of array detectors, should allow for significant advances in the time required to collect an accurate CPL spectrum. This will, of course, help to eliminate some observed photochemical degradation which is a common problem when trying to measure the CPL from systems using indirect UV excitation. It is certainly possible to envisage the development of circularly polarized luminescence imaging as an important tool for biological applications, and the detection of circular polarization in the luminescence from single molecules containing luminescent lanthanide ions could provide a unique probe of individual molecular dynamics.

As described throughout this chapter, CPL is becoming increasingly useful as a probe of the existence of chiral lanthanide structures, and as an indicator of changes in chiral structure. However, there are currently no reliable correlations relating specific aspects of chiral structure to CPL measurements. The development of such spectra-structure correlations is key to the advancement of this technique as a useful probe of the stereochemistry of chiral lanthanide systems.

Acknowledgements

The authors gratefully acknowledge the many students and collaborators who have contributed to our research efforts in the CPL spectroscopy of lanthanide systems. Special recognition is given to Dr. Frederick L. Richardson, Dr. Harry P.J.M. Dekkers, Dr. Janina Legendziewicz, and Dr. David Parker for many years of fruitful collaboration. G.M. express his gratitude to Dr. Jean-Claude G. Bünzli for his support and guidance during his doctoral studies, and JPR wishes to recognize the contributions of his former doctoral students Dr. Christine L. Maupin, Dr. Nursen Çoruh, and Dr. Gary L. Hilmes. The authors also thank Mrs. Françoise C. Muller for her technical help in the preparation of the manuscript.
References

Chapter 221

LANTHANIDE-CONTAINING COORDINATION POLYMERS

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List of acronyms

\begin{tabular}{ll}
\text{bdc} & 1,4 benzene dicarboxylate \\
\text{BTB} & 4,4′,4″-benzene-1,3,5-triyl-trienzoate \\
\text{btc} & 1,3,5 benzene tricarboxylate \\
\text{btc} & 1,2,4,5 benzene tetracarboxylate \\
\text{bhc} & benzene hexacarboxylate \\
\text{IRMOS} & Iso Reticular Metal Organic Framework \\
\text{meso-L} & meso-1,2-bis(ethyisulfinyl)ethane \\
\text{ndc} & 2,6-naphthalenedicarboxylate \\
\text{ox} & oxalate \\
pda & 1,4-phenyleniacetate \\
pdc & 3,5-pyrazoledicarboxylate \\
ppa & N-(4-pyridinylmethyl)-4-pyrydinecarboxamid \\
rac-L & rac-1,2-bis(ethylisulfinyl)ethane \\
xrd & X-Ray Diffraction \\
tga & ThermoGravimetric Analysis \\
tdxd & Temperature Dependent X-ray Diffraction \\
 BET & Brunauer Emmett Teller \\
\end{tabular}

* Corresponding author.
1. Introduction

The assembly of metal–organic infinite framework via the coordination of metal ions by multifunctional organic ligands is a field of increasing interest. Work along this line is motivated by the concept that molecular-based coordination polymers have potential technological applications such as opto-electronic or magnetic devices and microporous materials for shape- and size-selective separations and catalysis. The advantage of these metal-organic open frameworks is to allow a wide choice in various parameters including diverse electronic properties and coordination geometry of the metal ions as well as versatile functions and topologies of the organic ligands. So, specific electronic or structural characteristics of the metal ion can be translated into bulk properties for solid state compounds. In this context, there has been current interest in synthesizing stable polymeric coordination open frameworks with transition metal ions. The lanthanide ions owning to their very similar chemical and structural properties but very different physical properties from one to the other could lead to materials exhibiting tunable properties via the choice of the rare earth ion.

We would like to report here the state of the art in this rapidly emerging field of research. This review is devoted to coordination polymers. These are built up from metal ions and multi-functional organic ligands. Examples of such polymeric coordination frameworks are known for a long time. Until recently they have been generally studied because of their interesting molecular topologies and crystal packing motifs (Batten and Robson, 1998; O’Keeffe et al., 2000), their interest as molecular precursor for condensed phases, or their expected magnetic properties. As an illustration of this, one may consider the oxalate based coordination polymers. The well-known honeycomb-like molecular motif of the hydrated oxalato lanthanide(III) is drawn on fig. 1.

Actually, these oxalato lanthanide compounds have been extensively studied for structural (Hansson, 1970, 1973; Hansson and Albertsson, 1968; Michaelides et al., 1988; Kahwa et al., 1984; Watanabe and Nagashima, 1971; Bataille et al., 1999; Bataille and Louër, 1999), magnetic (Larionova et al., 1998; Decurtins et al., 1998; Mörtl et al., 2000) purposes as well as for their interest as molecular precursors for condensed phases usable as magnets (Nag and Roy, 1976a, 1976b), high temperature superconducting ceramics (Jongen et al., 1999) or pigments (Trollet et al., 1997; Romero et al., 1995).

Due to the variety of the existing coordination polymers and of their potential applications, the literature describing them is very abundant and dispersed. Therefore performing an exhaustive review of all the reported coordination polymers is out of the scope of this article.

There is currently a renewed interest in that field partly because of the nano-technologies emergence. Actually the design of nano-porous open frameworks is a factor of this revival (Yaghi et al., 1996, 2004; Serpaggi et al., 1999) because these compounds are anticipated to exhibit good efficiency as far as size selective separation, catalysis and gas storage are concerned (Eddaoudi et al., 2002).

One of the major challenges in that field is to maintain the host framework integrity in the absence of guest molecules in order to allow reversible access to the cavities within the host. As one can notice, the requirements for the choice of the binding organic ligand are
its chemical and thermal robustness, its topological properties and its functionality. Those requirements are fulfilled by the benzene-polycarboxylate ligands (see scheme 1). Indeed, these ligands are rigid, planar, very stable thermally and chemically and they have non negligible structuring effect thanks to \( \pi \)-stacking. Furthermore their carboxylato functional groups present a very good affinity with most of metallic cations and, due to their numerous possible coordination modes (see scheme 2), they allow one to expect a great structural diversity. At last, most of them are commercially available and cheap what is an asset for further industrial applications.

This review is essentially devoted to the coordination polymers which have been synthesized for their potential technologic applications or which could obviously be of interest as far as technological applications are concerned. As a consequence, polymeric complexes based on benzene-poly-carboxylates ligands and their derivatives will be described in more details. The paragraph dealing with coordination polymers containing transition metal ions is mainly
2. Coordination polymers containing transition metal ions

Actually very numerous transition elements ions containing coordination polymers have been described. Some of them are listed in table 1.

From this table, it is clear that this class of ligands has led to numerous structurally different compounds exhibiting various physical properties. Describing each of them is out of the scope of this article and we will focus only on the noticeable \([\text{Cu}_3(\text{btc})_2(\text{H}_2\text{O})_3]_n\) tri-dimensional compound reported by Chui et al. in 1999 (Chui et al., 1999). This compound, thermally stable up to 240 °C, present a 3D system of channels with a pore diameter of 1 nanometer and an
Table 1

<table>
<thead>
<tr>
<th>Chemical formula</th>
<th>Metal ion</th>
<th>Dimensionality</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,4 benzene dicarboxylate (bdc)</td>
<td>V(OH)((bdc)\cdot0.75(bdcH_2))</td>
<td>V$^{3+}$</td>
<td>3D</td>
</tr>
<tr>
<td></td>
<td>Zn$_3$(bdc)$_2\cdot6(CH_3OH)$</td>
<td>Zn$^{2+}$</td>
<td>3D</td>
</tr>
<tr>
<td>1,3,5 benzene tricarboxylate (btc)</td>
<td>Zn$_2$(btc)(NO$_3$)(H$_2$O)$_2$·(C$_2$H$_5$OH)$_5$Zn$_2$+</td>
<td>Zn$^{2+}$</td>
<td>3D</td>
</tr>
<tr>
<td></td>
<td>Ni$_5$(btc)$_2$(Hbtc)$_2$(C$_5$H$_5$N)$_2$.2/3C$_5$H$_5$N</td>
<td>Ni$^{2+}$</td>
<td>1D</td>
</tr>
<tr>
<td></td>
<td>Co$_2$(btc)(C$_5$H$_5$N)$_2$.2$(H$^2$btc)$\cdot$2C$_5$H$_5$N</td>
<td>Ce$^{2+}$</td>
<td>2D</td>
</tr>
<tr>
<td></td>
<td>Mn$_3$(btc)$_2$, $\times$H$_2$O·yC$_4$H$_9$OH·zC$_5$H$_5$NNi$_2$+</td>
<td>Mn$^{2+}$</td>
<td>3D</td>
</tr>
<tr>
<td></td>
<td>M(Hbtc)$\cdot$2H$_2$O·0.5Sr$_2$+ Ba$_2$+</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ni$_3$(btc)$_2$(H$<em>2$O)$</em>{12}$</td>
<td>Ni$^{2+}$</td>
<td>1D</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1,2,4,5 benzene tetracarboxylate (btc)</td>
<td>V$_2$(OH)$_4$[C$_6$H$_2$(CO$_2$)$_4$]·4H$_2$O</td>
<td>V$^{3+}$</td>
<td>3D</td>
</tr>
<tr>
<td></td>
<td>V(OH)[C$_6$H$_2$(CO$_2$)$_2$(COOH)$_2$]·H$_2$O</td>
<td>V$^{3+}$</td>
<td>3D</td>
</tr>
<tr>
<td></td>
<td>Cu$_2$(btc)·2H$_2$O</td>
<td>Cu$^{2+}$</td>
<td>2D</td>
</tr>
<tr>
<td></td>
<td>Ni$_5$(2',2'-bipy)$_2$(OH)$_2$(H$^2$btc)</td>
<td>Ni$^{2+}$</td>
<td>1D</td>
</tr>
<tr>
<td></td>
<td>Co$_2$(btc)(H$<em>2$O)$</em>{18}$</td>
<td>Co$^{2+}$</td>
<td>1D</td>
</tr>
<tr>
<td>benzene hexacarboxylate (bhc)</td>
<td>Co$_3$(bhc)$\cdot$18H$_2$O</td>
<td>Co$^{2+}$</td>
<td>1D</td>
</tr>
<tr>
<td></td>
<td>Al$_2$(bhc)$\cdot$16H$_2$O</td>
<td>Al$^{3+}$</td>
<td>3D</td>
</tr>
</tbody>
</table>

accessible porosity of nearly 40% in the solid. Unlike classical zeolites, the channel linings can be chemically functionalized. An extended view of its asymmetric unit and a projection view of the polymer framework are shown in fig. 2.

This example emphasizes the potentiality of the benzene polycarboxylate ligands. However some other coordination polymers containing transition metal ions deserve to be highlighted.

So is the compound Cu$_3$(BTB)$_2$(H$_2$O)$_3$·(DMF)$_9$(H$_2$O)$_2$ described in 2001 by O.M. Yaghi et al. (Chen et al., 2001, where H$_3$BTB stands for 4,4',4''-benzene-1,3,5-triyl-tribenzoic acid. This compound present large circular pores (16.4 Å in diameter) in which voluminous amounts of gases and organic solvents can be reversibly sorbed. It is thermally stable up to 250 °C. A view of the polymeric framework is reported in fig. 3.

The H$_3$BTB ligand is topologically rather similar to benzene polycarboxylate. In particular it is rigid. Some other coordination polymers built with flexible ligands are worth noticing. That is in particular the case of the two 3D networks obtained by reaction of Ag$^{+}$ with pyridyl type ligands (Tong et al., 2002). A perspective view of the 3D network of the compound
[Ag₂(ppa)₂(ox)]₉H₂O where ppa stands for N-(4-pyridinylmethyl)-4-pyridinecarboxamide and ox for oxalate is drawn in fig. 4.

This compound can be described as 2D [Ag(ppa)]ₙ infinite layers pillared by (ox) ligands as connectors. Unfortunately this compound is only structurally described and there is no information about its thermal stability nor about its microporous character.

To complete this short overview about coordination polymers containing transition metal ion, it is worthwhile pointing to recent works demonstrating that the construction of coordination polymers by employing rigid organic building-blocks binding metallic clusters provide an effective strategy to form robust hosts possessing porosity (Eddaoudi et al., 2001; Zhao et al., 2002). For instance, the third compound reported in fig. 5 is one of the highest porous metal-organic frameworks reported so far (Eddaoudi et al., 2002): 91% of the crystal volume is open space.

This compound which can be described as a cubic 3D network of zinc–oxygen clusters connected by molecular struts exhibits a density lower than any previously reported crystalline material (around 0.2 g cm⁻³) and has been synthesized for its predicted useful properties for gas storage. It belongs to a family of sixteen metal-organic frameworks with the same network topology but various functionalities and pore sizes. This family has been systematically designed and for instance, the pore size is varied from 3.8 Å and 28.8 Å. One member of
the family exhibits a high capacity for methane storage. All the sixteen compounds are rather stable thermally (up to 400°C).

3. Lanthanide-containing coordination polymers

Despite the impressive development of the field of coordination polymers containing transition metal ions, the analogous chemistry involving the lanthanide ions remains much less developed. This is a priori rather surprising because the lanthanide ions are the largest family of chemically similar elements in the periodic table. So in principle, it must be possible to design coordination polymers with tunable physical properties by an appropriate choice of the lanthanide ion. However the relatively high cost of the lanthanide ions implies the design of materials with very specific properties. Indeed if the substitution of the lanthanide ion by a main group or a transition metal ion is possible without changing the physical properties, then there is no economical interest. For instance it seems worthless to synthesize lanthanide containing functionalized coordination polymers except if the lanthanide ion plays an essen-
tial role in the chemical process. That is why, before designing a material one should think about the targeted physical properties.

Three physical properties are generally targeted in the field of lanthanide coordination polymers: the magnetic properties, the optical properties and the catalytic properties.

Much work has been done in the field of extended networks involving rare-earth ions and exhibiting interesting magnetic properties. These works were based on the observations that some lanthanide ions can present the higher local spin moments and the higher anisotropies of the periodic table. Unfortunately the inner character of the lanthanide ion magnetic orbitals leads to very poor overlap and extremely low coupling as far as coordination compounds are concerned (Decurtins et al., 1998; Daiguebonne et al., 1998, 2001b; Guillou et al., 1992a, 1992b, 1992c; Kahn and Guillou, 1993; Oushoorn et al., 1996). This field of research remains active and interesting because the mechanisms of the 4f-4f and 4f-3d interactions are still

Fig. 4. Projection view from the b-axis direction of the 3D coordination network of \([\text{Ag}_2(ppa)_2(\text{ox})]\cdot9\text{H}_2\text{O}:\) monoclinic, C2/c, \(a = 28.947(8)\) Å, \(b = 8.617(3)\) Å, \(c = 16.307(6)\) Å, \(\beta = 121.07(1)\)°. Redrawn from Tong et al. (2002).
misunderstood (Decurtins et al., 1998; Sutter et al., 1998, 1999; Costes et al., 1998; Liu et al., 1998; Sanada et al., 1998; Rizzi et al., 2002) (for a recent review see reference (Benelli and Gatteschi, 2002)). So, to the best of our knowledge, magnetic materials seems not to be a driving force for the design of lanthanide based coordination polymers.
On the other hand, lanthanide ions are well known for their unique optical properties. These properties are extensively used in numerous fields such as laser amplifiers (Adam, 2002; Kuriki et al., 2002) or electroluminescent materials (Kido and Okamoto, 2002) for instance. Coordination polymers could be very interesting as far as optical properties are concerned. Indeed they are transparent and the rare earth ions distribution can be perfectly controlled by an appropriate choice of the ligand while in plastic or glasses they are generally statistically dispersed. The prerequisites of such coordination polymers exhibiting optical properties are obviously the following: the material must be thermally rather stable, solvent molecules must be avoided and the inter-metallic distances must be carefully adjusted in order to allow the best efficiency for the targeted application.

Some lanthanide ions have also well-known catalytic properties. For instance, the Sm(II) ion is extensively used in asymmetric catalysis (Ilanaga et al., 2002) and cerium and praseodymium, thanks to their III/IV valences, are used in many depollution processes (“Catalysis by Ceria and Related Materials”, 2002). This second class of application seems to be very promising as far as coordination polymers are concerned. The prerequisites for these applications are clearly a high thermal stability, a porous network and a chemical availability of the lanthanide ions.

From this rapid overview it appears that the benzene polycarboxylates and their derivatives could be good ligands for the design of 3D networks exhibiting catalytic or optical properties depending on the inserted lanthanide ion(s). Indeed benzene polycarboxylates are thermally stable, their carboxylato coordinating groups have good affinity with the hard acid character (according to Pearson’s definition) of the lanthanide ions. That is probably why they are involved in many studies related to lanthanide coordination polymers (see table 2).

From the above table it appears that it is rather difficult to control the dimensionality of the obtained material. This is inherent to the use of both lanthanide ions and ligands presenting coordinating carboxylate groups.

Indeed, contrary to transition metal ions, lanthanide ions, due to the inner character of their 4f valence orbitals, exhibit very little preference in bonding direction (Karraker, 1970). The bonds between lanthanide ions and ligands are essentially ionic and the spatial arrangement of the ligand around the lanthanide ions are mainly due to steric hindrance and inter-ligand interactions. Further more it is well known that the carboxylate ions presents several coordination modes (see scheme 2) (Ouchi, 1988).

The lack of structuring effect from lanthanide ions and the flexibility of the coordinating carboxylate groups are responsible of the structural diversity observed.

3.1. Coordination polymers based on the \((btc)^{3-}\) ligand

As an illustration of this structural diversity one can consider the five Er\(^{3+}\)-containing coordination polymers which are reported in table 2. All five can be obtained in aqueous media by reacting an Er\(^{3+}\) salt with the sodium salt of 1,3,5-benzene-tricarboxylic acid (Daiguebonne et al., 2000b). As can be seen in fig. 6, where the molecular motifs of these five compounds are reported, despite their similar chemical formulae they present very different structural types.

The first compound, namely Er\((btc)\cdot6\text{H}_2\text{O}\) (fig. 6(a)) has been synthesized by slow diffusion through water and presents a structure consisting in the juxtaposition of parallel ribbon-
Table 2
List of lanthanide based coordination polymers with benzene polycarboxylate ligands. Only structurally characterized compounds have been reported.

<table>
<thead>
<tr>
<th>Chemical formula</th>
<th>Lanthanide</th>
<th>Dimensionality</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,4 benzene dicarboxylate (bdc)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Er$_4$(bdc)$_6$.6H$_2$O</td>
<td>Er</td>
<td>3D</td>
<td>Pan et al., 2001</td>
</tr>
<tr>
<td>Ln$_2$(bdc)$_3$.4H$_2$O</td>
<td>Tb and Er</td>
<td>3D</td>
<td>Reneke et al., 1999a; Li, H. et al., 1998; Pan et al., 2001</td>
</tr>
<tr>
<td>Ln$_2$(bdc)$_3$(H$_2$O)$_6$</td>
<td>Er</td>
<td>3D</td>
<td>Deluzet et al., 2003</td>
</tr>
<tr>
<td>Tb(bdc)(NO$_3$).2DMF</td>
<td>Tb</td>
<td>3D</td>
<td>Reneke et al., 1999b</td>
</tr>
<tr>
<td>1,3,5 benzene tricarboxylate (btc)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Er(btc)(H$_2$O)$_3$.H$_2$O</td>
<td>Er, Sm</td>
<td>1D</td>
<td>Zhi-Bang et al., 1990; Daiguebonne et al., 2002</td>
</tr>
<tr>
<td>Lu(btc)(H$_2$O)$_3$.4H$_2$O</td>
<td>Lu</td>
<td>1D</td>
<td>Zhi-Bang et al., 1991</td>
</tr>
<tr>
<td>Gd(btc)(H$_2$O)$_3$.1.5H$_2$O</td>
<td>Gd, Er, Sm</td>
<td>2D</td>
<td>Daiguebonne et al., 1998, 2000a, 2002</td>
</tr>
<tr>
<td>Er(btc)(H$_2$O)$_3$.3.5H$_2$O</td>
<td>Er</td>
<td>1D</td>
<td>Daiguebonne et al., 1999</td>
</tr>
<tr>
<td>Y(btc)(H$_2$O)$_6$</td>
<td>Y, Er, Sm</td>
<td>1D</td>
<td>Daiguebonne et al., 1999, 2002</td>
</tr>
<tr>
<td>Er$_3$(btc)$_3$(H$_2$O)$_8$.4H$_2$O</td>
<td>Er</td>
<td>1D</td>
<td>Daiguebonne et al., 2000a</td>
</tr>
<tr>
<td>1,2,4,5 benzene tetracarboxylate (bttc)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Eu$_2$(bttc)(H$_2$btc)(H$_2$O).4H$_2$O</td>
<td>Eu</td>
<td>3D</td>
<td>Wu, C.D. et al., 2002</td>
</tr>
<tr>
<td>Yb$_3$(bttc)$_3$(H$_2$O)$_3$.6H$_2$O</td>
<td>Yb</td>
<td>3D</td>
<td>Cao et al., 2002b</td>
</tr>
<tr>
<td>Er$_8$(bttc)$_6$(H$<em>2$O)$</em>{16}$.16H$_2$O</td>
<td>Er</td>
<td>3D</td>
<td>Cao et al., 2002b</td>
</tr>
<tr>
<td>Tb$_2$(bttc)(H$_2$btc)(H$_2$O)$_2$.4H$_2$O</td>
<td>Tb</td>
<td>3D</td>
<td>Cao et al., 2002b</td>
</tr>
<tr>
<td>Ce(Hbttc).H$_2$O</td>
<td>Ce</td>
<td>1D</td>
<td>Yaghi et al., 1997b</td>
</tr>
<tr>
<td>benzene hexacarboxylate (bhc)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ln$_2$(bhc)(H$<em>2$O)$</em>{10}$.4H$_2$O</td>
<td>Y, Er</td>
<td>1D</td>
<td>Deluzet and Guillou, 2003; Robl and Hentschel, 1992</td>
</tr>
<tr>
<td>Ln$_2$(bhc)(H$_2$O)$_6$</td>
<td>La–Er</td>
<td>3D</td>
<td>Chui et al., 2001</td>
</tr>
<tr>
<td>Ln$_2$(bhc)(H$_2$O)$_4$.4H$_2$O</td>
<td>Tm–Lu</td>
<td>3D</td>
<td>Chui et al., 2001</td>
</tr>
<tr>
<td>Tm$_2$(bhc)(µ-H$_2$O)$_2$(H$_2$O)$_2$</td>
<td>Tm</td>
<td>3D</td>
<td>Chui et al., 2001</td>
</tr>
<tr>
<td>La$_2$(bhc)(H$_2$O)$_9$.2H$_2$O</td>
<td>La</td>
<td>3D</td>
<td>Wu, L.P. et al., 1996</td>
</tr>
</tbody>
</table>

Like molecular motifs spreading along the $\vec{a}$ direction. The Er$^{3+}$ ion is nine coordinated by six oxygen atoms from water molecules as well as by three oxygen atoms from three carboxylato groups. Free carboxylato groups are facing Er$^{3+}$ ions from adjacent ribbons forming an hydrogen bonds network with coordinated water molecules and so ensuring the cohesion of the crystal packing. One may note that there is no crystallization water molecule in this compound.

An other 1D coordination polymer has been described which presents the same global chemical formula Er(btc)(H$_2$O)$_5$.H$_2$O (Zhi-Bang et al., 1990). In this compound there are only five coordinated water molecules, the sixth one being a crystallization water molecule whereas in the previous compound the six water molecules were coordinated to the lanthanide ion. The structure can be described as parallel chains spreading along the $\vec{b}$ axis (see fig. 6(b)). The molecular chains stacks in such a way that phenyl groups superimpose along the $\vec{b}$ axis.
Fig. 6. Molecular motifs of the five Er(btc)-nH₂O coordination polymers already reported. Redrawn from Daiguebonne et al. (2000b). (a) Perspective view along the c-axis of Er(btc)-6H₂O: monoclinic, Cc, a = 11.373(2) Å, b = 17.850(3) Å, c = 7.130(1) Å, β = 119.14(3)°. (b) Projection view along the a-axis of the chain-like molecular motif of Er(btc)(H₂O)₃·H₂O: triclinic, P₁, a = 7.631(2) Å, b = 9.965(2) Å, c = 11.219(2) Å, α = 106.98(2)°, β = 103.52(2)°, γ = 107.50(2)°. (c) Projection along the c-axis of Er(btc)(H₂O)₃·3.5H₂O: monoclinic, C2/c, a = 14.740(2) Å, b = 16.987(2) Å, c = 14.459(1) Å, β = 118.754(9)°. (d) Projection along the a-axis of Er(btc)(H₂O)₃·1.5H₂O: monoclinic, C2/c, a = 20.454(2) Å, b = 9.973(1) Å, c = 15.251(2) Å, β = 125.68(1)°. (e) Projection along the a-axis of Er₃(btc)₃(H₂O)₉·4H₂O: triclinic, P₁, a = 9.988(2) Å, b = 15.656(3) Å, c = 19.698(3) Å, α = 110.77(2)°, β = 100.63(2)°, γ = 100.58(2)°.
The Er$^{3+}$ ions are nine coordinated by four oxygen atoms from two bidentate carboxylato groups and by five oxygen atoms from water molecules. The third mono-dimensional compound, Er(btc)(H$_2$O)$_3$:3.5(H$_2$O) (fig. 6(c)), has been obtained as single crystals by slow diffusion in an agarose gel. It can also be obtained as a precipitated powder by direct reaction between Er$^{3+}$ and (btc)$^{3-}$ in water. Its structure consists in the juxtaposition of molecular zig-zag chain-like motifs spreading along the $\vec{b}$ axis. The Er$^{3+}$ ion is only eight coordinated by five oxygen atoms from water molecules as well as by three oxygen atoms from carboxylato groups. The cohesion of the crystal structure is once more ensured by an hydrogen bond network but the chains are disposed in such a way that the $\pi$ systems of the btc$^{3-}$ ligands overlap. Crystallization water molecules are filling the inter-chain space as one can see on fig. 7.

This compound, when dehydrated by heating, leads to a bi-dimensional compound with a chemical formula Er(btc)(H$_2$O)$_3$:1.5(H$_2$O) (fig. 6(d)). It presents a crystal structure which consists in the juxtaposition of bi-dimensional honeycomb-like molecular motifs spreading parallel to the ($\vec{b}$, $\vec{a} + \vec{c}$) plane where one corner out of two is occupied by an Er$^{3+}$ ion and the other by the center of a phenyl ring. The Er$^{3+}$ ions are nine coordinated by three oxygen atoms from water molecules and six oxygen atoms from three carboxylato groups. In this structure it is worth noting that all the carboxylato groups are bidentate (mode II). The planes are connected together via an hydrogen bond network and the phenyl rings are superimposed in such a way that their $\pi$ systems interact. This compound may also be obtained via a slow diffusion through an agarose gel medium.

Dehydrating further this compound leads to Er$_3$(btc)$_3$(H$_2$O)$_8$:4H$_2$O. Its structure consists in the juxtaposition of parallel tubes with hexagonal section spreading along the $a$-axis (fig. 6(e)). Each tube is constituted by cyclic honeycomb-like molecular motifs similar to those already described for the previous compound. Tubes are held together via an hydrogen bonds network. The chemical formula of this compound is a little bit more complicated than those previously described because there is three crystallographically different Er$^{3+}$ ions in the structure. Two out of the three are nine coordinated by six oxygen atoms from three carboxylato groups (mode II) and three oxygen atoms from coordinated water molecules, while the third Er$^{3+}$ ion is only eight coordinated by six oxygen atoms from three carboxylato groups and two water molecules. The crystallization water molecules are localized inside the tubes. The diameter of the tubes is roughly 11 Å. This compound can also be obtained thanks to a gel diffusion method.

As one can notice from these five structural descriptions, the structuring role of the ligand is predominant and the topology of the crystal structure is mainly due to hydrogen bonds and overlaps of the $\pi$ systems. The structural diversity seems also to be a consequence of the versatility of the coordinating carboxylato groups. As an example the different coordinating modes encountered in these five structures are listed in table 3.

It can also be noticed that the third compound described, namely Er(btc)(H$_2$O)$_3$:3.5(H$_2$O), presents a pseudo 3D structure via hydrogen bonds. However it is not a microporous solid precursor. Indeed, when dehydrated this compound reversibly rearrange leading successively to the honeycomb-like compound and then to the hexagonal-tube like structure. This last compound finally leads to an anhydrous compound which unfortunately has not yet
been structurally characterized. By re-hydration of this anhydrous compound one can re-obtain the zig-zag chain-like structure. This cycle of hydration/dehydration is symbolized in fig. 8.
Table 3
Coordination modes of the carboxylato groups in the previously described benzene dicarboxylate based coordination polymers. See scheme 2 for a definition of the coordination modes I, II and III.

<table>
<thead>
<tr>
<th>Complexes</th>
<th>Number of oxygen atoms involved in bonding mode</th>
<th>Coordination numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Er(btc)(H₂O)₃·1.5H₂O</td>
<td>0 6 0</td>
<td>9</td>
</tr>
<tr>
<td>Er(btc)(H₂O)₅·3H₂O</td>
<td>1 2 0</td>
<td>8</td>
</tr>
<tr>
<td>Er(btc)(H₂O)₃·H₂O</td>
<td>0 2 0</td>
<td>9</td>
</tr>
<tr>
<td>Er(btc)(H₂O)₆</td>
<td>1 0 2</td>
<td>9</td>
</tr>
<tr>
<td>Er₂(btc)₃(H₂O)₈·4H₂O</td>
<td>0 6 0</td>
<td>8</td>
</tr>
<tr>
<td>Er1</td>
<td>0 6 0</td>
<td>9</td>
</tr>
<tr>
<td>Er2</td>
<td>0 6 0</td>
<td>9</td>
</tr>
<tr>
<td>Er3</td>
<td>0 6 0</td>
<td>9</td>
</tr>
</tbody>
</table>

Fig. 8. Hydration/dehydration cycle of the Er(btc)-nH₂O compounds.
The anhydrous compound, whose chemical formula is Er(btc), has not been structurally characterized because of its lack of crystallinity and its physical properties have not been explored in details. This family is characteristic of the problem encountered when trying to construct molecular based microporous materials, that is obstruction of the pores by interpenetration of networks or collapse of the structure upon removal of the guest molecules intercalated in the cavities. This problem is, of course, also present for coordination polymers based on transition elements but it is much more crucial for lanthanide-based materials. Indeed, lanthanide ions present a high coordination number (generally ranging between 7 and 12) and their coordination sphere most often contains some solvent molecules. These coordination water molecules, when removed upon dehydration, lead to structural rearrangements and, sometimes, to a loss of the microporous character.

Another compound with chemical formula Eu₃(H₂O)(OH)₆(btc)·3H₂O has been reported recently (Serre and Férey, 2002) (see fig. 9). It has been obtained via an hydrothermal synthesis. The crystal structure has been solved on the basis of an XRD diagram of the polycrystalline powder. It displays a 3D structure and consists in inorganic layers linked together by

![Fig. 9. Projection along the b-axis of Eu(H₂O)(OH)₆(btc)·3H₂O: monoclinic, P2₁, a = 17.640(1) Å, b = 3.689(2) Å, c = 12.385(1) Å, β = 91.247(4)°. Redrawn from Serre and Férey (2002).](image-url)
(btc)$^{3-}$ ions. The inorganic layers are built up from chains of europium polyhedra consisting of face-sharing europium polyhedra linked through $\mu_3$-hydroxo groups or oxygen atoms from carboxylates. Two out of the three crystallographically independent Eu$^{3+}$ ions are surrounded by seven $\mu_3$-hydroxo groups and two oxygen atoms from a bidentate carboxylato group (mode II) while the third Eu$^{3+}$ ion is surrounded by seven $\mu_3$-hydroxo groups and an oxygen atom from a monodentate carboxylato group (mode I). It completes its coordination sphere by a $\mu_2$-oxygen atom from a water molecule.

The thermal behavior of this compound has been studied by TGA and TDXD. Several phase transitions have been observed but none of these phases has been characterized. The removal of the crystallization and coordination water molecules is reversible. A BET nitrogen adsorption measurement has been performed on the solid dehydrated at 250°C and revealed a low surface area ($\cong 15 \text{ m}^2 \text{ g}^{-1}$) probably due to a loss of the initial crystal structure upon dehydration.

Very recently, the same group has submitted another paper describing the synthesis and the crystal structure of a three-dimensional anhydrous compound with chemical formula: $R(btc)$ with $R = \text{Pr–Er or Y}$ (Serre et al., 2004). This compound has been obtained via hydrothermal synthesis and the crystal structure has been solved upon X-ray powder diffraction diagram data. The rare-earth ions are eight coordinated by eight oxygen atoms from carboxylato groups, that is, no coordination water molecule is present. All oxygen atoms from btc$^{3-}$ are bound to $R^{3+}$ ions. Two out of the three carboxylato groups present coordination mode IV while the third one is bridging (mode III) (see fig. 10). This compound presents a one-dimensional inorganic sub-network related to the 3D network via (btc)$^{3-}$ ligand.

As all the oxygen atoms are already bound and no water molecule belongs to the $R^{3+}$ ion coordination sphere, there is no structural rearrangement upon heating and the crystal structure remains unchanged upon 500°C. At last, the optical emissions of doped isostructural compounds ($Y_{1-x}R_x(btc)$ with $R = \text{Eu, Tb or Dy and } x \sim 2\%$) have been studied. Very efficient red, green and blue emission respectively have been observed. As a concluding remark, one should notice that this anhydrous phase is different from the one obtained by de-hydration of the tube-like compound previously described.

### 3.2. Coordination polymers based on the (bdc)$^{2-}$ ligand

The benzene-1,4-dicarboxylic acid ($\text{H}_2\text{bdc}$) is a bifunctional rod-like ligand which has lead to numerous interesting compounds by association with transition metal ions. Some interesting coordination polymers have been reported as well. The first one has been described in 1999 (Reneike et al., 1999a) and its chemical formula is $\text{Tb}_2(\text{bdc})_3(\text{H}_2\text{O})_4$. It has been synthesized via an hydrothermal method. Its crystal structure is three-dimensional. As can be seen in fig. 11, each Tb$^{3+}$ ion is eight coordinated by six oxygen atoms from six bridging carboxylato groups (mode III) and two oxygen atoms from two coordination water molecules. Each (bdc)$^{2-}$ ligand is coordinated to four Tb$^{3+}$ ion. Therefore, all oxygen atoms from carboxylato groups are bounded to Tb$^{3+}$ ions.

Furthermore, there is no crystallization water molecule inside the structure. The projection view along the $b$-axis reported in fig. 11 emphasize the porosity of the structure. The
microporous character of this compound as been checked by TGA and TDXD. These measurements reveal that the coordination water molecules can be evacuated under vacuum at 115 °C and that the resulting anhydrous solid is thermally stable up to 450 °C. Unfortunately neither the crystal structure of the anhydrous compound nor its porosity have been reported.

The luminescent lifetime of the anhydrous compound has also been studied but no definitive result was obtained. The authors concluded this paper saying that the anhydrous microporous phase presenting 1D channels can be potentially considered as luminescent probe for detecting small molecules. That is the reason why two years later another group published another paper on this family. In this paper (Pan et al., 2001) the synthesis, via hydrothermal method, and the crystal structure (see fig. 12) of a compound with chemical formula $\text{Er}_4(bdc)_6 \cdot 6\text{H}_2\text{O}$ are described.
Fig. 11. (Left) extended asymmetric unit of Tb$_2$(bdc)$_3$·4H$_2$O: triclinic, P$\overline{1}$, $a = 6.1420(2)$ Å, $b = 10.0694(1)$ Å, $c = 10.0956(3)$ Å, $\alpha = 102.247(2)^\circ$, $\beta = 91.118(1)^\circ$, $\gamma = 101.518(2)^\circ$. Redrawn from Reneike et al. (1999a).
(Right) projection view along the $b$-axis of Tb$_2$(bdc)$_3$·4H$_2$O.

In this structure there are four crystallographically independent Er$^{3+}$ ions. The local coordination geometry around the first one is a dodecahedron which consists in six oxygen atoms from six (bdc)$^{2-}$ groups and two from water molecules. The coordination polyhedron around the second Er$^{3+}$ ion is a capped trigonal prism which contains five carboxylate oxygen and two water oxygen atoms. The coordination environments of the last two Er$^{3+}$ ions are pentagonal pyramidal. The pentagonal planes are composed of four carboxylate oxygen atoms and a water oxygen atom, and the two apical sites are occupied by two oxygen atoms from another carboxylate ion. A projection view along the $a$-axis of the polymeric network is presented in fig. 13.

The thermal behavior has been studied by TGA and TDXD measurements which revealed that Er$_4$(bdc)$_6$·6H$_2$O transforms upon dehydration (up to 300°C) into Er$_4$(bdc)$_6$. This anhydrous compound then re-hydrates in Er$_2$(bdc)$_3$·4H$_2$O which is isostructural with the Tb(III) compound previously described. The anhydrous compound is stable up to 500°C but its crystal structure has not been characterized.

Another compound with formula Er$_2$(bdc)$_3$(H$_2$O)$_6$ has also been reported (Deluzet et al., 2003). Contrary to the two previous ones, it has been obtained by slow diffusion through gel. Its crystal structure is three-dimensional and can be described as the succession of erbium ion planes linked by terephthalato organic groups (see fig. 14).
The Er\(^{3+}\) ions are eight coordinated by three oxygen atoms from coordination water molecules and five oxygen atoms from carboxylate groups. There is no crystallization water molecule in this structure. Unfortunately this interesting compound has not yet been thermally studied because of its poor availability. Indeed it has only been obtained as single crystals dispersed in gel.

Another terephtalate-based coordination polymer has been described (Reneike et al., 1999b). It has been obtained once more thanks to hydrothermal methods in methanol/DMF medium. Its chemical formula is Tb(bdc)(NO\(_3\))\(_2\)DMF. Its crystal structure is reported in fig. 15. The Tb\(^{3+}\) ions are eight-coordinated by eight oxygen atoms. Four out of the eight are from four carboxylato monodentate groups belonging to four different (bdc)\(^2-\) ligand, two out of the eight are from coordinated DMF and the last two are from a bidentate NO\(_3\)\(^-\) ion.

The framework can be described as chains spreading along the c-axis. In these chains the Tb\(^{3+}\) ions are linked together via bridging carboxylato groups and the chains are crossed-linked by (bdc)\(^2-\) entities to form a three-dimensional framework. As can be seen from fig. 15, this three-dimensional network presents no void. In order to create some the authors have removed the coordinated DMF molecules. The X-ray powder diffra-
tion diagram show significantly broadened lines and did not allow any structural characterization. However this desolvated phase shows a rather interesting thermal stability (up to 300°C) and presents some interesting properties as far as gas sorption is considered.

Some other terephthalate-containing compounds have been obtained thanks to hydrothermal synthesis. The chemical formulae of these compound are Eu$_2$(H$_2$O)$_2$(OH)$_4$(bdc) and Eu$_2$(OH)$_4$(bdc) (Serre et al., 2002). Their crystal structures (see fig. 16) are closely related to those of Eu(OH)$_3$. Oxygen atoms of the inorganic sheets are either water molecules, hydroxo groups or oxygen atoms from the dicarboxylate ligand. In both structure elongated hydrophobic channels are present but with no real porosity. By TGA and TDXD it has been shown that both structures are related by an hydration/dehydration process. During the dehydration the coordination water molecules are removed and the monodentate carboxylato groups become bidentate. This results in a slight contraction of the Eu–Eu distances (Eu–Eu distances are roughly 13.8 Å in the hydrated phase and 11.4 Å in the dehydrated one).
3.3. Coordination polymers based on the (bttc)$^{4-}$ ligand

Several 1,2,4,5-benzenetetracarboxylate containing coordination polymers have already been described. The oldest one was reported in a very short crystallographic paper (Yaghi et al., 1997b). It has been obtained via hydrothermal synthesis. Its chemical formula is Ce(Hbtco)-H$_2$O and its crystal structure is represented in fig. 17. The Ce$^{3+}$ ions are surrounded by nine oxygen atoms. One out of the nine belongs to the coordinated water molecule while the eight others are from carboxylato groups from four different (Hbtco)$^{3-}$ entities. The co-

Fig. 14. Projection view along the $b$-axis of Er$_2$(bdc)$_3$(H$_2$O)$_6$: triclinic, $P\bar{1}$, $a = 7.8373(1)$ Å, $b = 9.5854(2)$ Å, $c = 10.6931(2)$ Å, $\alpha = 68.78(1)^\circ$, $\beta = 70.87(2)^\circ$, $\gamma = 75.33(1)^\circ$. Redrawn from Deluzet et al. (2003).
ordination modes adopted by the carboxylato groups are modes II, III and IV. Despite its three-dimensionality and the absence of crystallization water molecules, no further study has been undertaken until now.

More recently two other groups have published the synthesis and crystal structures of several other compounds. All of them have been obtained via hydrothermal methods and are three-dimensional.

The chemical formula of the first one is \([\text{Eu}_2(\text{bttc})(\text{H}_2\text{bttc})\text{H}_2\text{O}]\cdot4\text{H}_2\text{O}\) (Wu, C.D. et al., 2002). Its crystal structure is reported in fig. 18. Eu\(^{3+}\) ions are nine coordinated by eight oxygen atoms from carboxylato groups belonging to six different (bttc\(^{4-}\)) or (H\(_2\)bttc\(^{2-}\)) ligands and the other one from a water molecule. The 3D structure presents channels spreading along the b-axis and filled by eight crystallization water molecules per unit cell. The magnetic properties have been studied but present no particular interest, as expected. The thermal behavior has also been studied. It shows that an anhydrous phase obtained upon heating is stable up to 400 °C. Unfortunately no crystal structure of this anhydrous phase has been solved.
At last, three other coordination polymers based on 1,2,4,5-benzene-tetracarboxylate have been reported in the same publication by Cao et al. (2002b), namely Yb$_4$(bttc)$_3$(H$_2$O)$_8$·6H$_2$O, Er$_8$(bttc)$_6$(H$_2$O)$_{16}$·16H$_2$O and Tb$_2$(bttc)(H$_2$ttc)(H$_2$O)$_2$·4H$_2$O. All three have been obtained via hydrothermal methods and all three present a three-dimensional network with channels accommodating guest water molecules (see fig. 19). The thermal behavior of these compounds have been studied by TGA. In all cases it is possible to remove the water molecules and the resulting anhydrous phases are fairly stable. No crystal structure of these anhydrous phases has been reported.

As a concluding remark regarding (H$_2$bttc)$^{2-}$-containing coordination polymers, we would like to underline that the usual versatility of the carboxylate groups is reinforced in this ligand by its partial protonation. It is also worth noting that up to now, all the reported lanthanide-based coordination polymers obtained with this ligand are 3D.

3.4. Coordination polymers based on the (bhc)$_6^{-}$ ligand

To conclude this overview of the benzene-polycarboxylate based coordination polymers we now consider compounds built from benzene-hexacarboxylate containing compounds. As can be seen from table 2, there are several examples of such compounds. One is mono-dimensional and its chemical formula is R$_2$(bhc)(H$_2$O)$_{10}$·4H$_2$O (Deluzet and Guillou, 2003; Robl and Hentschel, 1992). It has been obtained twice, with Y$^{3+}$ and Er$^{3+}$, in gel media. Its crystal structure can be described as chain-like molecular motifs spreading along the $\vec{a}$ axis. The rare-earth ion is eight coordinated by five oxygen atoms from water molecules and three oxygen atoms from three monodentate carboxylate groups. The six carboxylato groups are monodentate and the second oxygen atom remains free. The cohesion of the crystal structure is insured both by a complex network of hydrogen bonds and by interactions between the phenyl rings (see fig. 20).
Another compound has been structurally described (Wu, L.P. et al., 1996). Its chemical formula is \([\text{La}_2(\text{bhc})(\text{H}_2\text{O})_9]\cdot2\text{H}_2\text{O}\). It has been obtained via a soft hydrothermal synthesis (the solution was sealed in a glass tube which was left for two days in a water bath at 40°C). Its crystal structure is 3D. There are two crystallographically independent \(\text{La}^{3+}\) ions. Both are nine coordinated. One by five water molecules plus four oxygen atoms from carboxylate groups while the other is linked to only four water molecules and five oxygen atoms from carboxylate groups. One out of the six mellitate carboxylate groups is free while the five others are bidentate or monodentate (see fig. 21). Two crystallization water molecules

Fig. 17. Projection view along the \(a\)-axis of the three-dimensional network of \(\text{Ce(Hbttc)}\cdot\text{H}_2\text{O}\): triclinic, \(\text{P}\bar{1}\), \(a = 6.419(2)\) Å, \(b = 9.414(2)\) Å, \(c = 9.604(3)\) Å, \(\alpha = 88.49(2)^\circ\), \(\beta = 74.68(2)^\circ\), \(\gamma = 76.68(2)^\circ\). Redrawn from Yaghi et al. (1997b).
are localized inside the cavities. No thermal study has been performed. However, even if this hydrated crystal structure seems promising (it is 3D and contains a light lanthanide ion) for catalytic purposes, one should notice that three out of the twelve oxygen atoms of the mellitate anion are free and available for a crystallographic rearrangement upon dehydration.

At last three other 3D frameworks containing rare-earth mellitates have recently been published (Chui et al., 2001). They all have been obtained by hydrothermal methods. Indeed, the authors of this publication think that hydrothermal conditions can lead to compounds with a lower level of hydration compared to ambient conditions. Actually the three new coordination polymers therein described present at most only one crystallization water molecule as evidenced by their chemical formulae: $\text{Ln}_2(bhc)(\text{H}_2\text{O})_6$ with $\text{Ln} = \text{La–Er}$, $\text{Ln}_2(bhc)(\text{H}_2\text{O})_4(\text{H}_2\text{O})$ with $\text{Ln} = \text{Tm–Lu}$ and $\text{Tm}_2(bhc)(\mu_2-\text{H}_2\text{O})_2(\text{H}_2\text{O})_2$. Projection views of their molecular networks are presented in fig. 22.

The nine coordinated lanthanide ion in compounds $\text{R}_2(bhc)(\text{H}_2\text{O})_6$ with $\text{Ln} = \text{La–Er}$ (left) is surrounded by three coordination water molecules and four carboxylato groups from four different mellitate anions (two bidentate and two mono dentate). The crystal structure present some channels spreading along the $\vec{c}$-axis filled with coordinated water molecules. Dehydration of these polymers is impossible because they loose their crystallinity above 100 °C.

More interesting are the compounds $\text{R}_2(bhc)(\text{H}_2\text{O})_4(\text{H}_2\text{O})$ with $\text{R} = \text{Tm–Lu}$ (middle). They crystallize in a tetragonal space group and the smaller lanthanide ions are only seven coordinated by two water molecules and five monodentate carboxylato groups from four dif-
Fig. 19. (Left) projection view along the c-axis of the polymeric network of Yb₄(bttc)₃(H₂O)₈·6H₂O: monoclinic, C2/m, \(a = 20.8119(5)\ \text{Å}, \ b = 17.6174(1)\ \text{Å}, \ c = 5.7252(2)\ \text{Å}, \ \beta = 92.32(1)^\circ\). Redrawn from Cao et al. (2002b). (Middle) projection view along the a-axis of the polymeric network of Er₈(bttc)₆(H₂O)₁₆·16H₂O: triclinic, P\(\bar{1}\), \(a = 9.6739(5)\ \text{Å}, \ b = 11.0039(5)\ \text{Å}, \ c = 11.5523(6)\ \text{Å}, \ \alpha = 104.83(1)^\circ, \ \beta = 91.0(1)^\circ, \ \gamma = 114.26(1)^\circ\). Redrawn from Cao et al. (2002b). (Right) projection view along the b-axis of the polymeric network of Tb₂(bttc)(H₂bttc)(H₂O)₂·4H₂O: monoclinic, P\(\text{2}_1/c\), \(a = 10.725(1)\ \text{Å}, \ b = 7.1693(9)\ \text{Å}, \ c = 17.158(2)\ \text{Å}, \ \beta = 97.11(2)^\circ\). Redrawn from Cao et al. (2002b).
different mellitate anions. The bound water molecules point toward the center of the channels spreading along the \( \vec{c} \)-axis. The crystallization water molecule is disordered and lies at the center of the largest cavity. Once more the framework is lost upon dehydration.

Finally, the eight coordinated \( \text{Tm}^{3+} \) ion in the compound \( \text{Tm}_2 \text{(bhc)}(\mu_2\text{H}_2\text{O})_2(\text{H}_2\text{O})_2 \) (right) is surrounded by one terminal coordinated water molecule, five monodentate carboxylate groups and two bridging coordination water molecules. This last water molecule leads to dimeric units \([\text{Tm}_2(\mu_2\text{H}_2\text{O})_2]\). No dehydration experiment is reported.

As can be seen from the previous examples much work is in progress in the field of lanthanide-containing coordination polymers involving benzene polycarboxylates as ligands.

Before concluding this section we would like to cite two other interesting results involving ligands which can be considered as derivatives of these benzene polycarboxylates.

3.5. Coordination polymers based on benzene polycarboxylate derivatives

In the above-described crystal structures the intermetallic distances are a little bit too short if optical properties are to be targeted. Furthermore the porosity of the three-dimensional compounds should be enhanced for gas storage or molecular separation applications. The design of the two compounds described hereafter has been motivated by these observations.

The first one has been obtained thanks to gel diffusion methods by reacting an \( \text{Er}^{3+} \) ion with 2,6-naphtalenedicarboxylate (see scheme 3) (Deluzet et al., 2003). Its chemical formula is \( \text{Er}_2(\text{ndc})_3(\text{H}_2\text{O})_6 \) where \( \text{(ndc)}^{2-} \) stands for 2,6-naphtalenedicarboxylate.
Fig. 21. (Left) the coordination environment of the \((\text{bhc})^6^-\) ligand in \(\text{La}_2(\text{bhc})(\text{H}_2\text{O})_9\cdot2\text{H}_2\text{O}\). (Right) projection view along the \(b\)-axis of the crystal structure of \(\text{La}_2(\text{bhc})(\text{H}_2\text{O})_9\cdot2\text{H}_2\text{O}\): monoclinic, \(P2_1/n\), \(a = 6.723(4)\ \text{Å}, b = 11.368(5)\ \text{Å}, c = 29.813(4)\ \text{Å}, \beta = 95.851(3)^\circ\). Redrawn from Wu, L.P. et al. (1996).

Its crystal structure is exactly the same than the one of the compound obtained in similar conditions with terephthalate (see fig. 23), namely \(\text{Er}_2(\text{bdc})_3(\text{H}_2\text{O})_6\). The crystal structure can also be described as a succession of \(\text{Er}^{3+}\) ions planes connected by organic ligand planes. The only difference between the two structures lies in the inter inorganic plane distances. As an example the distances between two \(\text{Er}^{3+}\) ions connected through a 2,6-naphtalenedicarboxylate ligand is roughly 12 \(\text{Å}\) while the distances between two \(\text{Er}^{3+}\) ions connected through a terephthalate ligand was 10 \(\text{Å}\). This result illustrates the tunability of the crystallographic arrangement by the choice of the ligand. However, in this particular case, the goal was not reached. Indeed, the intermetallic distances in the inorganic planes remains almost unchanged because of the bridging coordination mode of the carboxylato groups.

The second compound (Pan et al., 2003) has been obtained via hydrothermal methods by reacting a lanthanide ion with 1,4-phenylenediacetate (see scheme 3). The chemical formula of the obtained coordination polymer is \(\text{Ln}_2(\text{pda})_3(\text{H}_2\text{O})_2\cdot2\text{H}_2\text{O}\) with \(\text{Ln} = \text{La} or \text{Er}\) and \((\text{pda})^2^- = 1,4\)-phenylenediacetate. Its extended structure consists of \(\text{Ln}–\text{COO}\) triple helixes cross-linked through \(-\text{CH}_2\text{C}_6\text{H}_4\text{CH}_2–\) spacers of the \((\text{pda})^2^-\) anions, showing 1D open chan-
Fig. 22. (Left) projection view along the $c$-axis of the crystal structure of \( \text{Ln}_2(\text{bhc})(\text{H}_2\text{O})_6 \): orthorhombic, Pnmm, $a = 13.642(2)$ Å, $b = 6.769(1)$ Å, $c = 10.321(2)$ Å. Redrawn from Chui et al. (2001). (Middle) projection view along the $c$-axis of the crystal structure of \( \text{Ln}_2(\text{bhc})(\text{H}_2\text{O})_4(\text{H}_2\text{O}) \): tetragonal, P4/ncc, $a = 18.443(3)$ Å, $c = 10.374(2)$ Å. Redrawn from Chui et al. (2001). (Right) projection view along the $a$-axis of the crystal structure of \( \text{Tm}_2(\text{bhc})(\mu\text{-H}_2\text{O})_2(\text{H}_2\text{O})_2 \): monoclinic, P2$_1$/c, $a = 5.781(1)$ Å, $b = 14.840(1)$ Å, $c = 8.920(1)$ Å, $\beta = 93.46(1)^\circ$. Redrawn from Chui et al. (2001).
channels along the crystallographic c-axis (see fig. 24). These channels accommodate the guest and coordinated water molecules. This compound can be dehydrated leading to Ln2(pda); which has been assumed to present almost the same crystal structure than its hydrated precursor on the basis of X-ray powder diffraction diagrams. It reversibly bind water reforming Ln2(pda)3(H2O)-2H2O. This dehydrated phase is stable up to 450°C. The effective pore window size has been estimated to roughly 3.4 Å. These results demonstrate the feasibility of achieving well defined porous and stable lanthanide-organic framework with coordinatively unsaturated metal sites. Such systems would be well suited for the study of size- and shape-selective catalysis of organic reactions known to be catalyzed by lanthanide Lewis acids (Shibasaki et al., 1999; Imamoto, 1994).

3.6. Coordination polymers based on other ligands

Numerous other lanthanide coordination polymers have been designed with ligands which are neither rigid, nor unsaturated (Starynowicz and Meyer, 2000; Wu, Y. et al., 2002; Long et al., 2001; Alleyne et al., 2001; Wan et al., 2003; Feil-Jenkins et al., 1995). Some interesting results are highlighted in this section.

3.6.1. Coordination polymers with carboxylate ligands

Let us so consider [R(C9H9O6)(H2O)]2·5H2O where C9H9O6 stands for the cis,cis-1,3,5-cyclohexanetricarboxylate anion (Pan et al., 2000a; Daiguebonne et al., 2001b) and R for Y and Dy–Yb. This ligand is rather interesting because it can be compared with the chemically similar 1,3,5-benzene-tricarboxylate. Comparing this structure with the various crystal structures obtained with (btc)3− emphasizes the important role of the ligand. Indeed, whereas all the crystal structures involving (btc)3− were mono- or bi-dimensional, the crystal structure of this compound is three-dimensional (see fig. 25). Unfortunately, analogous compounds with the lighter lanthanides have not been structurally characterized.

These compounds exhibit large channels along the three crystallographic axes filled with crystallization water molecules. The guest water molecules can be removed and the host structure has been claimed to remain unchanged. Unfortunately there is no direct evidence of this because of a lack of crystallinity. The anhydrous compound is thermally reasonably stable (up to 250°C). Nevertheless the dehydrated compound can reversibly bind water.
Another example is the \([\text{La}_2(\text{adipate})_3(\text{H}_2\text{O})_4]\cdot6\text{H}_2\text{O}\)_\(n\) in which the adipate ligand is a long chain dicarboxylate (Kiritsis et al., 1998) (C\(_6\)H\(_8\)O\(_4\)) (see fig. 26).

This compound is remarkable because it is one of the first 3D molecular-based microporous materials using flexible multidentate ligand. Furthermore it is not irreversibly destroyed upon removal of its crystallization solvent molecules. Unfortunately adsorption studies showed that the specific area of the dehydrated compound was only 8 m\(^2\) kg\(^{-1}\) and therefore that the porous host structure was not retained. The dehydrated compound is stable up to 300 °C.
This compound is rather interesting in the sense that it is one of the rare three-dimensional compounds based on a flexible ligand.

Very recently the same groups have reported the synthesis and the crystal structure of analogous compounds involving smaller lanthanide ions (Nd$^{3+}$–Yb$^{3+}$) (Dimos et al., 2002). The chemical formulae of these compounds are $R_2$(adipate)$_3$(H$_2$O)$_4$·$x$H$_2$O with $x$ ranging from 1.5 to 0. All the compounds were obtained thanks to slow diffusion in silica gel media. They also report, in the same paper, the crystal structure of La(C$_7$H$_{10}$O$_4$)(C$_7$H$_{11}$O$_4$)(H$_2$O)·H$_2$O. All these compounds are three-dimensional and present channels filled with crystallization water molecules that can be reversibly removed. The thermal stability of these compound is unfortunately relatively low.

Much work as also been performed with the succinate ([O$_2$C–(CH$_2$)$_2$–CO$_2$]$^{2−}$) or glutarate ([O$_2$C–(CH$_2$)$_3$–CO$_2$]$^{2−}$) ligands (Serpaggi and Férey, 1998, 1999, 2003). All of them have been obtained via hydrothermal methods. As can be seen on fig. 27, these compounds exhibit rather large channels filled with water molecules. There are, however, very few information about their thermal behavior.

The same group has very recently synthesized and structurally characterized a new three-dimensional porous rare-earth carboxylate with chemical formula Eu$_2$(O$_2$C–C$_{10}$H$_{14}$–CO$_2$)$_3$ (Millange et al., 2004). This compound has been obtained under hydrothermal conditions.
Fig. 25. (Left) projection view along the $a$-axis of $\left[\text{Er(C}_9\text{H}_9\text{O}_6)(\text{H}_2\text{O})_2\right]_2\cdot5\text{H}_2\text{O}$: monoclinic, C2/c, $a = 13.394(2)$ Å, $b = 12.2501(9)$ Å, $c = 15.922(2)$ Å, $\beta = 92.09(1)^\circ$. Crystallization water molecules have been omitted for emphasizing the cavities. Redrawn from Daiguebonne et al. (2001a). (Middle) projection view along the $b$-axis of $\left[\text{Er(C}_9\text{H}_9\text{O}_6)(\text{H}_2\text{O})_2\right]_2\cdot5\text{H}_2\text{O}$. Crystallization water molecules have been omitted for emphasizing the cavities. (Right) projection view along the $c$-axis of $\left[\text{Er(C}_9\text{H}_9\text{O}_6)(\text{H}_2\text{O})_2\right]_2\cdot5\text{H}_2\text{O}$. Crystallization water molecules have been omitted for emphasizing the cavities.

from aqueous mixtures of europium nitrate and 1,3-adamantanedicarboxylic acid. Its three-dimensional structure consists in chains built up from face-sharing nine-coordinated europium polyhedra connected by 1,3-adamantanedicarboxylate ligands. It exhibits rather large ($\approx 17$ Å$^2$) anhydrous channels (see fig. 28).
Fig. 26. (Left) view of the coordination environment of \( [\text{La}_2(\text{adipate})_3(\text{H}_2\text{O})_4\cdot 6\text{H}_2\text{O}]_n \); Triclinic, \( \text{P}\bar{1} \), \( a = 9.239(2) \ \text{Å}, b = 9.790(2) \ \text{Å}, c = 7.09(1) \ \text{Å}, \alpha = 68.04(1)^\circ, \beta = 84.49(1)^\circ, \gamma = 62.01(1)^\circ \). Redrawn from Kiritsis et al. (1998). (Right) projection view along the \( a \)-axis of the crystal packing of \( [\text{La}_2(\text{adipate})_3(\text{H}_2\text{O})_4\cdot 6\text{H}_2\text{O}]_n \). Crystallization water molecules have been omitted for clarity.

The inner part of these channels is made of adamantane molecules which stack along the \( \bar{c} \)-axis and point toward the center of the channels. The anhydrous character of this compound, which is probably due to the hydrophobic character of the adamantane, implies several original properties. In particular, as there is no water molecule, the crystal structure remains unchanged up to 300°C. A Langmuir surface area of 65 m\textsuperscript{2} g\textsuperscript{-1} has been measured, in agreement with the presence of a small accessible porosity. The isostructural \( \text{Y}^{3+} \) compound doped with \( \text{Eu}^{3+} \) has been prepared and the characteristic blue luminescence of \( \text{Eu}^{3+} \) ions has been observed at room temperature.

Finally, we turn our attention to the compounds synthesized via hydrothermal methods by reaction of a lanthanide nitrate with 3,5-pyrazoledicarboxylic acid (hereafter denoted \( \text{H}_3\text{pdc} \)) (Pan et al., 2000b). See scheme 4.

Three families of isostructural compounds have been obtained depending on the lanthanide ion involved. The most interesting feature in that beautiful crystallo-chemical work is that the highly asymmetric coordination patterns of the ligand leads to crystal structures belonging to acentric space groups. The authors of this publication suggest that it should be possible
Fig. 27. (Left) projection view along the $b$-axis of the crystal packing of $\text{[Pr(H}_2\text{O}]_2[\text{O}_2\text{C(CH}_2\text{)}_2\text{CO}_2\text{]}_3\cdot\text{H}_2\text{O}}$: monoclinic, $\text{C}_2\text{/c}$, $a = 20.2586(1)$ Å, $b = 7.9489(1)$ Å, $c = 13.9716(4)$ Å, $\beta = 121.64(2)^\circ$. (Right) projection view along the $a$-axis of the crystal packing of $\text{[Ln(H}_2\text{O}]_2[\text{O}_2\text{C(CH}_2\text{)}_3\text{CO}_2\text{]}_3\cdot4\text{H}_2\text{O}}$: monoclinic, $\text{C}_2\text{/c}$, $a = 8.1174(1)$ Å, $b = 15.1841(2)$ Å, $c = 19.8803(4)$ Å, $\beta = 93.76(2)^\circ$. Redrawn from Serpaggi and Férey (1998, 1999, 2003).
Fig. 28. (Left) coordination modes of the carboxylato groups in Eu$_2$(O$_2$C–C$_{10}$H$_{14}$–CO$_2$)$_3$: hexagonal, P6$_3$ cm, $a = 16.4660(1)$ Å, $c = 7.7375(1)$ Å. (Right) projection view along the $c$-axis of the network of Eu$_2$(O$_2$C–C$_{10}$H$_{14}$–CO$_2$)$_3$. The Ln$^{3+}$ ion are represented in polyhedral representation. Redrawn from Millange et al. (2004).

Scheme 4. 3,5-Pyrazoledicarboxilic acid.

to design new materials for non-linear optical devices using lanthanides in association with an asymmetric ligand. Actually, the little preference in bond direction of the lanthanide ions constitute an asset for the design of acentric coordination polymers.

3.6.2. Coordination polymers with sulfoxyde ligands
It is also worth citing the work of X.-H. Bu et al. devoted to the synthesis of lanthanide containing coordination polymers with flexible disulfoxide ligands (Bu et al., 2002a, 2002b; Li, J.R. et al., 2004a, 2004b). The use of this type of flexible ligands leads to unprecedented molecular networks. Some of these ligands are represented in scheme 5.

This work actually offers new perspectives for the development of novel coordination polymers with unusual properties. The magnetic properties of one of these compounds has been measured. As expected they are not very exciting. On the opposite the luminescent properties of some compounds have been studied and seems to be rather interesting. As an example the structure of one of these compounds with chemical formula
Scheme 5. Some sulfinyl ligands used in (Bu et al., 2002a, 2002b; Li, J.R. et al., 2004a, 2004b).

Fig. 29. (Left) projection view of the crystal structure of the $[\text{Tb}(\mu\text{-meso-L})(\text{rac-L})_2(\text{CH}_3\text{OH})_2](\text{ClO}_4)_3$: monoclinic, C2/c, $a = 19.976(8) \ \text{Å}$, $b = 11.584(5) \ \text{Å}$, $c = 19.242(8) \ \text{Å}$, $\beta = 105.83(1)^\circ$. Redrawn from Li, J.R. et al. (2004a). (Right) scheme of the ($\mu$-meso-L) ligand.

$[\text{Tb}(\mu\text{-meso-L})(\text{rac-L})_2(\text{CH}_3\text{OH})_2](\text{ClO}_4)_3$ with meso-L = meso-1,2-bis(ethylsulfinyl)ethane is presented in fig. 29.

This structure can be described as the juxtaposition of chain-like molecular motifs. The emission spectrum of this compound contains four bands. The intensity of the hypersensitive $^5\text{D}_4 \rightarrow ^7\text{F}_5$ transition is close to six times that of the other transitions. These results, according to the authors, show that the complex has excellent single color properties. Unfortunately the thermal stability of this class of compounds is rather low and decomposition occurs upon heating around 200°C.

3.7. Synthetic routes

Two main synthesis methods have been used in order to design lanthanide-containing coordination polymers, namely the diffusion through gels and hydrothermal conditions. We would like to briefly described both and try to highlight their respective advantages and drawbacks.

Let us first consider the more commonly used method: the synthesis under hydrothermal conditions. There is an increasing interest for this technique (Feng and Xu, 2000). This interest is essentially due to inherent advantages such as a high reactivity of reactants, an easy control of solution or interface reactions, the formation of metastable and original condensed phases, a reduced air pollution and a low energy consumption.

Most of the above described coordination polymers have been obtained thanks to hydrothermal methods. Typically, they consists in synthesizing the coordination polymers in a Teflon-lined Parr bomb under relatively high temperatures. The reaction occurs at high temperature (typically between 150°C and 250°C) and elevated pressure (autogenous pressure). As far as lanthanide coordination polymers are concerned, this synthetic method seems to lead to materials exhibiting high dimensionality and low hydration rate (Cao et al., 2002b). That is why
these methods have been up to now extensively used in order to obtain molecular precursors for three-dimensional microporous materials.

The main drawback of this method is the difficult separation of the targeted product(s) from the resulting powder that may sometimes also contains remaining reactants and by-products. Moreover, it is sometimes impossible to get single crystal and the structural characterization is therefore difficult.

The alternative way of synthesis is the slow diffusion of the reactants through a gel medium. This slow diffusion is necessary because of the inherent insolubility of the coordination polymers.

Typically the gel is poured in the bottom of a U-shaped tube. Then solutions of the reactants are poured into the vertical parts of the U-shaped tube and allowed to slowly diffuse at room temperature and ambient pressure. The gel can be either a physical gel such as agarose or a chemical gel such as silica gel.

This technique presents great advantages as long as single crystals of metastable phases are needed. Actually, for the systems which have been explored both by hydrothermal and gel diffusion methods, the latter leads to more numerous polymorphic phases than the former. As an illustration of its efficiency one can consider the three-dimensional \( \text{La}_2(\text{C}_2\text{O}_4)_3 \cdot 18\text{H}_2\text{O} \) which is represented in fig. 30. This compound has been first synthesized by Watanabe et al. in 1971 (Watanabe and Nagashima, 1971) but has never been structurally described despite the great number of works devoted to lanthanide oxalate compounds. Its crystal structure has been solved only recently (Camara et al., 2003) thanks to single crystals obtained by slow diffusion in gels.

This method has also allowed obtaining single crystal of all the phases encountered during the reversible dehydration process of \( \text{Ln} \text{btc}(\text{H}_2\text{O})_{5.5}\cdot3\cdot5\text{H}_2\text{O} \) compounds except, of course, the resulting anhydrous one. These three different phases have been obtained by varying only the density of the gel medium (Daiguebonne et al., 2000b) (see fig. 31).

The obvious main drawback of this method is the separation of the products. Indeed, it is sometimes difficult to isolate the desired product from the gel medium. This problem is overcome when the desired phase can be obtained by a thermal treatment of the hydrated phase obtained by direct precipitation. In such cases, the gel methods are only used for structural characterization purposes and the availability of the material is ensured.

3.8. Coordination polymers containing poly-hydroxo-lanthanide complexes

As for coordination polymers containing transition metal ions, it could be of interest to design coordination polymers in which the metallic sites would be no longer occupied by simple ions but by polynuclear species. This could result in an increase of the porosity, as is the case for transition metal ions. Furthermore, the topology of the polynuclear species themselves could play a structuring role. To the best of our knowledge, there is only very few examples of such materials. This is essentially due to the difficult synthesis of lanthanide only containing polynuclear species. Actually the first poly-hydroxo-lanthanide complexes have been reported (Zak et al., 1994; Giester et al., 1997) and there is now several other poly-hydroxo-lanthanide
complexes described in the literature and having various nuclearity (Zheng and Wang, 2000; Zheng, 2003). These species have been obtained by controlled hydrolysis. This control is, in most cases, ensured by organic ligands such as aminoacids. In some cases the use of bifunctionalized amino acids has led to extended frameworks in one pot syntheses (Ma et al., 2000a, 2000b). As an illustration of these beautiful works one can cite (Wang et al., 2001) the [Er₄(µ₃-OH)₄(C₅H₉NO₄)₃(H₂O)₈](ClO₄)₅·6H₂O where C₅H₉NO₄ is glutamic acid (see scheme 6). A projection view of the 3D molecular framework is presented in fig. 32.
Fig. 31. Predominance phase diagram versus the concentration of the gel medium and of the solutions of reactants. Redrawn from Daiguebonne et al. (2000b).

Zwiterionic structure of glutamic acid. Tetranuclear lanthanide-hydroxo species.

Scheme 6.
Fig. 32. Projection view along the c-axis of the crystal structure of \([\text{Er}_4(\mu_3-\text{OH})_4(\text{C}_5\text{H}_9\text{NO}_4)_3(\text{H}_2\text{O})_8](\text{ClO}_4)_5\cdot6\text{H}_2\text{O})\): orthorhombic, \(P2_12_12_1\), \(a = 14.6190(2)\ \text{Å}, \ b = 17.2110(1)\ \text{Å}, \ c = 21.2150(1)\ \text{Å}\). Redrawn from Wang et al. (2001).

This compound has been obtained in water by a self assembling strategy. Its extended solid state structure is composed of the cubane-like \([\text{Er}(\mu_3-\text{OH})_4]^{8+}\) cluster (see scheme 6) interlinked by the carboxylate groups of the glutamate ligands. The molecular networks is positively charged and five perchlorate anions per asymmetric unit are ensuring electric neutrality of the crystal arrangement. These perchlorate anions are localized in the channels as well as the six guest water molecules. The aperture of the channels has been evaluated to be around 35 Å².
The main drawbacks of this compound are obviously the high positive charges borne by the molecular network as well as by the tetranuclear cluster core. Indeed, the perchlorate anions localized inside the channels cannot be removed upon heating and therefore this compound is definitely not porous. Moreover, the high positive charge (8+) of the tetranuclear species induces a very low chemical stability. As a matter of fact, these tetranuclear species are highly unstable and their use as isolated precursors is impossible. That is why the reported extended networks involving such poly-hydroxo-lanthanide clusters have all been obtained via a one-pot synthesis. This is an important disadvantage as long as molecular design is concerned.

However, some poly-hydroxo-lanthanides with a smaller charge have been synthesized (Zak et al., 1994; Giester et al., 1997; Chane-Ching et al., 2002). Their general chemical formula is \( \text{Ln}_6(\mu_6\text{-O})(\mu_3\text{-OH})_6(\text{NO}_3)_6(\text{H}_2\text{O})_{12} \cdot (\text{NO}_3)_2(\text{H}_2\text{O})_n \). They are stable in non-protogenous polar solvents such as DMF or DMSO and may so be used as chemical precursors (Mahé et al., 2004). Such an hexanuclear cluster, as well as a 1D coordination polymer involving it are presented on fig. 33. This result shows that it is possible to build extended molecular networks involving poly-hydroxo-lanthanide complexes as metallic cores. Unfortunately, these hexanuclear species are not stable in water or less toxic solvents and this constitutes an important disadvantage as far as industrial applications are targeted.

4. Conclusion and outlooks

This review demonstrates that despite the rapid emergence of metal-organic frameworks much remains to do in the field of lanthanide-containing coordination polymers. Actually, to the best of our knowledge, there is no compound fully characterized structurally, chemically and physically. However, there are numerous interesting and promising results which demonstrate the great potentiality of these materials. Their more promising potential applications, namely optical or catalytic applications, will be, for sure, strong driving forces for the development of this field. Indeed, even if, up to now, there is no identified industrially interesting such material, the works described here show that it must be possible to design some. Before concluding this review, we would like to emphasize some potential applications for the lanthanide-containing coordination polymers.
From a synthetic point of view, the design of highly porous lanthanide coordination polymers, stable at reasonably high temperature and containing metallic sites available for catalytic reactions, remains a challenge. Such a compound could find its application in depollution processes for instance. Several synthetic strategy may be used: the use of longer ligands of course but also the use of polynuclear metallic species or of complex bridging units involving, for instance, transition metal ion or main group metal ions complexes.

3D compounds could also be of interest for the design of optical devices. Indeed, in coordination polymers, inter-lanthanide distances can be controlled. This allows the synthesis of compounds with a high content in lanthanide ions. Furthermore the chemical likeness of the lanthanide ions allows the synthesis of hetero di-lanthanide compounds with a very good random distribution of the two rare earths. This property could be used for diluting the optically active lanthanide in a given framework or for enhancing optical properties thanks to up-conversion phenomena for instance.

Finally, the use of flexible ligands for designing 1D lanthanide coordination polymers could be of interest as far as the synthesis of polymers with interesting optical properties and exhibiting a thermoplastic behavior is considered.

References

Chapter 222

CUTTING DNA AND RNA

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1. Introduction

1.1. Roles of DNA and RNA in living organisms

As most of the readers probably know well, DNA and RNA are important biopolymers which are responsible for the storage and expression of genetic information. Both are composed of nucleosides and phosphate, which are connected alternately (fig. 1). In RNA, there exists a hydroxyl group at the 2′-position of ribose. In DNA, however, it is replaced by hydrogen atom. In both DNA and RNA, a nucleobase is attached to the 1′-position of ribose. The nucleobases used for DNA are adenine (A), guanine (G), thymine (T), and cytosine (C). In RNA, uracil (U) is used in place of T, and the other three nucleobases are the same as those used for DNA (A, G, and C). Significantly, adenine selectively forms a Watson–Crick base pair with T (or U in RNA), and G selectively forms a pair with C. This is the most important rule in molecular biology, and most of the functions of DNA and RNA are interpretable in terms of this selective base-pair formation. In living organisms, DNA stores the information on

(i) what kinds of amino acids are used to compose a protein,
(ii) how many are they, and
(iii) in what order are they arranged.

This information determines the structures and properties of the protein. Typical examples of important proteins are enzymes, receptors, and antibodies, which carry out the desired biofunctions. When the genetic information of DNA is transferred to proteins, the necessary small part of DNA is copied to a kind of RNA (messenger RNA), which is ultimately used as template for producing the protein. In addition to messenger RNA, there are two other important RNAs named transfer RNA and ribosomal RNA.

When we prepare a desired protein by using biotechnology, the DNA of a living organism (e.g., Escherichia coli) is altered and the information on our target protein is incorporated into this DNA. There are two key steps in this technique: (i) site-selective scission of DNA and (ii) connection of DNA fragments. At present, both steps are successfully carried out by using naturally occurring enzymes. In the near future, however, we will need new tools that can accomplish even what these enzymes cannot do. As described in this manuscript, rare earth metal ions show remarkable activity for the hydrolysis of DNA and RNA, and should play very important roles in this field.

1.2. Importance of non-enzymatic scission of DNA and RNA

Non-enzymatic hydrolysis of DNA and RNA has been attracting interest, mainly because it is essential for further developments in biotechnology, molecular biology, therapy, and rel-
Fig. 1. Structures of DNA and RNA.
order to widen the scope of applications in the near future, however, we must deal with the DNAs of higher animals and plants. Can we directly apply the present method thereto? The answer is absolutely “no”. One of the most critical points is too low site-specificity of natural restriction enzymes. Most of them recognize a specific sequence comprised of 4 or 6 DNA bases. On average, the scission site of these restriction enzymes should be present at every $4^4 (= 256)$ or $4^6 (= 4096)$ DNA bases. These site-specificities are too low to cut the huge DNAs of higher living organisms at the desired site. For example, the DNA of human beings is composed of more than $10^9$ DNA bases. If this DNA is treated with a 6-base recognizing restriction enzyme, scission will occur at more than $10^5$ sites. Thus, artificial restriction enzymes with much higher specificity and able to selectively hydrolyze huge DNAs at the target positions, are crucially important (Komiyama et al., 2001, 1999; Komiyama and Sumaoka, 1998; Komiyama, 1995).

Artificial enzymes for sequence-selective RNA scission are also important. If only one RNA can be selected from many others in cells and is cleaved at the desired site, this should open the way to new RNA science. Regulation of the expression of a specific gene in cells, advanced therapy, and RNA manipulation are promising. However, none of the naturally occurring ribonucleases shows such a high sequence-selectivity. The so-called ribozymes are well suited for these purposes (Cech et al., 1981). However, these “semi-artificial” enzymes, based on RNA framework, do not necessarily fulfill all the requirements for versatile applications. The following factors sometimes impose limitations:

1. ribozymes are easily destroyed by the ribonucleases in living cells,
2. the scission occurs only at the specific sequences in RNA, and
3. RNAs having complicated tertiary structures are difficult to cleave (ribozymes are highly ordered macromolecules and cause significant steric hindrance against them).

In contrast, “completely artificial ribonucleases” are designed and synthesized according to the functions and properties we need, and thus factors (1)–(3) can be minimized. Furthermore, even non-natural functional groups can be easily incorporated in these man-made catalysts.

In order to prepare such artificial enzymes, catalysts for the scission of DNA and RNA are necessary. However, the phosphodiester linkages in them are enormously stable. The intrinsic half-life of the phosphodiester linkage in DNA at pH 7 and $25 \degree C$ is estimated to be 200 million years (Takasaki and Chin, 1993)! Although RNA is more reactive than DNA due to intramolecular attack by $2'-OH$, the corresponding value is as large as 1000 years. In order to hydrolyze these stable linkages without using natural enzymes, a great deal of efforts have been made in various laboratories. Varieties of organic and inorganic catalysts for the hydrolysis of activated phosphoesters (e.g., bis(p-nitrophenyl)phosphate) were designed and studied in detail in 1960–1990, and the information on the mechanisms of their catalysis was accumulated. Yet, non-enzymatic hydrolysis of DNA was impossible, and RNA could not be hydrolyzed at reasonable rates.

1.3. Lanthanide ions for molecular biology and biotechnology

Some of the readers might wonder how and why lanthanide ions are related to biotechnology. Certainly, there was little interaction with this field until the early 1990s. In 1992–1993, it
Table 1
Remarkable catalytic activities of lanthanide ions for the hydrolysis of DNA and RNA

<table>
<thead>
<tr>
<th>Nucleic acid</th>
<th>Half-life of phosphodiester linkage Without catalyst</th>
<th>Half-life of phosphodiester linkage With lanthanide ion</th>
<th>The most active species</th>
</tr>
</thead>
<tbody>
<tr>
<td>DNA</td>
<td>200 million years</td>
<td>a few hours</td>
<td>Ce(IV)</td>
</tr>
<tr>
<td>RNA</td>
<td>1000 years</td>
<td>a few minutes</td>
<td>Tm(III), Yb(III), Lu(III)</td>
</tr>
</tbody>
</table>

was found that these metal ions are highly effective for the hydrolysis of DNA (Matsumoto and Komiyama, 1992; Komiyama et al., 1993; Shiiba et al., 1993) and RNA (Matsumoto and Komiyama, 1992; Morrow et al., 1992). The results are shown in table 1 (Komiyama et al., 1994a, 1995). These findings represented a breakthrough in the relevant fields, since we could for the first time obtain very efficient scissors for cutting DNA and RNA. Accordingly, this opened the way to the preparation of artificial enzymes for the sequence-selective scission of DNA and RNA. Significantly, only lanthanide ions exhibit these remarkable catalytic properties, indicating that rare earth metals have more specific properties than previously understood. Why are lanthanide ions so eminent for the hydrolysis of DNA and RNA? How can they be applied to biotechnology? What can we expect in the future? These are the main themes of this review.

2. Discovery of DNA hydrolysis by Ce(IV)

2.1. The finding of DNA hydrolysis by lanthanide(III) trichloride

In 1992, the authors found that various lanthanide(III) salts are active for the hydrolysis of phosphodiester linkages in DNA (Matsumoto and Komiyama, 1992). Even linear (non-supercoiled) DNA was hydrolyzed at pH 7. This catalytic activity was quite important and attractive, since none of the catalysts previously reported could hydrolyze linear DNAs (several catalysts could hydrolyze supercoiled plasmid DNA, but they were inactive for the hydrolysis of linear DNAs: vide infra). Non-lanthanide ions, e.g., Mg(II), Zn(II), Co(II), Al(III), Fe(III), showed no measurable activity, and thus the remarkable catalysis for the hydrolysis of DNA is restricted to lanthanide ions. These findings built a bridge between biochemistry and rare earth chemistry.

More importantly, among the lanthanide(III) salts investigated, CeCl₃ was overwhelmingly (> 10³ fold) more active than were the trichloride salts of other lanthanide(III) ions. Only this Ce(III) salt hydrolyzes non-supercoiled DNA under physiological conditions within a day. At pH 7 and 50 °C, for example, the half-life of phosphodiester linkage in dinucleotides (e.g., TpT, GpG, and others) is decreased to only a few hours. Thus, the acceleration of DNA hydrolysis by this metal salt is 10¹² fold, which is enormous. This is the first hydrolysis of non-supercoiled DNA which was accomplished at reasonable rates without using naturally occurring enzymes. The rate of DNA hydrolysis was almost independent of the kind of nucleic acid bases in substrate DNA, showing that they do not directly participate in the catalysis. Consistently, oligonucleotides were hydrolyzed almost randomly throughout their strands.
without no significant base-preference. Both single-stranded DNAs and double-stranded ones were hydrolyzed at almost the same rates.

2.2. Remarkable catalysis by Ce(IV)

Soon after the DNA hydrolysis by CeCl₃ was discovered, its catalytic mechanism was investigated in detail (Komiyama et al., 1994a, 1995; Takasaki and Chin, 1994). First, it was found that molecular oxygen is necessary for this metal salt to show remarkable catalytic activity. When oxygen was removed by repeated freeze-and-thaw cycles and the reaction with CeCl₃ was carried out under nitrogen atmosphere, the DNA substrate was hardly hydrolyzed (the scission rate was similar to the value for other lanthanide trichlorides). Furthermore, the ionic state of the Ce in the reaction mixtures was investigated by redox-titration experiments. A given amount of FeSO₄ was added to the mixtures, and the resultant solutions were back titrated with Ce(NH₄)₄(SO₄)₄ using 8-phenanthroline as an indicator (its Fe(II) complex is red, while the Fe(III) complex is light-purple). It was shown that significant amounts of Ce(IV) were present in the reaction mixtures and that the rate of DNA hydrolysis linearly increased with increasing concentration of Ce(IV). Apparently, the active species is this tetravalent ion, which is formed by the oxidation of the Ce(III) with molecular oxygen. Furthermore, this fact satisfactorily explained why CeCl₃ was overwhelmingly more active than the other lanthanide(III) salts.¹ As is well known, only Ce can take both the trivalent state and the tetravalent state which are stable enough to exist in water under normal conditions.

It was also found that Ce(IV) salts (e.g., Ce(NH₄)₂(NO₃)₆) are also sufficiently active for DNA hydrolysis. The catalytic activity was comparable with that accomplished by CeCl₃ in presence of molecular oxygen. Still more importantly, molecular oxygen was not required at all when these salts were used as starting materials, and the catalysis was efficient even in its total absence. Thus we could conclude that, in the DNA hydrolysis by CeCl₃, molecular oxygen is used only for the preparation of the catalytically active Ce(IV) species from the Ce(III) in the starting materials. The catalytic processes themselves do not require molecular oxygen at all. The salt Ce(NH₄)₂(NO₃)₆ is commercially available and easily soluble in water, and thus very convenient for practical purposes. After the finding of DNA hydrolysis by Ce(IV), various metal ions, including Ti(IV) and Zr(IV), were investigated in terms of their activities for DNA hydrolysis. They were found to be active but their catalytic effect was not sufficient. Even today, Ce(IV) is still the sole catalyst which can promptly hydrolyze linear DNAs irrespective of the DNA sequences.

Monophosphates such as adenosine 3′-monophosphate and guanosine 5′-monophosphate are also hydrolyzed by Ce(IV) (Miyama et al., 1997). The reactions are still faster than

¹ On the catalysis by CeCl₃/O₂ system, another mechanism involving dinuclear Ce(IV) species as the active species was proposed (Takasaki and Chin, 1994). There, the two Ce(IV) ions are bridged by –O–O– group which is derived from molecular oxygen, and this group is the nucleophile for the reaction. However, this mechanism is unlikely, since molecular oxygen is not necessary to the catalytic process as described in text. Note that this mechanism is entirely different from the one described in sect. 5, although both involve dinuclear Ce(IV) species. In the mechanism proposed in sect. 5, the metal-bound hydroxide is the nucleophile and thus no molecular oxygen is required for the catalysis.
DNA hydrolysis. As an application of this catalysis, the phosphomonoesters at the 3′- or 5′-termini of DNA were removed by Ce(IV). Here, the internal phosphodiester linkages were kept rather intact. The selectivity of the terminal monoester hydrolysis, with respect to the phosphodiester hydrolysis, was notably increased by using concentrated Hepes and Tris buffer solutions. In doing so, the terminal phosphates were selectively removed from DNA with the main strands kept intact. The removal of the terminal monophosphate is one of the most fundamental processes in biotechnology and is currently accomplished by using naturally occurring phosphomonoesterase. The lanthanide(III) ions are far less active than Ce(IV). Lanthanide(III) ions also hydrolyze the phosphate esters other than DNA and DNA (Hay and Govan, 1990; Breslow and Zhang, 1994; Morrow et al., 1995; Matsumura and Komiyama, 1997).

2.3. Evidence for the hydrolytic scission of DNA

Although Ce(IV) is a well-known oxidant, the scission of DNA by Ce(IV) proceeds totally via hydrolysis of the phosphodiester linkages. No oxidative damage of either the nucleobases or the ribose occurs during the scission. This is really the most important advantage of the Ce(IV) catalysis, since the resultant fragments can be directly applicable to biotechnology. There, DNA is cut by a hydrolytic process with restriction enzymes, and the resultant fragments are transformed and/or combined by using enzymes. If the scission is accomplished by oxidative cleavage of ribose, for example, DNA can be certainly cut, but the fragments do not accept various enzymatic reactions and thus cannot be easily used any more.

The first important evidence is the results of 1H NMR spectroscopy and HPLC analysis on the products. In the cleavage of dinucleotides (TpT, GpG, and others), the products were only nucleosides. No release of nucleobases to the solutions was detected although it should take place if the reaction were involving oxidative cleavage of the ribose. It is conclusive that the phosphodiester linkages were hydrolyzed by Ce(IV) and the monophosphates at the termini of the resultant fragments were further removed by the phosphomonoesterase activity of Ce(IV). In polyacrylamide gel electrophoresis, the fragments obtained by the scission of oligonucleotides with Ce(IV) completely comigrated with the fragments which were prepared by the digestion with naturally occurring nucleases.

Direct evidence of hydrolytic scission of oligonucleotides is presented in fig. 2 where the DNA fragments are manipulated using enzymes (Sumaoka et al., 1998). When oligonucleotides were labeled at 5′-end by radioactive 32P and cleaved by Ce(NH4)2(NO3)6, a pair of fragments were formed for the scission at each of the phosphodiester linkages in the DNA substrate (see lane 3 in fig. 2(A)). One of these fragments has a monophosphate at its terminus and the other has an OH terminus. A pair of fragments were isolated from the gel, and each of them was treated by terminal deoxynucleotidyl transferase (TdT), as shown in fig. 2(B). This enzyme attaches a nucleoside to the fragment if an OH terminus is present at the 3′-end. When the slower fragment (5′P*-T17-3′s) was incubated with unlabeled 2′, 3′-dideoxyadenosine-5′-triphosphate (ddCTP) in the presence of this enzyme, a new band having a smaller mobility than 5′P*-T17-3′s was quantitatively formed (compare lane 2 with lane 3 which refers to the fragment before the enzymatic treatment). Apparently, this fragment has a 3′-OH terminus
Fig. 2. Autoradiographs of a denaturing 20% polyacrylamide gel electrophoresis for Ce(IV)-induced hydrolysis of 22-mer DNA (A) and for the enzymatic treatment of the resultant fragments (B) and (C): the scales on the left-hand side show the nucleotides at the termini. (A): The 5' end of the 32P-labeled DNA oligomer. Lane 1, control; lane 2, Maxam–Gilbert A + G; lane 3, reaction with Ce(NH4)2(NO3)6; lane 4, DNase I. A typical pair of the fast and the slow fragments is designated by open arrows. Reaction conditions for lane 3: [Ce(IV)] = 1 mM at pH 7 (Hepes buffer) and 37°C for 24 h. (B): The 5' end of the 32P-labeled fragments. Lane 1, Maxam–Gilbert A + G; lanes 2 and 3, 5'P*-T17-3'; lanes 4 and 5, 5'P*-C16-3'; lanes 6 and 7, 5'P*-C16-3'. Lanes 2, 4, and 6, treated with TdT; lanes 3, 5, and 7, untreated. (C): The 3' end of the 32P-labeled fragments. Lane 1, Maxam–Gilbert C + T; lanes 2–4, 5'I-G10-3'P'; lanes 5–7, 5'S-G10-3'P'. Lanes 2 and 5, untreated; lanes 3 and 6, treated with BAP; lanes 4 and 7, treated with T4 kinase.

and a 2', 3'-dideoxycytidine moiety was enzymatically introduced thereto (the upper part of fig. 3(A)). In contrast, the slower fragment (5'P*-T17-3') was not susceptible to the enzymatic reaction, since it has a 3'-phosphate terminus (compare lanes 4 and 5 in fig. 2(B): see also the lower part of fig. 3(A)). Totally consistently, the slower fragment 5'P*-C16-3's (lane 7), which is one-nucleotide shorter than 5'P*-T17-3's and has a 3'-OH terminus, was elongated by the treatment with this enzyme (lane 6). The mobility of the product was virtually the same as that of 5'P*-T17-3's.

In figs. 2(C) and 3(B), the fragments were labeled at the 3'-end, and the susceptibility of the fragments to two enzymes was examined. As was the case in the 5'-end labeled oligonucleotides, a pair of fragments, corresponding to the monophosphate terminus and
the OH terminus, were formed for the scission at each of the phosphodiester linkages. The faster fragment $5'f-G_{10}-3'P^*$ was treated with bacterial alkaline phosphatase (BAP) which removes the terminal monophosphate if any. A new band of smaller mobility was quantitatively produced (compare lane 3 with lane 2 in fig. 2(C)). The mobility of the band was identical with that of the slower counterpart $5's-G_{10}-3'P^*$ (lane 5). In contrast, the mobility of $5's-G_{10}-3'P^*$ was unchanged on the BAP treatment (lane 5 vs. lane 6). Apparently, $5'f-G_{10}-3'P^*$ has a phosphate residue at the $5'$-end, and it is removed by BAP. On the other hand, $5's-G_{10}-3'P^*$ has an OH terminus. These arguments were furthermore confirmed by the following results. When $5's-G_{10}-3'P^*$ (lane 5) was reacted with ATP in the presence of T4 polynucleotide kinase (T4 kinase), a phosphate residue was successfully attached to its $5'$-end (lane 7). Note that the fragment flows faster since a negative charge is attached. The mobility of the product was exactly identical with that of $5'f-G_{10}-3'P^*$ in lane 2. As expected, the mobility of $5'f-G_{10}-3'P^*$ was unchanged, when it was treated with the combination of T4 kinase and ATP (compare lanes 2 and 4). Accordingly, upon the treatment with three enzymes (alkaline phosphatase, polynucleotide kinase, and terminal deoxynucleotidyl transferase), the DNA fragments obtained by using Ce(IV) were quantitatively transformed into the form expected for each of the enzymes (fig. 3). It is clear that the DNA fragment is completely free from any chemical damages (e.g., oxidative cleavage of the ribose) during the scission by Ce(IV). Otherwise, the fragments should not be susceptible to these natural enzymes, since enzymes strictly discriminate between normal
scission-products and abnormal ones. These results establish the potential of Ce(IV) as a tool in biotechnology, since the DNA fragments can be modulated at will by using various enzymes. As described above, the enzymatic transformation of the fragments, prepared by using restriction enzymes, into the desired forms is the key in the current and future biotechnology.

2.4. **Cooperation of Ce(IV) and Pr(III) for faster DNA hydrolysis**

Interestingly, the activity of Ce(NH$_4$)$_2$(NO$_3$)$_6$ for DNA hydrolysis is substantially enhanced when PrCl$_3$ is added to the mixture (Takeda et al., 1996). At [Ce(NH$_4$)$_2$(NO$_3$)$_6$]/[PrCl$_3$] = 2, the DNA hydrolysis is 10 times as fast as that induced by Ce(NH$_4$)$_2$(NO$_3$)$_6$ alone (the Pr(III) itself is virtually inactive under the reaction conditions employed). Notable synergetic cooperation is operative. This bimetallic system is the most active among all the catalysts for DNA hydrolysis ever reported. Mixed hydroxo-clusters are formed in situ from Ce(IV) and Pr(III) in the reaction mixtures, and the catalysis occurs therein. Assumedly, the Pr(III) ion in this mixed cluster provides its metal-bound water as the acid catalyst, and promotes the removal of the leaving OH of the DNA fragment. Under the reaction conditions (pH 7), the Pr(III)-bound water ($pK_a \sim 9$) mostly remains undissociated, and is superior as acid catalyst to the Ce(IV)-bound water. Although the [Ce$^{IV}$(H$_2$O)$_n$]$^{4+}$ ion is intrinsically a very strong acid (the $pK_a$ for the first deprotonation is around 0), it loses three protons at pH 0–4 and is only a weak acid at pH 7 where the reactions are achieved.

Nd(III) can be also used, and significant cooperativity with Ce(IV) is observed. However, other lanthanide(III) ions are completely ineffective for this purpose, and rather suppress the catalytic activity of the Ce(IV). All non-lanthanide ions show no measurable effects on the catalytic activity of Ce(IV).

2.5. **Cleavage of plasmid DNA**

Plasmid DNA has a super-coiled structure and is activated by strain and other factors. The cleavage experiments of these DNAs are very convenient to achieve in chemical laboratories and we can learn much from them (Basile et al., 1987; Schnaith et al., 1994; Hashimoto and Nakamura, 1996; Hettich and Schneider, 1997). However, there are many catalysts which cleave plasmid DNA but cannot cut linear DNA. For example, lanthanide(III) ions are very active for the hydrolysis of this DNA, but cannot hydrolyze linear DNA so efficiently. It will take many years to hydrolyze linear DNA completely. On the contrary, the Ce(IV) ion does not cleave plasmid DNA so efficiently, although it rapidly hydrolyzes linear DNA as described above. Thus, sufficient care must be paid when the results on the catalysis for plasmid cleavage are extrapolated to the catalysis for the hydrolysis of linear DNA.

3. **Homogeneous Ce(IV) complexes for DNA hydrolysis**

Although Ce(IV) ion is very active for DNA hydrolysis, it easily forms metal hydroxide gel at physiological pH, and makes the system heterogeneous. This property has been imposing sig-
significant limitations to practical applications of its catalytic effect. To solve this problem, many attempts were made to prepare Ce(IV) complexes which are homogeneous and catalytically active at pH 7.

3.1. Ce(IV)/saccharide complexes

Selected sugar derivatives form homogeneous complexes with Ce(IV) (Kajimura et al., 1998). Ribose, isomaltose, sugar alcohols, and dextran are especially effective. These sugar derivatives are electrically neutral and, probably because of this fact, the catalytic activity of Ce(IV) is little deteriorated by the complex formation. Similarly as in the catalysis by Ce(IV) hydroxide gel, the products of the hydrolysis of dinucleotides are mostly nucleosides (the monophosphates in the nucleoside phosphates are removed by the phosphomonoesterase activity of the Ce(IV) complex).

Quite interestingly and importantly, the synergism between Ce(IV) and Pr(III) described above (sect. 2.4) is also operating in these homogeneous solutions, and the catalytic activity of homogeneous Ce(IV)/dextran system was synergistically promoted by Pr(III). The activity for DNA hydrolysis shows a maximum at the [Ce(IV)]/[Pr(III)] ratio of 2. Other lanthanide ions also synergistically promoted the DNA hydrolysis by the Ce(IV)/dextran system.

3.2. Ce(IV)/EDTA complex and its equivalents

The homogeneous Ce(IV) complex was prepared by using ethylenediamine-N,N,N′,N′-tetraacetate (EDTA), which is being most widely used as metal-binding ligand (Igawa et al., 2000). Another negatively charged ligand 5-methyl-2-hydroxy-1,3-xylenediamine-N,N,N′,N′-tetraacetate was also usable (Branum et al., 2001). These complexes are sufficiently stable around pH 7 and exhibit notable catalytic activity for DNA hydrolysis.

It is noteworthy that the Ce(IV)/EDTA complex shows remarkable substrate specificity (see table 2). The catalysis of this complex is effective only for the hydrolysis of polynucleotides and oligonucleotides which are longer than tetranucleotides. Neither dinucleotides nor trinucleotides are hydrolyzed at all. Because of this fact, this complex had been for a long time believed to be inactive for DNA hydrolysis. Its catalytic effect for the hydrolysis of oligonucleotides was discovered in 2000 (Igawa et al., 2000). In addition to size-specificity, this complex clearly differentiates between single-stranded DNA and double-stranded one. Thus, single-stranded DNA is efficiently hydrolyzed by this complex, but double-stranded DNA is hardly cleaved. As described in sect. 12, these specificities are the origin of site-selective scission of DNA by using gap- or bulge-structures. In these structures, the target sites in the substrate DNA are kept single-stranded but the other parts form double-strands with appropriate oligonucleotide additives. When these structures are treated with the Ce(IV)/EDTA complex, the target sites are selectively hydrolyzed and thus the desired site-selective scission of DNA is successfully accomplished.

The remarkable substrate specificity of the Ce(IV)/EDTA complex is highly in contrast with non-specific properties of heterogeneous gels of Ce(IV) hydroxide, obtained by the addition
Table 2

<table>
<thead>
<tr>
<th>Substrate</th>
<th>Catalyst</th>
<th>Ce(IV)/EDTA complex</th>
<th>Ce(IV) hydroxide gel</th>
</tr>
</thead>
<tbody>
<tr>
<td>XpX</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>XpXpX</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>pXpXpXpX</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>XpXpXpX</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>larger oligonucleotide</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>double-strands</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
</tbody>
</table>

aX = nucleoside.

of Ce(NH$_4$)$_2$(NO$_3$)$_6$ to buffered solutions in absence of EDTA. These heterogeneous gels efficiently hydrolyze all dinucleotides, trinucleotides, tetranaucleotides, and polynucleotides. They hardly distinguish between single-stranded DNA and double-stranded one, and hydrolyze both of them at almost the same rates (see table 2).

The catalytic ability of the Ce(IV)/EDTA complex is notably promoted by cooperative effect with oligoamines such as ethylenediamine, diethylenetriamine, triethylenetetramine, spermine, and spermidine (Sumaoka et al., 2001). The synergism is conclusive, since all the oligoamines are completely inactive for DNA hydrolysis in the absence of the Ce(IV) complex. For example, 100 mM of ethylenediamine accelerated the DNA hydrolysis by more than 50 fold. Spermine is still more efficient for this cooperation in that it is effective at much smaller concentrations (e.g., 1 mM). The concentration of ethylenediamine must be 10 mM or larger in order to accelerate the reaction sufficiently. In a proposed mechanism, the positively charged oligoamines bind to the DNA and stabilize the negatively charged transition state of DNA hydrolysis through Coulomb interactions.

4. Mechanistic analysis of DNA hydrolysis by Ce(IV)

4.1. Kinetic analysis

One of the most significant obstacles for detailed kinetic analysis of DNA hydrolysis by Ce(NH$_4$)$_2$(NO$_3$)$_6$ is the formation of Ce(IV) hydroxide gel at physiological pH. Accordingly, the species responsible for the catalysis can not be easily pinned down and the mechanism of catalysis by Ce(IV) is not clear. However, we found that homogeneous Ce(IV) solutions can be obtained from Ce(NH$_4$)$_2$(NO$_3$)$_6$ when the pH is 2.5 or lower. The absence of any colloidal particles in these solutions was confirmed by light-scattering photometry. At least, there were no particles of diameter 15 Å or larger. Even in these acidic solutions, DNA hydrolysis by Ce(IV) proceeds efficiently and the rate is not much different from that at pH 7. The rate constant for the hydrolysis of thymidylyl(3’-5’)thymidine (TpT) by Ce(NH$_4$)$_2$(NO$_3$)$_6$ (10 mM) at pH 2.0 and 50 °C is $6.3 \times 10^{-4}$ min$^{-1}$, whereas the value at pH 7.0 is $3.2 \times 10^{-3}$ min$^{-1}$. In both cases, the product is only thymidine. No oxidative cleavage was observed as long as the
pH was not too low (e.g., pH < 1). Apparently, essential features of the hydrolysis of TpT by Ce(NH₄)₂(NO₃)₆ in these acidic solutions are similar to those at pH 7, and thus the catalytic mechanism in acidic solutions should be comparable with that in neutral solutions. The redox titration showed that all the Ce(IV) ions in the solutions retain their tetravalent states throughout the DNA hydrolysis. This result further confirmed the absence of any redox reaction in the mixtures.

In the reaction mixtures, the solvolysis of Ce(IV) occurs as shown in eqs. (1)–(6), and there exist six Ce(IV)-derived species ([CeIV]⁺⁺, [CeIV(OH)]⁺⁺, [CeIV(OH)₂]⁺⁺⁺, [CeIV₂(OH)₂]⁺⁺, [CeIV₂(OH)₃]⁺⁺⁺, [CeIV₂(OH)₄]⁺⁺⁺, and [CeIV₆(OH)₁₂]⁺⁺⁺). By using the equilibrium constants available, the concentrations of all these species were calculated, and the dependence of the rate of DNA hydrolysis on these values was investigated.

\[
\begin{align*}
\text{Ce}^{IV} + \text{H}_2\text{O} &\rightleftharpoons [\text{Ce}^{IV}(\text{OH})]^3+ + \text{H}^+, \quad \log Q_{1,1} = 1.1, \quad (1) \\
\text{Ce}^{IV} + 2\text{H}_2\text{O} &\rightleftharpoons [\text{Ce}^{IV}(\text{OH})_2]^2+ + 2\text{H}^+, \quad \log Q_{1,2} = 0.3, \quad (2) \\
2\text{Ce}^{IV} + 2\text{H}_2\text{O} &\rightleftharpoons [\text{Ce}^{IV}_2(\text{OH})_2]^{6+} + 2\text{H}^+, \quad \log Q_{2,2} = 3.6, \quad (3) \\
2\text{Ce}^{IV} + 2\text{H}_2\text{O} &\rightleftharpoons [\text{Ce}^{IV}_2(\text{OH})_3]^{5+} + 3\text{H}^+, \quad \log Q_{2,3} = 4.1, \quad (4) \\
2\text{Ce}^{IV} + 2\text{H}_2\text{O} &\rightleftharpoons [\text{Ce}^{IV}_2(\text{OH})_4]^{4+} + 4\text{H}^+, \quad \log Q_{2,4} = 3.5, \quad (5) \\
6\text{Ce}^{IV} + 12\text{H}_2\text{O} &\rightleftharpoons [\text{Ce}^{IV}_6(\text{OH})_{12}]^{12+} + 12\text{H}^+, \quad \log Q_{6,12} = 15.4. \quad (6)
\end{align*}
\]

Under typical reaction conditions, the concentrations of these species are in the following order: [CeIV(OH)₂]²⁺ > [CeIV₂(OH)₄]⁴⁺ > [CeIV(OH)]⁺⁺ > [CeIV₂(OH)₃]⁵⁺ > [CeIV₆(OH)₁₂]¹²⁺ > [CeIV₂(OH)₂]⁶⁺ ≈ [CeIV]⁺⁺⁺.

The pseudo-first-order rate constant \(k_{obs}\) monotonously increases with increase in the initial concentration of Ce(NH₄)₂(NO₃)₆ (cf. the open circles in fig. 4(A)). Here, the pH was kept constant at 2.0. All the experimental points fairly fit solid line (vi) corresponding to the equilibrium concentration of the bimetallic hydroxo-cluster [CeIV₂(OH)₄]⁴⁺. The dotted line (iv) showing the equilibrium concentration of [CeIV₂(OH)₂]⁶⁺ has a similar shape as line (vi). These two species are the candidates for the catalytically active species. On the other hand, line (i) for [CeIV]⁺⁺⁺, line (ii) for [CeIV(OH)]⁺⁺, line (iii) for [CeIV(OH)₂]²⁺, and line (v) for [CeIV₂(OH)₃]⁵⁺ are too flat to model the experimental data, while line (vii) for [CeIV₆(OH)₁₂]¹²⁺ is too steep. These species cannot be the active species. Note that the shapes of these lines are concretely determined only by the \(Q_{x,y}\) values, pH, and [Ce(NH₄)₂(NO₃)₆]. On the other hand, the positions of lines are vertically movable by varying the catalytic rate constants of the corresponding species.

The pH dependence of the rate constant of hydrolysis is shown by the open circles in fig. 4(B) ([Ce(NH₄)₂(NO₃)₆] is kept constant at 10 mM). The profile is rather flat from pH 1.4 to 2.1. All the experimental points are consistent with the shapes of the theoretical lines for the equilibrium concentrations of [CeIV(OH)₂]²⁺ (iii), [CeIV₂(OH)₄]⁴⁺ (vi), and [CeIV₆(OH)₁₂]¹²⁺ (vii). The concentrations of the other four species rapidly decrease with increasing pH and are inconsistent with the experimental data. Of the seven Ce(IV) derived
Fig. 4. Logarithm plots of the pseudo first-order rate constant of the TpT hydrolysis at 50 °C vs. (A) the initial concentration of Ce(NH$_4$)$_2$(NO$_3$)$_6$ and (B) pH. The pH was kept constant at 2.0 in (A), whereas [Ce(NH$_4$)$_2$(NO$_3$)$_6$]$_0$ was constant at 10 mM in (B). The solid line (vi) is the theoretical one showing the concentration of the proposed active species [Ce IV$_2$(OH)$_4$]$_4^{4+}$ (right ordinate). The dotted lines are the theoretical ones for (i) [Ce]$^{4+}$; (ii) [Ce IV(OH)$_2$]$^{2+}$; (iii) [Ce IV(OH)$_2$]$_2^{2+}$; (iv) [Ce$_2$ IV-OH]$_2$$_{6}^{6+}$; (v) [Ce$_2$ IV(OH)$_3$]$_3^{3+}$; and (vii) [Ce$_6$ IV(OH)$_{12}$]$_{12}^{12+}$. All theoretical lines were calculated using the equilibrium constants in eqs. (1)–(6).

species in the mixtures, only the tetracationic bimetallic hydroxo-cluster [Ce IV$_2$(OH)$_4$]$_4^{4+}$ agrees with both independent analyses described in figs. 4(A) and 4(B). It is concluded that this species is not probably the active catalytic species in the DNA hydrolysis promoted by Ce(NH$_4$)$_2$(NO$_3$)$_6$. 


A similar kinetic analysis of the hydrolysis of cyclic adenosine 3',5'-monophosphate (cAMP) by Ce(NH₄)₂(NO₃)₆ indicated that the [Ce⁴⁺₂(OH)₄]⁴⁺ cluster is also the active catalytic species. This cyclic phosphate is an important biomaterial which is responsible for cell-to-cell communication. The hydrolysis of cAMP has many common features with DNA hydrolysis. First, both cAMP and DNA are enormously stable (the half-lives of the phospho-

diester linkages in the absence of catalysts are estimated to be 400 thousand (Chin and Zou, 1987) and 200 million years, respectively). Second, the hydrolyses of both compounds are remarkably accelerated by Ce(IV) ion by 10¹²–10¹³ fold (Sumaoka et al., 1992, 1994). Thirdly, only Ce(IV) is effective for these reactions. Fourthly, both reactions proceed via the attack by OH⁻ (or its relevant species) as external nucleophile. The only notable difference between them is the fact that cAMP is hydrolyzed more promptly than DNA because of its ring strain. These results further confirm the proposal that the dinuclear Ce(IV) cluster [Ce⁴⁺₂(OH)₄]⁴⁺ should be responsible for the catalytic DNA hydrolysis.

4.2. Spectroscopic studies

Why is Ce(IV) so active for DNA hydrolysis? What factors differentiate this metal ion from other lanthanide ions and non-lanthanide ions? Do the f-orbitals of Ce(IV) take significant roles in the catalysis? These questions are critically important for practical applications of this catalysis and also from the viewpoints of pure rare earth chemistry. In order to answer them, core-level photoelectron spectroscopy (Shigekawa et al., 1996), as well as EXAFS (extended X-ray absorption fine structure) and XANES (X-ray absorption near edge structure) measurements (Shigekawa et al., 1999), were carried out. The spectroscopic analysis was simplified by using diphenyl phosphate (DPP) in place of DNA, and the EXAFS and the XANES measurements were carried out on the samples frozen in liquid nitrogen.

Core-level spectroscopy was made on the phosphorus atom of DPP in various lanthanide complexes. This method provides the energy levels of atomic orbitals of the P atom. When the electron density on this atom is decreased and the atomic orbitals are stabilized, the corresponding binding energy should be increased. As shown in fig. 5, the binding energy of the 2p orbitals in the Ce(IV) complex is considerably greater than those for the complexes with La(III), Eu(III), Lu(III), and other non-lanthanide ions. Apparently, the Ce(IV) ion withdraws electrons from the phosphodiester linkage to a far greater extent than do the other metal ions, and increases the binding energy of the orbitals of the phosphorus atom far more drastically. As a result, the electrophilicity of the phosphodiester linkage in the Ce(IV) complex is exceptionally promoted. The superiority of Ce(IV) for the activation of DNA has been for the first time spectroscopically evidenced. Undoubtedly, this factor should be one of the origins of remarkable catalysis of Ce(IV) for DNA hydrolysis.

Detailed analysis of these core-level spectra further indicates that the phosphodiester is simultaneously coordinated to two Ce(IV) ions in the Ce(IV) complex, which is highly activated. As depicted in fig. 6, the P2p signals of the DPP in the Ce(IV) complexes are composed of two components (P₁ and P₂: each peak is further split into a spin-doublet). When the Ce(IV)/DPP ratio is 1:1, the peak P₂ of the lower energy is predominant (b). Here, the phosphodiester linkage is bound to one Ce(IV) ion. Thus it is activated but only slightly.
Consistently, DNA hydrolysis hardly takes place under these conditions. As the Ce(IV)/DPP ratio increases (c)–(e), the intensity of the P_H signal, which has a higher energy, gradually increases. This signal corresponds to the DPP which is coordinated to two Ce(IV) ions in the hydroxo-cluster and is exceptionally activated by the two metal ions. Exactly as expected, the remarkable DNA hydrolysis occurs under conditions where Ce(IV) is in excess with respect to DNA (twofold or more). These arguments are further substantiated by the EXAFS spectra reported on fig. 7. In the absence of DPP ((a) and (b)), the Ce(IV) ions form metal hydroxo-clusters (and gel as they aggregate). Accordingly, the signal for the Ce–Ce distance is clearly observed at 3.6 Å (cf. the broken line). The signal at around 2.0 Å is assignable to the Ce–O bond. On addition of DPP, the signal for the Ce–Ce distance rapidly weakens and virtually vanishes for a Ce/DPP ratio of 1:1 (c). Since the 1:1 Ce(IV)/DPP complex exists as an independent species, without aggregation, the Ce–Ce signal at 3.6 Å is absent. When the Ce(IV)/DPP ratio becomes larger than one, however, the Ce–Ce signal appears again and its intensity gradually increases with increasing Ce/DPP ratio ((d) and (e)). Here, two Ce(IV) ions are simultaneously interacting with one DPP molecule, and located in a close proximity. Thus, these two Ce(IV) ions are bound to each other, giving rise to the Ce–Ce signal. These spectroscopic analyses completely agree with the conclusion of a kinetic study showing that [Ce^{IV}_{2}(OH)_{4}]^{4+} is the active species for DNA hydrolysis (see sect. 4.1). Furthermore, these results are also consistent with the conclusion of quantum-chemical studies.

That electron-transfer from phosphate to the Ce(IV) ion occurs is further supported by L3-XANES spectroscopy. In the Ce(IV)/DPP complex, there is 0.67 electron in the 4f orbitals of the Ce ion, although these orbitals should be empty in completely tetravalent Ce. This electron probably flows into the 4f orbitals from the surroundings. Consistently, the corresponding electron-transfer to the metal(III) ions is not observed in the DPP complexes of Ce(III) and other lanthanide(III) ions. As described above, the enormous activity of the Ce(IV) for DNA
hydrolysis is primarily ascribed to its dominant electron-accepting capability. These analyses further show that this specific property is derived from the stability of the trivalent state of Ce. The lanthanide(III) ions cannot efficiently accept the electrons from DNA, since their divalent states are too unstable.

When the Ce(IV)/DPP complex is formed, a new state appears near the Fermi level (fig. 8). Neither free DPP nor Ce(IV) hydroxo cluster (without DPP) has any density of state around the Fermi level. This suggests that the electron-transfer from the phosphate to the Ce(IV) ion occurs, at least partially, through hybrid orbitals, which are formed from the 4f-orbitals of the Ce(IV) and the orbitals of the phosphorus atom (and/or of the oxygen atom) in the phosphate

Fig. 6. Core-level spectra of the P2p orbital of DPP in the Ce(IV)/DPP mixtures of different molar ratios. The peaks $P_H$ and $P_L$ correspond to the doublet signals of higher and lower binding energies, respectively.
Fig. 7. EXAFS spectra for the Ce(IV)/DPP systems. The dotted line corresponds to the Ce–Ce distance.

Fig. 8. X-ray photoelectron spectra near the Fermi-level in the Ce(IV)/DPP systems.
residue. Since the 4f orbitals of Ce(IV) are lower in energy than those of the lanthanide(III) ions, they can efficiently interact with the orbitals of the phosphate. The 5d orbitals of Ce(IV) could also participate to the hybrid orbitals. This mixing of the orbitals (as well as the resultant electron transfer) can be another factor for the activation of DNA by Ce(IV).

4.3. Quantum-chemical studies on the catalytically active \([\text{Ce}^{IV}_2(\text{OH})_4]^{4+}\) cluster and its complex with phosphodiester

The structures of various species responsible for DNA hydrolysis were investigated by using \textit{ab initio} quantum chemical calculations. The optimized structure of \([\text{Ce}^{IV}_2(\text{OH})_4]^{4+}\) (the catalytically active species) is presented in fig. 9(A). This species is sufficiently stable, since the two Ce(IV) ions therein are bridged by two hydroxide groups. This dinuclear cluster is highly symmetric; the two Ce(IV) atoms and the two bridging OH groups (both the oxygen atoms and the hydrogen atoms) are located in one plane. Furthermore, the whole structure of the cluster has \(C_2\) symmetry, the axis going through the two O-atoms of the OH groups. The distance between the two Ce(IV) atoms is 3.848 Å. This value is fairly in accord with the Ce–Ce distance (3.6 Å) in the 2:1 Ce(IV) complex of diphenyl phosphate (DPP), which was determined by the EXAFS study (see sect. 4.2). The structure of the catalytically active species, which was proposed above mainly in terms of both kinetic studies and spectroscopic analyses, has been further substantiated. It is noteworthy that a bimetallic Ce(IV) cluster of the same

![Fig. 9. The optimized structures determined by \textit{ab initio} quantum-chemical calculations. (A) \([\text{Ce}^{IV}_2(\text{OH})_4]^{4+}\) involving two OH bridges. (B) 1:1 complex between \(\text{H}_2\text{PO}_4^-\) (a model compound of the phosphodiester in DNA) and \([\text{Ce}^{IV}_2(\text{OH})_4]^{4+}\). Bond lengths and interatomic distances are given in Å.](image-url)
composition is quite unstable when there exists only one OH bridge between the two Ce(IV) atoms. Upon optimizing the geometry of this bimetallic cluster by quantum-chemical calculations, the OH-bridge was spontaneously broken and accordingly the cluster decomposed to two \([\text{Ce}^{IV}(\text{OH})_2]^{2+}\) molecules.

In order to obtain information on the catalytic process, the structure of the complex between DNA and this active species was also calculated. As a model compound of DNA, monoanionic phosphate \(\text{H}_2\text{PO}_4^-\) was employed. This choice is based on the fact that the phosphodiester group of DNA mostly exists as a monoanion under the reaction conditions where the kinetic studies were carried out (pH 1.4–2.5; sect. 4.1). The \(pK_a\) value of the phosphodiester linkage of TpT is 0.4, according to a titration monitored by \(^{31}\text{P}-\text{NMR.}\) Figure 9(B) shows the optimized structure of the complex between \(\text{H}_2\text{PO}_4^-\) and the \([\text{Ce}^{IV}_2(\text{OH})_4]^{4+}\) cluster (in fig. 9(A)). This complex is sufficiently stable, and the free energy change for its formation, evaluated by the theoretical calculations, is \(-20.31\text{ kcal mol}^{-1}\). The two oxygen atoms of \(\text{H}_2\text{PO}_4^-\) are coordinated to each of the two Ce(IV) atoms, and the Ce–O distance for both bonds is 2.13 Å. Upon complex formation, the Ce–Ce distance (3.73 Å) is almost unchanged, although the symmetric structure of the bimetallic Ce(IV) cluster is slightly distorted. These results are completely consistent with the core-level spectroscopy (fig. 6) showing that the activated Ce(IV) complex involves simultaneous coordination of phosphodiester to two Ce(IV) ions. The charge (+2.57) on the phosphorus atom in this complex is remarkably greater than the value for free \(\text{H}_2\text{PO}_4^-\) (+1.89), confirming that the electrons on this atom are notably withdrawn by the Ce(IV) ions. As a result, the electrophilicity of this reaction center is enhanced.

For the purpose of comparison, calculations were also carried out on the 1:2 complex between one \(\text{H}_2\text{PO}_4^-\) molecule and two \([\text{Ce}^{IV}(\text{OH})_2]^{2+}\) clusters. In the optimized structure of this ternary system, each of the two oxygen atoms of \(\text{H}_2\text{PO}_4^-\) is coordinated to either one of these two \([\text{Ce}^{IV}(\text{OH})_2]^{2+}\) molecules. The Ce–O distances are 2.20 Å. These two \([\text{Ce}^{IV}(\text{OH})_2]^{2+}\) clusters behave as independent species, the distance between these two Ce atoms being 6.63 Å. This 1:2 complex is quite unstable in comparison with the 1:1 complex between \(\text{H}_2\text{PO}_4^-\) and the dinuclear \([\text{Ce}^{IV}_2(\text{OH})_4]^{4+}\) cluster shown on fig. 9(B). The free energy change for its formation is only \(-11.04\text{ kcal mol}^{-1}\), whereas the corresponding value for the latter complex is \(-20.31\text{ kcal mol}^{-1}\). The charge on the P atom is +2.50, which is smaller by 0.07 unit than the value for the complex of the bimetallic cluster \([\text{Ce}^{IV}_2(\text{OH})_4]^{4+}\). This conclusion is consistent with the results presented in fig. 4. In order to achieve sufficient catalytic efficiency for DNA hydrolysis, the dinuclear Ce(IV) cluster \([\text{Ce}^{IV}_2(\text{OH})_4]^{4+}\) is essential both for the formation of a stable complex with the DNA substrate and for sufficient activation of the scissile phosphodiester linkage.

5. Proposed mechanism for the DNA hydrolysis by Ce(IV)

The hydrolysis of DNA proceeds as a two-step reaction. In the first step, one hydroxide ion attacks the phosphorus atom, forming a pentacoordinated intermediate. In the second step, the 5'-OH of 2'-deoxyribonucleotide is removed from the phosphorus atom through the scission of the P–O(2') bond (in non-enzymatic hydrolysis, the P–O(3') scission can also take place).
The Ce(IV) ion accelerates this process 10^{12} fold. The proposed mechanism of DNA hydrolysis by the bimetallic cluster $[\text{Ce}^\text{IV}_2(\text{OH})_4]^{4+}$ is schematically depicted on fig. 10. First, the phosphate residue is coordinated to the two Ce(IV) ions in $[\text{Ce}^\text{IV}_2(\text{OH})_4]^{4+}$; the apparent association constant between Ce(IV) and TpT was determined to be $10^3$ M$^{-1}$ at pH 2.0 and 50 °C. As clearly evidenced by both spectroscopic and theoretical studies (cf. sects. 4.2 and 4.3), the electrons of the scissile phosphodiester linkage are strongly withdrawn by the Ce(IV) ions. Furthermore, the orbitals of this phosphate are mixed with the orbitals of the Ce(IV), and form new hybrid-orbital(s). These two factors greatly activate the phosphodiester linkage and make it highly susceptible to the attack by various nucleophiles.

Then, this phosphodiester group is attacked by the hydroxide ion coordinated to one of the two Ce(IV) ions in the $[\text{Ce}^\text{IV}_2(\text{OH})_4]^{4+}$ cluster. Although this metal-bound hydroxide ion is a rather weak nucleophile, the phosphodiester group is so much activated that this reaction can be efficient. Furthermore, the metal-bound hydroxide ion is located in quite a suitable position for the nucleophilic attack. The two Ce(IV) ions in the bimetallic cluster have many coordination water molecules, and at least one of them lies most appropriately for the purpose. Finally, the positive charges, accumulated in the Ce(IV) hydroxo cluster, stabilize the negatively charged transition state of DNA during its hydrolysis (the transition state is more negatively charged than the initial state, and is stabilized to a greater extent by the adjacent positive charges). Because of these factors, the pentacoordinated intermediate is efficiently formed.

In the breakdown of the intermediate, the water bound to Ce(IV) functions as acid catalyst. With this catalysis, the alkoxide ion of 5'-OH can be promptly removed from the phosphorus atom. Otherwise, the leaving group is poor and hard to be removed. Consistently, the DNA hydrolysis by Ce(IV) is accompanied by a notable D$_2$O solvent isotope effect ($k_{\text{H}_2\text{O}}/k_{\text{D}_2\text{O}} = 2.2–2.4$). The proton transfer is, at least partially, rate limiting. The large coordination-number of the Ce(IV) ions in the bimetallic cluster is favorable for this acid catalysis, exactly as described above for the first step of DNA hydrolysis (formation of the pentacoordinated intermediate). This mechanism is proposed on the basis of the kinetic analysis in acidic solutions. When DNA hydrolysis is carried out at around pH 7, similar acid-base cooperation by Ce(IV)-bound water molecule and the corresponding hydroxide should occur in higher aggregates of $[\text{Ce}^\text{IV}_2(\text{OH})_4]^{4+}$. 

Fig. 10. Proposed mechanism for the hydrolysis of phosphodiester by the bimetallic hydroxo-cluster $[\text{Ce}^\text{IV}_2(\text{OH})_4]^{4+}$. 

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6. RNA hydrolysis by lanthanide ions and their complexes

6.1. Catalysis by lanthanide(III) ions

RNA bears a hydroxyl group at the 2′-position of ribose. When it is hydrolyzed, this group functions as intramolecular nucleophile, and attacks the adjacent phosphorus atom. As the result, a pentacoordinated intermediate is formed. Then, the 5′-OH of ribonucleotide is removed from the P atom in the intermediate, and 2′,3′-cyclic monophosphate is formed as the covalent intermediate. In the following step, this cyclic monophosphate is hydrolyzed, and the RNA hydrolysis is completed. Thus the reaction mechanism of RNA hydrolysis is entirely different from that of DNA hydrolysis which involves intermolecular attack by external nucleophile towards the P atom. The intramolecular reaction in RNA hydrolysis is so efficient that RNA hydrolysis is far faster (10^5–10^6 fold) than DNA hydrolysis. Nevertheless, RNA is sufficiently stable under physiological conditions, and it is not easy to hydrolyze it efficiently without using enzymes (in the absence of catalysts, the half-life of the phosphodiester linkage in RNA is estimated to be around 1000 years). The catalysts for RNA must be enormously active. In order to hydrolyze RNA within an hour, for example, the catalyst must accelerate the reaction by 10^7-fold or more. In spite of a number of attempts, sufficiently fast non-enzymatic hydrolysis of RNA was unsuccessful for a long period of time.

In 1992, it was discovered that lanthanide ions are extraordinarily active for RNA hydrolysis (Matsumura and Komiyama, 1992). With the use of lanthanide trichlorides, diribonucleotides are completely hydrolyzed at pH 7 within 30 min. Although several catalysts for RNA hydrolysis were previously reported, lanthanide(III) ions greatly exceed any of them in the catalytic activity. Thus these findings attracted significant interest from chemists and biochemists. All the lanthanide(III) ions are active, but the catalytic activity monotonically increases with increasing atomic number as depicted in fig. 11 (Matsumura and Komiyama, 1997). Thus, the last three lanthanide ions (Tm(III), Yb(III), and Lu(III)) are especially active. When [LuCl_3]_0 = 5 mM at pH 7 and 30°C, for example, more than half of adenylyl(3′-5′)adenosine (ApA) disappears in 5 min, and 1:1 mixture of adenosine (A) and its monophosphates (A_2′p + A_3′p) is formed. The reaction intermediate, adenosine 2′,3′-cyclic monophosphate (A > p), is rapidly hydrolyzed by the Lu(III) ion to A_2′p or A_3′p, and not much accumulated. The pseudo first-order rate constant for the disappearance of ApA is 3 × 10^{-3} s^{-1} (half-life = 3.8 min), and the acceleration is almost 10^8 fold. The reaction rate is not much dependent on the kind of substrate. The hydrolysis rate is proportional to [LuCl_3]_0. Interestingly, Ce(IV), which is overwhelmingly the best ion for DNA hydrolysis, is not exceptionally active for RNA hydrolysis (its activity is comparable with that of Tb(III)).

Oligoribonucleotides are also efficiently hydrolyzed by lanthanide(III) ions. The scission occurs randomly without any specific base-preference, as is the case in the DNA hydrolysis by Ce(IV). When [LuCl_3]_0 = 10 mM at pH 7.5 and 30°C, substrate RNAs are almost completely degraded into small fragments within 1 h. The termini of the RNA fragments are either the 2′- or 3′-monophosphates, and the 2′,3′-cyclic monophosphate termini are hardly formed because they are rapidly hydrolyzed to the monophosphates. It is noteworthy that the rate of RNA hydrolysis by lanthanide ions is drastically dependent on the pH of the solution. With LuCl_3,
the RNA hydrolysis at pH 8 is 300-fold faster than that at pH 7; the slope of the logarithm of rate constant vs. pH is around 2.5, reflecting the formation of catalytically active dinuclear metal hydroxide cluster (vide infra). In order to obtain reproducible results, the pH of the reaction mixture must be very carefully controlled.

6.2. Lanthanide complexes for RNA hydrolysis

Immediately after the remarkable catalytic ability of lanthanide(III) ions for RNA hydrolysis was found, a number of attempts were made to prepare their complexes and extend the scope of the applications of this catalysis. The lanthanide complexes of iminodiacetate (in fig. 12(A)) and acetylacetone are obtainable simply by mixing the metal salt with the corresponding ligand (Sumaoka et al., 1995). The lanthanide complexes of Shiff-base macrocycles (fig. 12(B)) are prepared by refluxing the corresponding diamine and dialdehyde in the presence of the metal salts (Morrow et al., 1992). So-called “texaphyrins” (expanded porphyrins involving five nitrogen atoms; fig. 12(C)) (Magda et al., 1994, 1997) and derivatives of tetraazacyclododecane (fig. 12(D)) (Chin and Morrow, 1994) are also available.

The mixtures of LaCl₃ and hydrogen peroxide efficiently hydrolyze RNA. The active species is a trimeric aggregate of the dinuclear complex [La(O−O)₃La] (Komiyama et al., 1996). In the hydrolysis of an activated phosphodiester [bis(4-nitrophenyl) phosphate (BNPP)] by the same combination, however, monomeric [La(O−O)₂La]²⁺ is the active species (Takasaki and Chin, 1995). The catalytic mechanism (and thus the catalytic species) is strongly dependent on the substrate.
7. Kinetic analysis on the RNA hydrolysis

The Nd(III)-induced hydrolysis of adenylyl(3′-5′)adenosine (ApA) has been kinetically analyzed in detail (Matsumura and Komiyama, 1997). In the reaction mixtures, solvolysis of Nd(III) occurs and there exist three Nd(III)-derived species: free Nd$^{3+}$, [Nd$^{III}$\((OH)\)^2$^+$, and [Nd$^{III}_2(OH)_2$]$^{4+}$ (see eqs. (7) and (8)). The equilibrium concentration of each species can be calculated by using the $K_{1,1}$ and $K_{2,2}$ values, and the dependence of the reaction rate on the concentration of these species has been investigated.

As shown in fig. 13, the logarithm of the rate of RNA hydrolysis steeply increases with increasing pH, up to pH 8, and then reaches a plateau. The slope in the steep region is greater than 2. The experimental points fairly fit the theoretical line (the solid line) which shows the equilibrium concentration of [Nd$^{III}_2(OH)_2$]$^{4+}$. The other species never satisfy the experimental results. Undoubtedly, this bimetallic cluster is the active species for the RNA hydrolysis. In order to hydrolyze RNA efficiently, the bimetallic structure is essential, as is the case in DNA hydrolysis. The concentration of this active species is strongly dependent on pH, since it is accompanied by the release of two protons. Because of this, the rate of RNA hydrolysis by lanthanide(III) ions is so highly pH dependent.

\[
Nd^{III} + H_2O \rightleftharpoons [Nd^{III}(OH)]^{2+} + H^+, \quad pK_{1,1} = 9.4.
\]

(7)
Fig. 13. The pH-dependence of the rate of ApA hydrolysis by NdCl$_3$ (1 mM) at 30$^\circ$C. The closed circles are the experimental points, whereas the curved lines show the equilibrium concentration of the Nd(III)-derived species.

Fig. 14. The pH-dependence of the rate of ApA hydrolysis by various lanthanide(III) chlorides (1 mM) at 30$^\circ$C. ○ – La(III); ● – Nd(III); □ – Tb(III); ○ – Lu(III). Only typical results are presented here, since the shapes of the profiles for all the lanthanide(III) ions are similar and their positions monotonically shift towards lower pH with increasing atomic number.

$$2\text{Nd}^{III} + 2\text{H}_2\text{O} \rightleftharpoons \text{Nd}^{III} \cdot \text{Nd}^{III} \cdot 2\text{(OH)}_2^{4+} + 2\text{H}^+, \quad pK_{2,2} = 13.9. \quad (8)$$

The pH-rate constant profiles for the RNA hydrolysis by all the other lanthanide(III) ions have similar shapes, and are composed of (i) a steep straight line at lower pH and (ii) a plateau at the higher pH values (see fig. 14). The straight line corresponds to the formation of the active species $[\text{R}^{III} \cdot \text{R}^{III} \cdot 2\text{(OH)}_2]^{4+}$, whereas its formation is completed in the plateau region at higher pH. As the atomic number of lanthanide ion increases, these profiles gradually shift towards the lower pH side. Accordingly, the concentration of the active species at pH 7 increases with
increasing atomic number. For example, Tm(III), Yb(III), and Lu(III) mostly exist under the form of bimetallic active species \([\text{R}^{\text{III}}_2\text{(OH)}_2]^{4+}\) at pH 7. As a result, these metal ions are quite active for RNA hydrolysis in neutral solutions. However, most of La(III) ions are in their monomeric form at pH 7, with only a small fraction incorporated into the active form \([\text{La}^{\text{III}}_2\text{(OH)}_2]^{4+}\). Thus, Tm(III), Yb(III), and Lu(III) are superior to the others, when RNA hydrolysis is carried out under physiological pH. In alkaline solutions, however, even La(III) ions mostly exist under the active form and are sufficiently active.

8. Mechanism of RNA hydrolysis by lanthanide ions (fig. 15)

The first step in RNA hydrolysis is the intramolecular nucleophilic attack of the phosphorus atom by the 2'-OH of ribose. This step is activated by the coordination of the phosphodiester linkage in RNA to the lanthanide(III) ion in the bimetallic cluster \([\text{R}^{\text{III}}_2\text{(OH)}_2]^{4+}\), since the electrons are withdrawn by the metal ion from the phosphorus atom. This electron withdrawal promotes the electrophilicity of the P atom, although it is not so drastic as the effect achieved by the Ce(IV) in DNA hydrolysis (cf. sect. 5). Furthermore, the hydroxide ion bound to another lanthanide(III) ion in the bimetallic cluster functions as a general base catalyst, and enhances the electrophilicity of the 2'-OH by removing its proton. Alternatively, the 2'-OH is directly coordinated to this metal ion, and its dissociation to alkoxide ion is facilitated. In this way, both the nucleophilic center (the oxygen in the 2'-OH) and the electrophilic center (the phosphorus atom) are simultaneously activated by the bimetallic cluster, and thus the intramolecular nucleophilic attack proceeds efficiently.

The breakdown of the resultant pentacoordinated intermediate is rate limiting, since the first step is an intramolecular reaction facilitated by a favorable activation entropy term. In this step, the metal-bound water functions as a general acid catalyst. The water bound to the lanthanide(III) ions has a \(pK_a\) in the range 8–9, which should be further decreased in the bimetallic clusters since the second trivalent ion should further withdraw electrons from the

Fig. 15. Proposed mechanism of the lanthanide(III)-induced RNA hydrolysis.
first one. This is quite appropriate for RNA hydrolysis carried out around pH 7. When the apparent activities of various acid catalysts are compared with each other at a predetermined pH, the activity increases as the $pK_a$ gets closer to the pH. Alternatively, the metal ion itself in the cluster acts as the acid catalyst. In this way, the general acid catalysis cooperates with the general base catalysis in the hydroxo clusters, resulting in the prompt hydrolysis of RNA.

9. DNA hydrolysis vs. RNA hydrolysis

Which factor makes the Ce(IV) ion best for DNA hydrolysis? Why are lanthanide(III) ions so efficient for RNA hydrolysis? The energy diagram for the uncatalyzed (alkaline) hydrolysis of DNA is virtually symmetric, and the heights of the first-barrier (for the formation of pentacoordinated intermediate) and the second-barrier (for the removal of leaving group from the reaction intermediate) are almost the same. Under these conditions, the primary requirement for an efficient catalysis is to lower the energy barrier of the first step (otherwise, the first step is rate limiting, even when the second step is accelerated). Furthermore, the catalyst can stabilize the pentacoordinated intermediate with respect to the initial state. The Ce(IV) ion satisfactorily fulfills both of these requirements, since it efficiently activates DNA by both (i) withdrawing the electrons from the phosphate in the “two-metal activation” and (ii) mixing its f-orbitals with those of the phosphate. Accordingly, the second-barrier is lowered with respect to the initial state. In contrast with these remarkable effects, the lanthanide(III) ions cannot withdraw electrons so efficiently and their activation of DNA is much poorer. Yet it is noteworthy that even lanthanide(III) ions are far better than non-lanthanide ions.

In RNA hydrolysis, however, the second energy barrier, the removal of leaving group from the pentacoordinated intermediate, is considerably higher than the first energy barrier. Here, the promotion of the second step should directly accelerate the whole reaction. The lanthanide(III) ions are suitable for promoting the second step as acid catalysts, since they have many (8 or 9) coordination water molecules with $pK_a$ values close to the pH of the reaction mixtures. These metal-coordinated water molecules exist in large amounts at pH 7 and have sufficiently large activity as acid catalyst. According to Brönsted rule for acid/base catalysis, the activity of an acid catalyst increases with decreasing $pK_a$. At the same time, these metal ions withdraw electrons and promote the first step, which of course also contributes to facilitate the RNA hydrolysis as is the case in DNA hydrolysis.

10. Strategy for site-selective scission of DNA and RNA

As described in the introduction section, one of the goals of non-enzymatic hydrolysis of DNA and RNA is to prepare artificial enzymes for site-selective scission of them. With these new tools, we can develop novel biotechnology and therapy, which are impossible with the use of naturally occurring enzymes. Now that we have lanthanide ions as efficient molecular scissors on DNA and RNA, what we should do next for the site-selective scission is to let these scissors work at the desired site in DNA and RNA substrates.
The strategies for sequence-selective scission of DNA and RNA hitherto proposed are divided into the following two categories. In the first one (“covalent strategy”), a catalyst for the hydrolysis of phosphodiester linkages in DNA and RNA is covalently attached to oligonucleotides that are complementary with DNA and RNA substrates (fig. 16(A)). Accordingly, the molecular scissors are placed near the target phosphodiester linkage, and cleave this linkage selectively. Peptide nucleic acids (PNAs), DNA-binding proteins, and other organic or inorganic molecules are also available as the sequence-recognizing moieties. If necessary, they can be chemically modified to provide the desired physicochemical and/or biological properties (e.g., increased permeability through cell membranes or strong resistance against enzymatic cleavage). There is no limitation in their molecular design.

Another methodology for sequence-selective scission is “non-covalent strategy”, in which the target phosphodiester linkage is differentiated from the other linkages in the substrate in terms of intrinsic reactivity (fig. 16(B)). For example, as described below, a single-stranded fragment in substrate DNA is much more susceptible to hydrolysis by the Ce(IV)/EDTA complex than is the double-stranded fragment. Accordingly, the target phosphodiester linkage is selectively hydrolyzed by placing this linkage in a single-stranded fragment and the other linkages in double-stranded fragments. Here we do not have to immobilize the molecular scissors on any sequence-recognizing moiety. The scission is selective because of the difference in the intrinsic reactivity, even though the scissors are freely moving around in the solutions. If the target site can be activated by some non-covalent interactions, the selectivity can be improved. The non-covalent activation of the target site in RNA by acridine-modified DNA (see sect. 13) is a typical example.

Fig. 16. Two strategies for site-selective scission of DNA and RNA: (A) “covalent strategy” in which lanthanide complexes are covalently linked near the target phosphodiester linkages by using oligonucleotides that are complementary with the DNA and RNA substrates, and (B) “non-covalent strategy”, in which the target phosphodiester linkage is activated by some non-covalent interactions and differentiated from the others in the substrate in terms of intrinsic reactivity. The black ribbons show the oligonucleotides (or their equivalents) used for the artificial enzymes to recognize the target sites.
11. Covalent strategy for site-selective scission of DNA

Artificial enzymes for site-selective scission of DNA were prepared by hybridizing the Ce(IV)/iminodiacetate complex with oligonucleotides (Fig. 17(A)) (Komiyama et al., 1994b, 1994c). First, the oligonucleotide which is complementary to the DNA substrate was prepared on an automated synthesizer, and an amino group was attached to its 5'-end by using the commercially available phosphoramidite monomer. This modified oligonucleotide was reacted with an activated urea derivative bearing a diethyl iminodiacetate. Finally, the ethyl esters in

Fig. 17. Typical artificial enzyme for sequence-selective DNA hydrolysis. (A) The procedure for its preparation and (B) the site-selective scission pattern.
the intermediate which was protecting the iminodiacetate group were hydrolyzed under alkaline conditions. By these procedures, an iminodiacetate group was bound to the 5′-end of the oligonucleotide via a urea linkage. As shown in fig. 17(B), the Ce(IV) complex in this artificial restriction enzyme selectively hydrolyzed the target phosphodiester linkage of the DNA substrate, since this linkage was placed exactly near the catalytically active species in the substrate/catalyst complex.

The length and the sequence of oligonucleotides used to recognize the target sequence in these artificial restriction enzymes are freely chosen so that the sequence-specificity of these artificial enzymes can be increased as needed. In the above example, a 19-mer oligonucleotide was used, and thus the target sequence should appear at every 419 nucleobases. This number is more than 50 fold greater than the number of total nucleobases in the DNA of human beings. In other words, even in this huge DNA, the scission site of this sequence-selective artificial restriction enzyme should come out at most only once. If necessary, site-specificity can be easily increased by increasing the length of oligonucleotide which is obtained by an automated synthesizer. The scission of DNA is totally hydrolytic, and thus the fragments obtained are successfully transformed into desired forms by using natural enzymes (e.g., alkaline phosphatase, polynucleotide kinase, and terminal deoxynucleotidyl transferase) (Sumaoka et al., 1998). The match between these artificial restriction enzymes and the current biotechnology is straightforward.

12. Non-covalent strategy for site-selective DNA scission

In contrast with “covalent strategy” described above, “non-covalent strategy” involves no covalent fixation of lanthanide complexes to oligonucleotides. The advantages of this strategy are (i) the simplicity of their preparation procedures and (ii) versatility and freedom of their molecular design. We do not need ligands which bind lanthanide ions to sequence-recognizing moiety. These factors are certainly advantageous for various applications, although notable efforts are necessary to achieve sufficiently high site-selectivity in the scission.

12.1. Gap strategy for site-selective DNA scission (Kitamura and Komiyama, 2002)

The Ce(IV)/EDTA complex preferentially hydrolyzes single-stranded DNA over double-stranded DNA (see sect. 3.2). This specificity indicates that the target site in the DNA substrate will be site-selectively hydrolyzed if this linkage is placed in single-stranded portion and the other linkages are kept in double-stranded portion. In order to satisfy this situation, gap structures were formed in the DNA substrate by using two shorter oligonucleotides which are complementary with some part of the substrate DNA. When these three components form duplexes in the reaction mixtures, most part of the DNA substrate is present in double strands but only a small designated part (the target site for the scission) is kept single-stranded.

Figure 18 shows typical gel electrophoresis patterns for the scission of gap structures by the Ce(IV)/EDTA complex. In absence of oligonucleotide additive, the whole part of the substrate DNA (50-mer) is single-stranded and thus this DNA is cut almost randomly without
Fig. 18. Gap strategy for site-selective hydrolysis of DNA by the Ce(IV)/EDTA complex. Lane 1, control; lane 2, single-stranded DNA; lane 3, completely double-stranded DNA; lane 4, part of the DNA substrate (bottom region in the gel) is kept single-stranded; lane 5, another part of the DNA substrate (top region in the gel) is kept single-stranded; lane 6, 3-base gap; lane 7, 5-base gap; lane 8, 10-base gap. The bars on the right-hand side designate the gap sites. Reaction conditions: [Ce(IV)/EDTA complex] = 500 µM, [NaCl]₀ = 100 mM, and [spermine] = 100 µM at pH 7.0 and 37 °C.
any specific base-preference (lane 2). When gap structures are formed by using two oligonucleotide additives, however, the scission is strictly restricted to the corresponding gap-site. In lane 6, the nucleotides C28, A29, and C30 in the DNA substrate are unpaired, and the others are forming Watson–Crick base pairs (the position of the gap site is shown by the bar in the right-hand side of the gel). The scission by the Ce(IV) complex selectively occurs at these three unpaired nucleotides. With the 5-base gap (lane 7) and the 10-base gap (lane 8), the scission takes place also satisfactorily within the gap (see the quantitative analysis in fig. 19). The double-stranded region is not hydrolyzed to a measurable extent. Both the site-selectivity and the scission-efficiency are not much dependent on the sequence of substrate. As expected, the scission site concurrently moves when the gap site is shifted to the desired position by using appropriate oligonucleotide additives. The selective scission can be freely modulated.

12.2. Bulge structures for site-selective DNA scission (Kitamura and Komiyama, 2002)

Site-selective DNA scission by the Ce(IV)/EDTA complex is also successful when bulge-structures are formed at the target site by using appropriate oligonucleotide additives (fig. 20). These additives are complementary with most of substrate DNA, but shorter than the substrate. Thus, some nucleotides in the substrate could not find their partners and remain single-stranded (this kind of structure is referred to as a “bulge”). As depicted in lanes 4–7, the scission selectively occurs at the bulge-site. The scission efficiency is the largest around the center of the bulge. With increasing bulge length (5-, 6-, 8-, and 10-base bulge in lanes 4, 5, 6, and 7, respectively), the scission efficiency monotonously increases. Even with large bulges, as seen in lanes 6 and 7, the scission is sufficiently restricted to the bulge-site. The addition of 3-base gap at the joint of a bulge causes no effect.

Significantly, the bulge-induced site-selective DNA hydrolysis by the Ce(IV)/EDTA complex is notably accelerated when an oligoamine (e.g., spermine, spermidine, and ethylenediamine) is added to the reaction mixture. The acceleration by these oligoamines of the gap-induced site-selective DNA hydrolysis is also evident. In both cases, double-stranded DNA is hardly hydrolyzed by the Ce(IV)/EDTA complex even in presence of oligoamines. Only the
Fig. 20. Bulge strategy for the site-selective DNA hydrolysis by the Ce(IV)/EDTA complex. Lane 1, control; lane 2, single-stranded DNA; lane 3, completely double-stranded DNA; lane 4, 5-base bulge; lane 5, 6-base bulge; lane 6, 8-base bulge; lane 7, 10-base bulge; lane 8, 10-base bulge + 3-base gap.
hydrolysis of either the gap or bulge structures is selectively promoted. These findings present various possibilities for further improvement of this strategy, and one of them is described in sect. 12.4.

12.3. Catalytic effect of a Ce(IV) hydroxide gel on the hydrolysis of gap and bulge sites

In contrast with the remarkable substrate specificity in the DNA hydrolysis by Ce(IV)/EDTA complex, heterogeneous gels of Ce(IV) hydroxide, obtained by the addition of Ce(NH₄)₂(NO₃)₆ to buffer solutions in the absence of EDTA, hardly distinguished between single-stranded DNA and double-stranded one. With these gels, both of these two types of DNA were hydrolyzed at almost the same rates. Accordingly, no site-selective scission took place, even when gap- or bulge-structures were formed in the DNA substrate. Homogeneous Ce(IV)/EDTA complex is absolutely necessary for the present site-selective reactions based on either gap-strategy or bulge-strategy.

As shown in table 2 and described in sect. 3.2, the catalysis by the Ce(IV)/EDTA complex is specific to polynucleotides which are longer than tetranucleotides. Trinucleotides are not hydrolyzed, but are subject to the catalysis when a monophosphate is attached to their termini. Apparently, the catalysis of this complex requires three or more phosphoesters in the substrate. On the basis of this remarkable substrate-specificity, it was proposed that two (or more) phosphodiester linkages of the DNA substrate are simultaneously coordinated to the Ce(IV) ion (Sumaoka et al., 2001). As a result of this coordination, the phosphodiester linkages are sufficiently activated and hydrolyzed under physiological conditions. Undoubtedly, these catalytic processes must be accompanied by notable conformational changes of the DNA backbone, since the metal ions are bound by two or more phosphoesters in the substrate. These factors are responsible for the structure-reactivity relationship in the present gap- and bulge-induced site-selective DNA scission. The gap strategy is successful only when the size of gap (the number of unpaired nucleotides) is greater than three, since otherwise the conformational change of the DNA main chain for the coordination to the Ce(IV) induces unacceptably large strain. The double-stranded portion in DNA is too rigid for this conformational change and is hardly hydrolyzed. In the bulge strategy, the bulge length must be 5-base or longer to provide the required molecular flexibility.

12.4. Covalent conjugates of oligoamine and acridine for the promotion of gap-selective DNA hydrolysis by Ce(IV)/EDTA complex (Yamamoto et al., 2003)

As described in the previous sections, both gap sites and bulge sites in substrate DNA are preferentially hydrolyzed by homogeneous Ce(IV)/EDTA complex under physiological conditions. Notable site-specificity in DNA hydrolysis has been for the first time substantiated by non-covalent strategy which involves no fixation of molecular scissors to sequence-recognition moieties. By using appropriate oligonucleotides as the additives, the sites of selective scission can be freely chosen according to our needs.

In order to further promote the scission rate and the site-selectivity, both (i) the accelerator of catalysis by the Ce(IV)/EDTA complex and (ii) a DNA-binding molecule were covalently conjugated. In typical examples, spermidine, dipropylentriamine, and propylenediamine (all
of them accelerate the catalysis of the DNA hydrolysis by the Ce(IV)/EDTA complex, see sect. 3.2) were bound to acridine, a well-known intercalator, via a hexamethylene linker. At pH 7.0, these conjugates have 4, 4, and 3 positive charges, respectively. The ring-nitrogen atoms of the acridine are mostly protonated there and all the aliphatic amino residues are also protonated. As shown on fig. 21, gap structures of desired length were formed in the DNA substrate by using appropriate oligonucleotides. In lanes 4 and 5, 10-base gap ranging from T40 to C49 is formed. In the presence of the spermidine-acridine conjugate, the gap site is preferentially hydrolyzed (lane 5). This gap-selective DNA scission assisted by the conjugate is far faster than the scission without the conjugate (lane 4). Similar synergistic cooperation between the Ce(IV)/EDTA complex and spermidine-acridine conjugate is also observed when 5-base gap or 15-base gap is formed in the DNA substrate (lanes 2–3 and lanes 6–7, respectively). The dipropylenetriamine-acridine conjugate and the propylenediamine-acridine conjugate also significantly enhance the selective DNA scission at the gap site. These conjugates themselves are entirely inactive for the DNA hydrolysis, so that the cooperation of the Ce(IV)/EDTA complex and these conjugates is evident.

Fig. 21. Autoradiographs for the hydrolysis of gaps of different lengths by the combination of spermidine-acridine conjugate and Ce(IV)/EDTA complex. Lane 1, control; lane 2, 5-base gap without the conjugate; lane 3, 5-base gap with the conjugate; lane 4, 10-base gap without the conjugate; lane 5, 10-base gap with 1; lane 6, 15-base gap without the conjugate; lane 7, 15-base gap with the conjugate; lane 8, 20-base gap without the conjugate; lane 9, 20-base gap with the conjugate; lane 10, 30-base gap without the conjugate; lane 11, 30-base gap with the conjugate.
In contrast with the remarkable acceleration by these oligoamine-acridine conjugates, quinacrine (an acridine derivative having only an alkylated exocyclic amino group) shows no measurable effect on the hydrolysis at the gap site under the conditions employed. Without being bound to acridine, spermidine, dipropylenetriamine, and propylenediamine yield only poor activity. The efficient promotion of the Ce(IV)/EDTA complex-induced hydrolysis of gap site absolutely requires the intramolecular cooperation of oligoamine and acridine. The rate of hydrolysis at the gap site monotonously increases with increasing concentration of these conjugates, whereas the hydrolysis of double-stranded portion is little affected. Accordingly, the gap-selective scission becomes much more clear-cut with increasing concentration of the conjugates.

12.5. Hybridization of covalent and non-covalent strategies for improved site-selective DNA scission (Arishima et al., 2003)

Still more efficient and clear-cut site-selective scission was achieved when Ce(IV)/EDTA complex was fixed near a gap in the DNA substrate. Oligonucleotides bearing ethylenediamine-\(N,N,N'\)-triacetate groups are remarkable cofactors for this purpose and enormously promote the selective hydrolysis at the gap site. At pH 7.0 and 37°C, a DNA substrate (41-mer) was treated with the Ce(IV)/EDTA complex in presence of various modified oligonucleotide additives. Figure 22(A) shows the typical gel electrophoresis patterns while the corresponding structures formed from the substrate and the additives are schematically depicted in fig. 22(B). In lane 2, one of the oligonucleotide additives used is complementary with the 5'-side half of the DNA substrate, and bears an ethylenediamine-\(N,N,N'\)-triacetate group on the X residue at its 5'-terminus. The other additive is complementary with the 3'-side half of the DNA substrate, and bears an ethylenediaminetriacetate group at its 3'-terminus. As the result, in the duplex formed from the DNA substrate and these two additives, two ethylenediamine-\(N,N,N'\)-triacetate groups are placed near T21 which remains unpaired at the gap site. Significantly, the gap site is efficiently hydrolyzed with high selectivity. When unmodified oligonucleotides baring no ethylenediaminetriacetate groups are used as additives under the same conditions, no DNA scission occurs. With these unmodified oligonucleotide additives, even 5-base gaps are hydrolyzed much less efficiently (lane 1). Note that the scission efficiency of the gap strategy (without ethylenediaminetriacetate groups) is evident only when the gap is 5-base or longer (see sect. 12.1). It is conclusive that the ethylenediaminetriacetate groups at the gap site enormously promote the selective DNA scission. The gap-selective scission, promoted by these modified oligonucleotides, is still more efficient when both of the additives bore two ethylenediaminetriacetate groups, and four ethylenediamine-\(N,N,N'\)-triacetate groups are placed near the gap site (lane 3). The system involving six ethylenediaminetriacetate groups at the gap site is also successful for site-selective scission (lane 4). Thus, highly selective DNA scission can be achieved at the gap site by attaching two or more ethylenediaminetriacetate groups to it. In a proposed mechanism, two (or more) Ce(IV)/EDTA complexes are bound by the ethylenediamine-\(N,N,N'\)-triacetate groups, and accumulated at the gap-site. These complexes show cooperative catalysis, resulting in unprecedentedly efficient site-selective DNA hydrolysis. Such a mechanism is reminiscent of the
Fig. 22. Combination of non-covalent and covalent strategies for improved site-selective DNA scission. Lane 1, 5-base gap without ethylenediamine-$N,N,N'$-triacetate groups; lane 2, 1-base gap with two ethylenediaminetriacetate groups at the gap site; lane 3, 1-base gap with four ethylenediaminetriacetate groups at the gap site; lane 4, 1-base gap with six ethylenediaminetriacetate groups at the gap site. The systems are schematically depicted in (B). The residue $X$ is a derivative of thymidine and bears an ethylenediamine-$N,N,N'$-triacetate group as presented in (C).
cooperation of two Ce(IV) ions in the \([\text{Ce}^{IV}_2(\text{OH})_4]^{4+}\) hydroxide cluster for DNA hydrolysis by Ce(IV) ions in absence of EDTA (see sect. 4). Consistently, the gap-selective DNA scission vanishes when all the carboxylate groups in the ethylenediaminetriacetate groups in these modified oligonucleotide additives are converted into their amides. Although ethylenediaminetriacetate is inferior to EDTA in metal-binding ability, the phosphodiester residues adjacent to the target linkage can assist the binding. The gap site is necessary for the scission, mainly because this catalytic process is accompanied by notable conformational change of the main chain of DNA, as is the case of reactions without ethylenediaminetriacetate groups.

13. Covalent strategy for site-selective RNA hydrolysis

Conjugates of an iminodiacetate and DNA oligomers were synthesized as shown in fig. 23(A) (Matsumura et al., 1994). First, the DNA oligomer, which is complementary with the RNA substrate (near the targeted scission site), was prepared on an automated synthesizer. In the final stage of the DNA synthesis, an amino linker was attached to the 5′-end of DNA by using the commercially available phosphoramidite monomer. This modified DNA was reacted with the p-nitrophenyl ester of N,N-bis(ethoxycarbonylmethyl)glycine, and then the ethyl esters were hydrolyzed under alkaline conditions. By these procedures, an iminodiacetate group was bound to the DNA via an amide linkage. The artificial enzymes were prepared in situ by mixing these conjugates with lanthanide(III) chloride in water. According to the polyacrylamide gel electrophoresis, the RNA scission occurred selectively at the target site where the lanthanide complex was fixed (fig. 23(B)). When \([\text{DNA-iminodiacetate conjugate}]_0 = [\text{Lu(III)}]_0 = 10 \mu\text{M at pH 8 and 37°C, the conversion for the sequence-selective RNA scission was around 20 mol% after 8 h. In order to achieve selective scission, the [Lu(III)]/[iminodiacetate] ratio must be 1.0 or smaller. Otherwise, the solution contains free Lu(III) ions and non-selective RNA scission by them concurrently occurs.}

In place of the iminodiacetate group, so-called “texaphyrins” (fig. 12(C)) can also be used as molecular scissors (Magda et al., 1997, 1994). In a typical example, the Dy(III)/texaphyrin complex was attached to the 5′-end of the 20-mer DNA, and an RNA substrate was treated with the conjugate of the Dy(III)-texaphyrin complex with this DNA at pH 7.5 and 37°C. The sequence-selectivity for the scission was satisfactorily high. The half-life of RNA cleavage depended on the length of the linker between the Dy(III)/texaphyrin complex and the DNA for sequence-recognition. In general, shorter linkers provided better results.²

In order to activate the target phosphodiester linkage and promote the site-selective scission, a bulge structure was intentionally formed in the DNA substrate (fig. 24) (Hall et al., 1996; Hüskens et al., 1996). The DNA oligomer, used as the sequence-recognizing moiety in these attempts, was complementary with the target RNA, but a few nucleotides in the middle were absent to form the bulge-structure. When the molecular scissors, the lanthanide complexes

² In the second-generation artificial ribonucleases, molecular scissors are incorporated into the inside of DNA strand in order to achieve efficient catalytic turnover. Upon scission, the RNA fragment used for the binding to DNA is divided into two small fragments. The stability of the RNA/DNA hetero-duplex decreases as the RNA becomes shorter, so that the small RNA fragments are spontaneously removed from the artificial enzymes.
Fig. 23. Lu(III)/DNA hybrid as sequence-selective ribonuclease: (A) synthetic procedure for DNA-iminodiacetate hybrid and (B) the RNA scission pattern.
of Shiff-base macrocycles (fig. 12(B)) were bound to this DNA oligomer, the phosphodiester linkages in the bulge were selectively hydrolyzed. The scission efficiency was increased by the bulge formation. In a proposed mechanism, the conformations of the ribose residues in the bulge-structures are more suitable for the intramolecular attack by the $2'$-OH towards the phosphorus atom.

14. **Non-covalent strategy for site-selective RNA scission**

14.1. **Novel RNA cutters**

The second generation artificial ribonucleases employ “non-covalent strategy”. Their site-selective scission ability is based on two factors: (i) the target site in the RNA substrate is activated by non-covalent interactions with the “activator”, and (ii) the RNA in RNA/DNA hetero-duplex is strongly resistant against the digestion by lanthanide ions used as the molecular scissors. In contrast with “covalent strategy”, the lanthanide ions are never bound to any sequence-recognizing moiety, and thus can develop their intrinsic activity. As expected, the scission efficiency is satisfactorily high. Typical examples are presented in fig. 25. These “non-covalent” artificial ribonucleases are composed of an RNA-activator (chemically modified oligonucleotide(s)) and a catalyst for RNA scission (Kuzuya and Komiyama, 2000; Kuzuya et al., 2002b). The type-I RNA-activator is an oligonucleotide which bears an acridine in the middle (fig. 25(A)). On the other hand, the type-II activator is the combination of an oligonucleotide bearing an acridine and an unmodified oligonucleotide (fig. 25(B)). When the RNA substrate forms a hetero-duplex with the oligonucleotide(s) in these RNA-activators, most of the ribonucleotides in the RNA form Watson–Crick base pairs with the counterpart nucleotides. However, the ribonucleotide, which is located in front of the acridine, remains unpaired. As a result, the phosphodiester linkages adjacent to this unpaired ribonucleotide are selectively activated, and preferentially hydrolyzed by lanthanide ions over the other linkages.

As shown in fig. 25(A), DNA$_{F1}$-Acr has an acridine moiety in its internal position, and is complementary with most part of the substrate RNA$_1$. In the RNA$_1$/DNA$_{F1}$-Acr hetero-duplex, only ribonucleotide U19 (in front of the acridine) is kept free from Watson–Crick base-pairing. When this system was treated with LuCl$_3$, the substrate RNA was site-selectively and efficiently hydrolyzed (lane 9 in fig. 26). The selective scission sites are the 5’-side of U-19 (major scission-site) and its 3’-side (minor scission-site), as schematically depicted in fig. 27(A). At pH 7 and 37°C, more than half of the substrate RNA was cleaved...
within 6 h. The site-selectivity is satisfactorily high throughout the whole process. In contrast, the scission is marginal when a trimethylene spacer is inserted into DNAF1 in front of U19 (DNAF1-S: lanes 2, 5, and 8). Thus the acridine residue is absolutely essential for the present site-selective scission. The activity of various lanthanide ions at pH 8 is as follows: Ho(III), Dy(III), Tb(III), Gd(III) > Lu(III), Yb(III), Tm(III), Er(III) > Sm(III), Eu(III) > La(III) > Ce(III) > Pr(III) > Nd(III).

Site-selective scission was also successful, when the type-I activator was divided into an oligonucleotide bearing an acridine and another unmodified oligonucleotide (type-II RNA-activator, fig. 27(B)). For example, DNA_L1-Acr is complementary with the 5' side portion of the substrate RNA and bears an acridine at the 5' end, while DNA_R1 is complementary with
Fig. 26. Site-selective RNA scission by combinations of the type-I activator and lanthanide(III) ion. Lane 1, La(III) only; lane 2, DNA_{F1} -S/La(III); lane 3, DNA_{F1} -Acr/La(III); lane 4, Eu(III) only; lane 5, DNA_{F1} -S/Eu(III); lane 6, DNA_{F1} -Acr/Eu(III); lane 7, Lu(III) only; lane 8, DNA_{F1} -S/La(III); lane 9, DNA_{F1} -Acr/Lu(III). At pH 8.0 and 37 °C for 2 h; [RNA\(_1\)]\(_0\) = 1, [DNA_{F1} -S]\(_0\) = [DNA_{F1} -Acr]\(_0\) = 10, [LaCl\(_3\)]\(_0\) = 100 µM; [NaCl]\(_0\) = 200 mM. R, RNA\(_1\) only; H, alkaline hydrolysis; T\(_1\), RNase T\(_1\) digestion; C, control reaction in buffer solution.

Fig. 27. Site-selective RNA scission by “non-covalent strategy”: (A) the system composed of the type-I activator (DNA_{F1} -Acr) and Lu(III) and (B) the system composed of the type II activator (DNA_{L1} -Acr + DNA_{R1}) and Lu(III).

the 3′-side portion of the RNA (see fig. 25(B)). Accordingly, only U-19 of the substrate RNA is unpaired. Upon addition of Lu(III), this RNA is selectively and efficiently hydrolyzed at the phosphodiester linkage in the 5′-side of U-19 (lane 3 in fig. 28). Another weak scission
occurs at its 3'-side. These scission sites are exactly identical with those using DNA_F1-Acr as the type-I activator. The site-selective RNA scission is also successful, when an unmodified oligonucleotide DNA_{L1} is combined with a modified oligonucleotide Acr-DNA_{R1} which bears an acridine at the 3'-end (lane 4 in fig. 28). The scission efficiency is comparable with that of the DNA_{L1}-Acr/DNA_{R1}/Lu(III) system. Thus, an acridine residue can be attached to either of the two oligonucleotides. When either one of DNA_{L1}-Acr and DNA_{R1} was used, however, the RNA was cleaved randomly throughout the region which remained single-stranded without the coverage by the oligonucleotides. No activation by the acridine occurred at either U-19 or the other sites. In order to activate RNA satisfactorily and achieve site-selective scission, both these oligonucleotides are necessary and, in addition, one of them must have an acridine residue at the end.
One of the most significant advantages of these non-covalent systems is the fact that any phosphodiester linkage in the RNA substrate is effectively hydrolyzed. There is no noticeable sequence-specificity for the scission (fig. 29). The site can be selectively cut by simple placing of an acridine in front of the target site. This is significant from the viewpoints of various applications, since ribozymes strictly choose a specific sequence (usually triad) at the scission site. In the DNA/RNA hetero-duplexes, the acridine groups are sandwiched by the two Watson–Crick base pairs (see fig. 30). The acridine is so large in size that it cannot be simultaneously accommodated together with the unpaired RNA base. As a result, the unpaired RNA base is pushed out of the hetero-duplex, perturbing the conformation of the target site in the RNA. The 2'-OH at the scissile linkage is located near the corresponding phosphorus atom, and its intramolecular nucleophilic attack is greatly promoted. Furthermore, the acridine, which is protonated under the reaction conditions, acts as an acid catalyst and promotes the removal of leaving 5'-OH of oligoribonucleotide (Kuzuya et al., 2002a, 2002b). Because of these two factors, stable phosphodiester linkages in RNA are promptly hydrolyzed.

14.2. Two-site RNA cutters for SNPs genotyping (Kuzuya et al., 2003)

By extending the non-covalent strategy described in sect. 14.1, new RNA cutters which selectively cut RNA substrates at two designated sites were prepared. With these tools, any portion of fragment can be clipped out of RNA substrates (fig. 31). These RNA cutters were prepared
by attaching two acridines to oligonucleotides (fig. 32(A)). When these modified oligonucleotides form duplexes with the sample RNA, the two phosphodiester linkages in front of the two acridines are selectively activated and cut by lanthanide ions (the scission sites are shown by arrows). The scission efficiency is hardly dependent on either the nature of the nucleobases in front of the acridines or the RNA sequence in their vicinity. Thus, any predetermined fragment of desired length can be clipped out from RNA substrates in high yields.

As an application of this method, designated RNA fragments are clipped out from human genomes and analyzed by mass spectrometry. This method is quite useful for genotyping of various kinds of single nucleotide polymorphisms (SNPs). SNPs are the alteration of one DNA base in genes of human beings. Since they are strongly related to hereditary diseases, the design of tailored drugs, and many other biomedical properties, precise and rapid detection of SNPs is one of the most attractive themes at present. In an example, RNA from the human genome was treated with the two-RNA cutter/Lu(III) system, and the products of the two-site RNA hydrolysis were analyzed by MALDI-TOF MS. As shown in fig. 32(B), three peaks were clearly observed. The signal at $m/z = 4168.8$ is assignable to the 13-mer fragment ranging from U298 to C286 (theoretical value = 4169.5). Two other peaks ($m/z = 3542.6$ and 4787.7) correspond to the 5′-side portion of the RNA substrate (G309-C299) and its 3′-side fragment.
Fig. 32. Typical MALDI-TOF MS spectrum for the products of site-selective scission of RNA from human gene by Lu(III) in presence of oligonucleotides bearing two acridines (the sequences are presented in (A)). The signal at $m/z = 4168.8$ corresponds to the 13-mer fragment (U298-C286) which involves a potent SNP site (the nucleotide at position 290). The signals at $m/z = 3542.6$ and 4787.7 are for the 5'-side fragment (G309-C299) and the 3'-side fragment (U285-A271), respectively.

Apparently, both target sites (5'-sides of U298 and U285) were efficiently activated and hydrolyzed by the Lu(III) ion. It was concluded that the nucleobases at the potent SNP site (number 290) in this sample is guanine (G). If this nucleobase is altered to adenine (A) in another patient, for example, this alternation should be directly detected by the difference (16.0) in the mass numbers of fragments. In the same way, various SNPs can be successfully analyzed. The present two-site RNA cutters have the following advantages:

1. the scission selectively takes place at two designated sites,
2. these scissions are prompt irrespective of the RNA sequence,
3. the distance between the two sites for the scission is easily controllable, and
(4) the RNA fragments, prepared by the two-site scission, are sufficiently protected from subsequent digestion. These methods should be also valuable for further developments of RNA science. For example, RNA fragments, which have complicated tertiary structures or involve modified nucleobases, are easily obtainable from naturally occurring RNAs. Preparation of chimera of two modules from two sources should be also greatly facilitated.

15. Conclusions

For a long period of time, rare-earth science did not impact much on biotechnology, and vice versa. About 10 years ago, however, it was found that lanthanide ions efficiently hydrolyze DNA and RNA under physiological conditions. These findings were critical for the developments of biotechnology and molecular biology in that we could for the first time cut DNA and RNA without using naturally-occurring enzymes. At the same time, they greatly contributed to close the gap between rare-earth science and biotechnology. The most significant point for rare-earth science is the fact that efficient catalytic effects for the hydrolysis of DNA and RNA are entirely specific to the lanthanide ions. At present, only Ce(IV) ion can hydrolyze non-supercoiled DNA with reasonable efficiency, apart from naturally occurring enzymes. The catalytic activities of lanthanide(III) ions for RNA hydrolysis are far greater than those of other non-enzymatic catalysts. Furthermore, Ce(IV) is also effective for the hydrolysis of proteins which are important biopolymers; these reactions are beyond the scope of this article so that the readers who are interested in them should refer to the published literature (Takarada et al., 2000; Komiyama and Takarada, 2003). Thus, from now on, lanthanide ions will be inevitable tools for biotechnology and molecular biology.

The remarkable activity of lanthanide ions has been further taken advantage of for site-selective scission of DNA and RNA, which is essential for the manipulation of huge DNAs of higher animals and higher plants and the relevant RNAs. Two strategies were used for the sequence-selective scission: (i) “covalent strategy” in which lanthanide complexes are covalently bound to the oligonucleotides that are complementary with the substrate DNA and RNA, and (ii) “non-covalent strategy”, in which the target phosphodiester linkage is differentiated from the others in the substrate in terms of intrinsic reactivity. In “non-covalent strategy”, lanthanide ions are not linked to any sequence-recognizing moiety. For site-selective hydrolysis of DNA, Ce(IV) ion and Ce(IV) complexes are being exclusively used, since they are the sole catalysts available for the hydrolysis of non-supercoiled DNA at present. For RNA hydrolysis, various lanthanide(III) ions and their complexes are widely used because of their far greater activities than any other catalysts. Sufficiently selective artificial enzymes have been already obtained, and further improvements of their molecular design are still being made.

Recent detailed studies have shed light on the mechanism of these exciting catalytic activities of lanthanide ions. For DNA hydrolysis, the tetracationic bimetallic hydroxo cluster \([\text{Ce}_{2}^{V}(\text{OH})_{4}]^{4+}\) is the active species. The Ce(IV) ion notably activates the phosphodiester linkages in DNA through electron-withdrawal from these linkages. The stability of the trivalent state of Ce is favorable for this electron-withdrawal. Furthermore, the orbitals of the
Ce(IV) being mixed with the orbitals of phosphate, the electrophilicity of the P atom is enhanced. As a result, the nucleophilic attack by Ce(IV)-bound hydroxide ion towards this P atom for the formation of pentacoordinated intermediate is remarkably promoted. The removal of the leaving group (5′-OH of oligonucleotide) from the intermediate is also facilitated by the acid catalysis by the bimetallic cluster. In RNA hydrolysis, $[\text{R}^{\text{III}}\text{II}_2\text{OH}_2]^{4+}$ is the active species. The breakdown of the pentacoordinated intermediate, which is rate-limiting, is accelerated by this bimetallic cluster. The activity at pH 7 increases with increasing atomic number of the lanthanide(III) ion, since the concentration of the active species increases in this order. The cooperation of two metal ions in DNA and RNA hydrolysis is reminiscent of the recent findings that binuclear and trinuclear metal complexes are far more active for the hydrolysis of phosphoesters than are the corresponding mononuclear metal complexes (Yashiro et al., 1995, 1997; Young and Chin, 1995; Molenveld et al., 1999). Lanthanide ions are suitable for the multi-metallic cooperation, since they automatically form aggregates of metal hydroxide in neutral solutions and in these aggregates two or more metal ions are placed nearby.

These mechanistic understandings on the catalysis of lanthanide ions should be valuable for the design of still more active catalysts for the hydrolysis of DNA and RNA, which should lead to the preparation of advanced artificial enzymes for site-selective scission of DNA and RNA. By accumulating more detailed knowledge on the mechanisms of the catalysts, the applications of lanthanide ions to biotechnology, molecular biology, gene therapy, and others should be further widened. Such attempts are currently being made in our laboratory and others.

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